Proximity Effect in a Disordered 2D Metal: interplay of the ZBA and the repulsion in the Cooper channel

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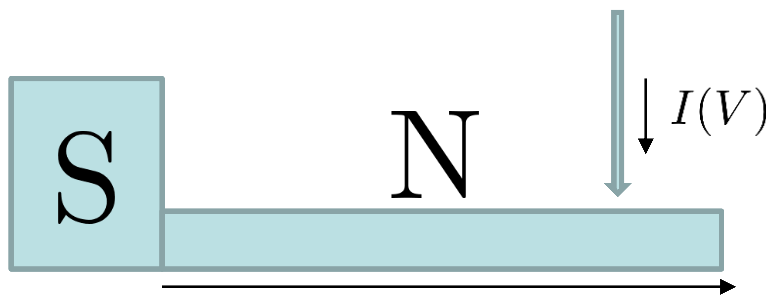
Outline

- General setup
- Proximity effect
- Zero-bias-anomaly
- Repulsion in the Cooper channel
- Bringing everything together

Setup: spectroscopy of proximity effect

Bulk superconductor in a good contact with normal film

Superconducting correlations in the film are probed by a tunneling tip



Normal film is relatively dirty: $\Delta_S^{-1} \gg \tau \gg 1/\epsilon_F$

We will be interested in interaction effects in the film as seen in the tunneling conductance:

- Superconductive pairing, induced by contact to the superconductor
- Long-range Coulomb repulsion
- Short-range repulsion in the Cooper channel

The most relevant for us regime is when $g = \frac{\hbar/e^2}{R_\Box} = \frac{4.1k\Omega}{R_\Box}$ is not too small, say $g \approx 1$.

Proximity effect

Proximity effect: SN geometry

Green function (Nambu space): $\hat{g} = \begin{pmatrix} \cos\theta & e^{i\varphi}\sin\theta \\ e^{-i\varphi}\sin\theta & -\cos\theta \end{pmatrix}$

We will consider equilibrium setup (no current), so $\phi \equiv 0$

Complex pairing angle θ is energy-and position dependent

- Normal metal, far from SC: $\theta(\epsilon) = 0$
- BCS superconductor $\theta(\epsilon = 0) = \frac{\pi}{2}$

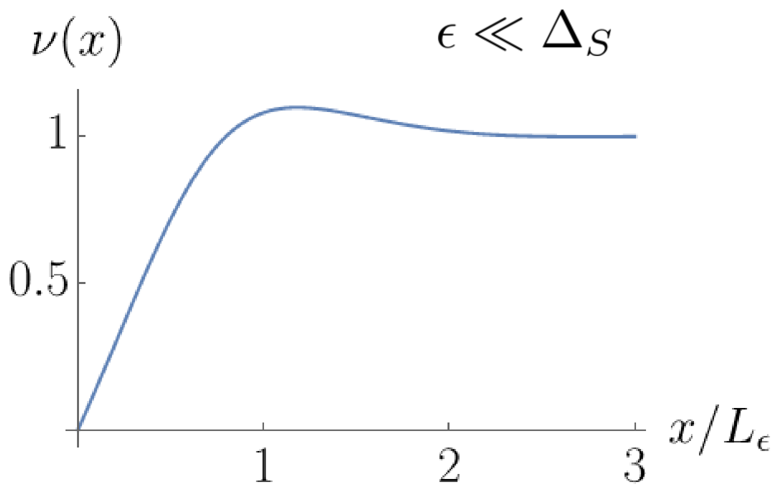
Pairing angle θ depends on energy ϵ and satisfies Usadel eq. $\frac{D}{2}\nabla^2\theta + i\epsilon\sin\theta = 0$

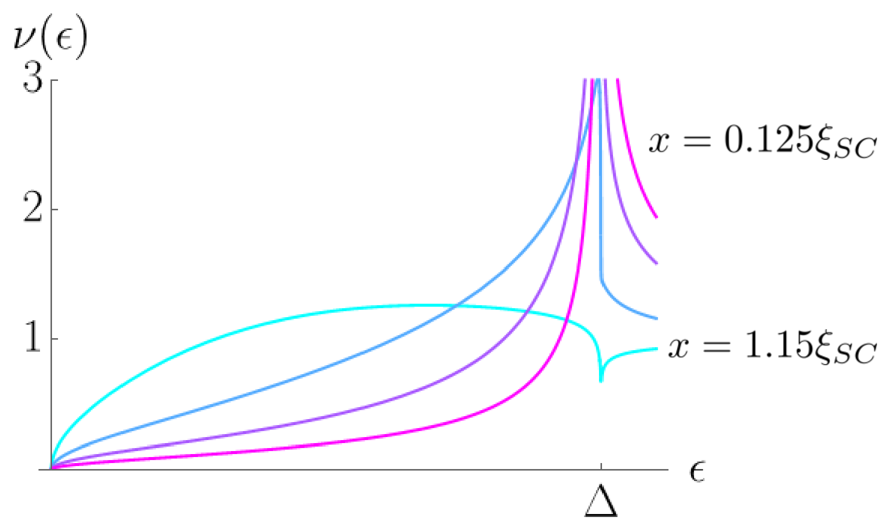
Proximity effect: SN geometry

$$\theta_{BCS}(\epsilon) = \arccos \frac{\epsilon^2}{\sqrt{\epsilon^2 - \Delta_S^2}}$$

$$\frac{D}{2}\nabla^2\theta + i\epsilon \sin\theta = 0 \qquad \nu(\epsilon, x) = \mathrm{Re}\cos\theta(\epsilon, x)$$

In the normal part of the wire: $\theta(\epsilon, x) = 4\arctan\left[e^{-x\sqrt{-2i\epsilon/D}}\tan\frac{\arccos\theta_{BCS}(\epsilon)}{4}\right]$





Experiment (1)

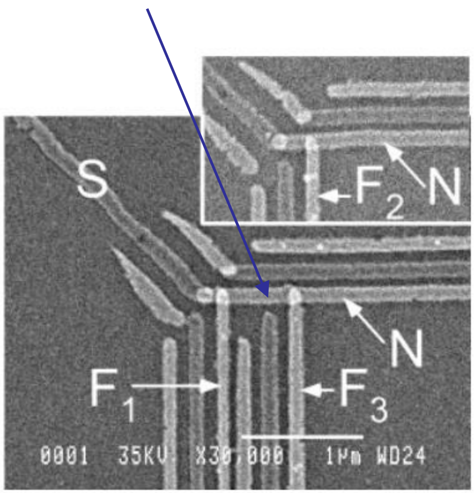
Superconducting Proximity Effect Probed on a Mesoscopic Length Scale

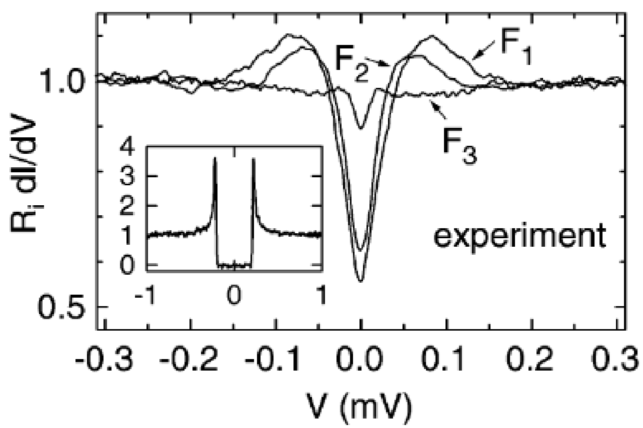
S. Guéron, H. Pothier, Norman O. Birge,* D. Esteve, and M. H. Devoret
Service de Physique de l'Etat Condensé, Commissariat à l'Energie Atomique, Saclay, F-91191 Gif-sur-Yvette Cedex, France
(Received 12 April 1996)

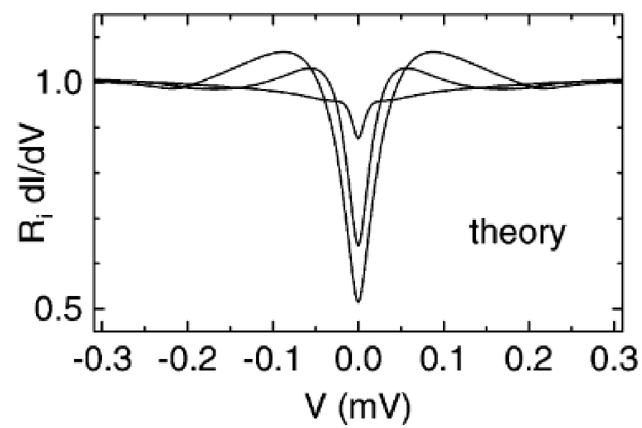
$$\frac{\hbar D}{2}\frac{\partial^2\theta}{\partial x^2} + (iE - \hbar\gamma_{\mathrm{sf}}\cos\theta)\sin\theta = 0 \qquad n(x,E) = N(0)\mathrm{Re}[\cos\theta(\mathrm{x},\mathrm{E})]$$

$$I = \frac{1}{eR_T}\int dEn(E)f(E)(f(E-V) - f(E+V))$$

Normal (copper) wires







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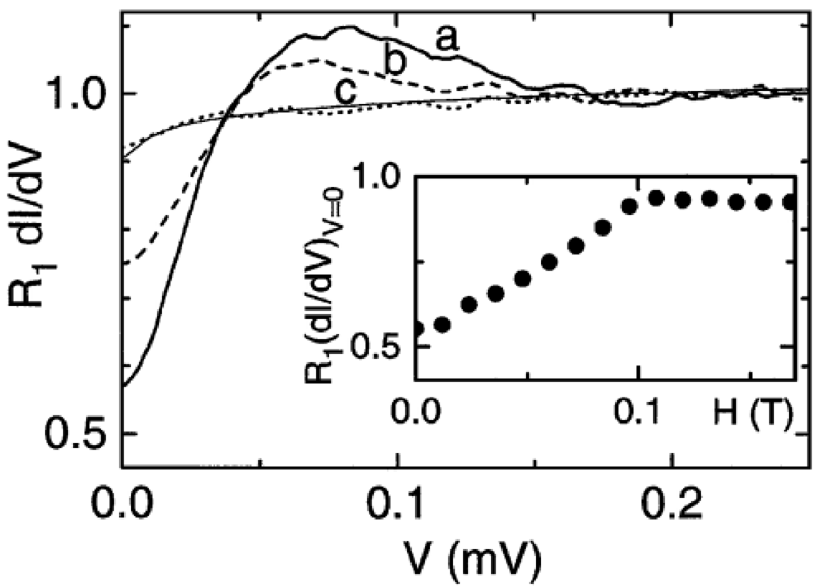
Even when proximity effect is suppressed by magnetic field, curve (c), the tunneling conductance is not trivial!



For example, at zero T:

$$\frac{dI}{dV} = \frac{1}{R_t} \int_0^{eV} n(x,E)P(eV - E)\,dE$$

$P(E)$: prob for environment to absorb ebergy E

Devoret et al 1990

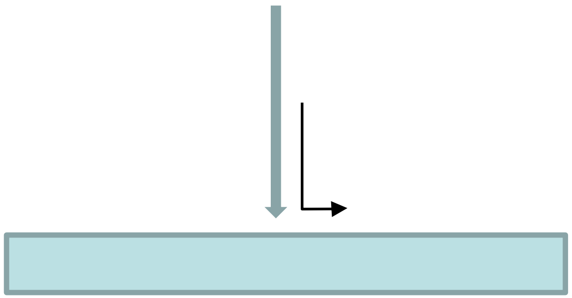
In this exp., the effect is relatively weak, $P(E)$ is fitted with a power-law

Altshuler-Aronov anomaly (ZBA)

Zero-bias anomaly: semiclassical computation

Coulomb Zero-Bias Anomaly: A Semiclassical Calculation.

L. S. Levitov[a,b] and A. V. Shytov[b]



$$\mathcal{J}(r,t) = e\delta(r)(\delta(t+\tau) - \delta(t-\tau))$$

(bounce instanton for charge tunneling over time τ)

The spreading of charge due to self-interaction is governed by:

$$\begin{cases} \dot{\rho} + \nabla \cdot \mathbf{j} = \mathcal{J}(r,t) \; ; \\ \mathbf{j} + D\nabla\rho = \hat{\sigma}(\omega, q)\mathbf{E} \; ; \\ \mathbf{E}(r,t) = -\nabla_r \int dr' \rho(r',t) U(|r-r'|) \end{cases}$$

$$\mathcal{S} = \frac{1}{2} \int\int d^4x_1 d^4x_2 \left[\underbrace{\mathbf{g}_1^T \hat{K}_{x_1-x_2} \mathbf{g}_2}_{\text{Ohmic action}} + \frac{\delta_{12}\rho_1\rho_2}{|r_1 - r_2|} \right]$$

$$S(\tau) = \frac{e^2}{8\pi^2\sigma} \ln\left(\frac{\tau}{\tau_{imp}}\right) \ln\left(\tau\tau_{imp}\sigma^2(\nu e^2)^2\right) - 2eV\tau \qquad \tau_* = \frac{e}{4\pi^2 V\sigma} \ln(\hbar\sigma\nu e/V)$$

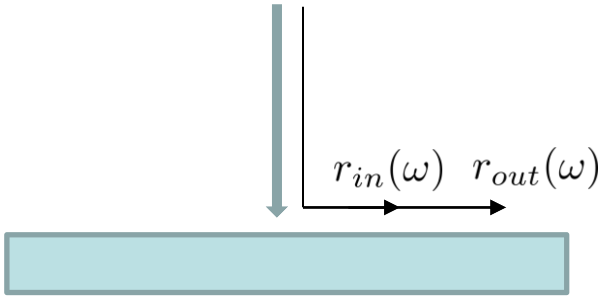
The tunnelling is supressed by a factor $\propto e^{-S(\tau_*)}$. In the perturbative regime, reduces to Altshuler-Aronov correction.

Zero-bias anomaly

Coulomb-induced suppression of tunneling DOS: $\nu(\epsilon) = \nu_0 e^{-S(V=\epsilon)}$

with $S(\epsilon) = \frac{2}{R_Q} \int_\epsilon^{1/\tau} \frac{d\omega}{\omega} R(\omega)$, $R_Q = h/e^2$ and $R(\omega)$ is spreading resistance

between diffusive length $r_{in}(\omega) = \sqrt{D/\omega}$ and EM length $r_{out}(\omega) = \sqrt{D\omega_P/\omega^2}$



In 2D: $R(\omega) = \frac{R_\square}{4\pi} \ln \frac{\omega_P}{\omega}$ leading to $S(\epsilon) = \frac{R_\square}{4\pi R_Q} \left(\ln^2 \frac{\omega_P}{\epsilon} - \ln^2 \omega_P \tau \right)$

which gives $\nu(\epsilon) \sim \epsilon^\alpha$ with weakly energy-dependent exponent

$$\alpha(\epsilon) = -\frac{\partial S}{\partial \ln \epsilon} = \frac{R_\square}{2\pi R_Q} \ln \frac{\omega_P}{\epsilon}$$

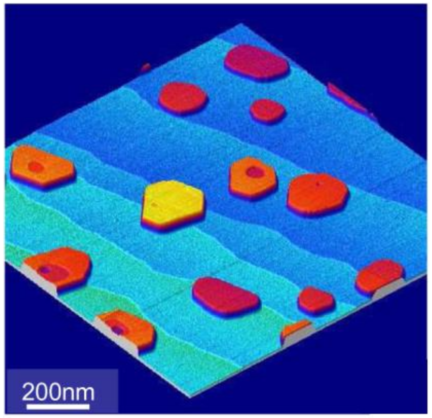
for $T \ll V$, this translates to power-law differential conductance $\frac{\partial I}{\partial V} \propto V^\alpha$

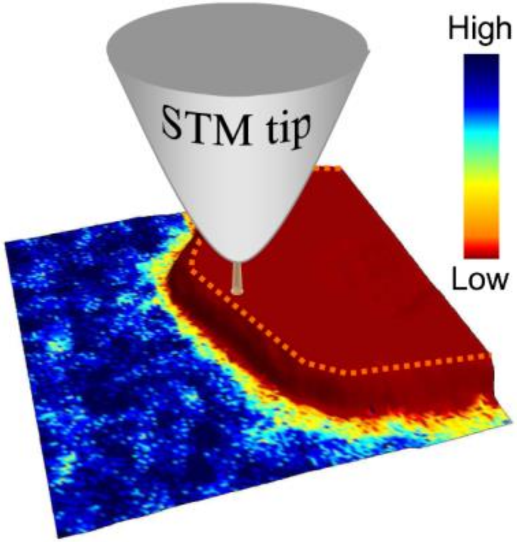
Experiment (2)

Scanning Tunneling Spectroscopy Study of the Proximity Effect in a Disordered Two-Dimensional Metal

L. Serrier-Garcia,[1] J. C. Cuevas,[2] T. Cren,[1,*] C. Brun,[1] V. Cherkez,[1] F. Debontridder,[1]
D. Fokin,[1,3] F. S. Bergeret,[4] and D. Roditchev[1]

Superconducting Pb islands on the wetting layer on normal Pb





Experiments are conducted at $T = 320mK$, when islands are superconducting but the thinner film is normal.

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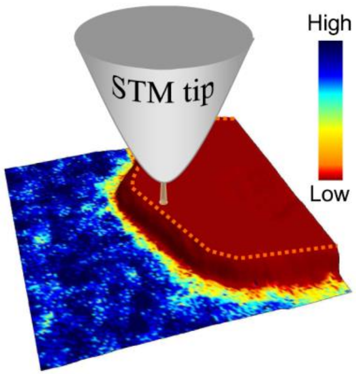

$$I = \frac{1}{eR_T}\int dE\,\nu(E) f(E)(f(E-V) - f(E+V))P(E)$$

- $P(E)$: probability for an electron to tunnel, emitting an energy E (accounts for ZBA)
- $\nu(E)$: density of states (accounts for the proximity effect)

$$P(E) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} dt\, \exp\left[J(t) + iEt/\hbar\right]$$

$$J(t) \;=\; 2\int_0^{\infty} \frac{d\omega}{\omega}\frac{\mathrm{Re}\, Z(\omega)}{R_{\mathrm{K}}}\left\{\coth\left(\frac{\hbar\omega}{2k_{\mathrm{B}}T}\right)\left[\cos(\omega t) - 1\right] - i\sin(\omega t)\right\}$$

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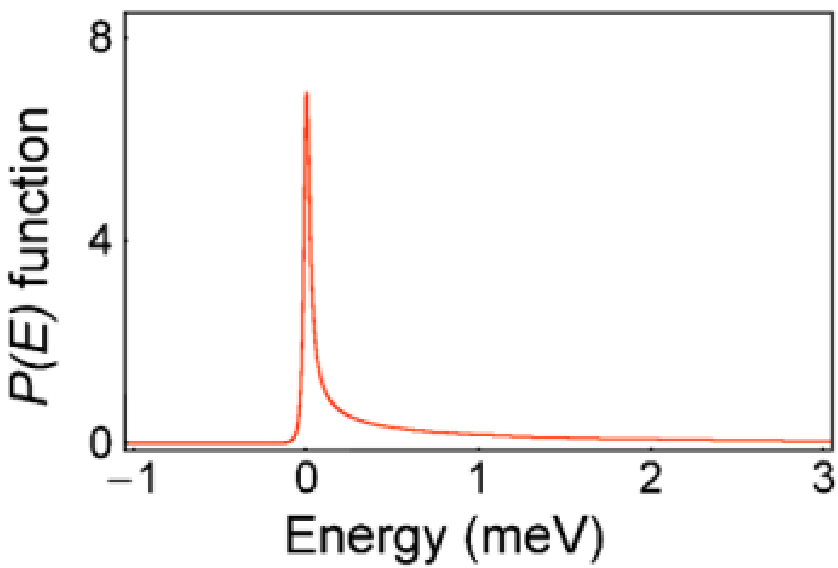

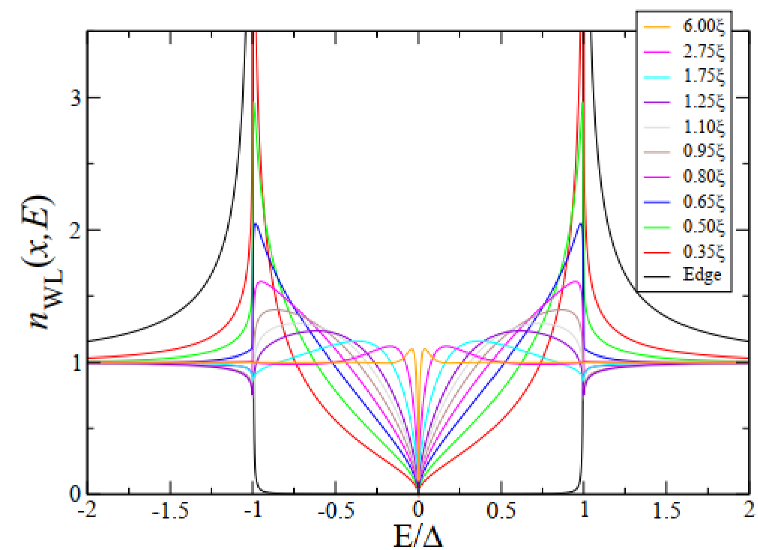

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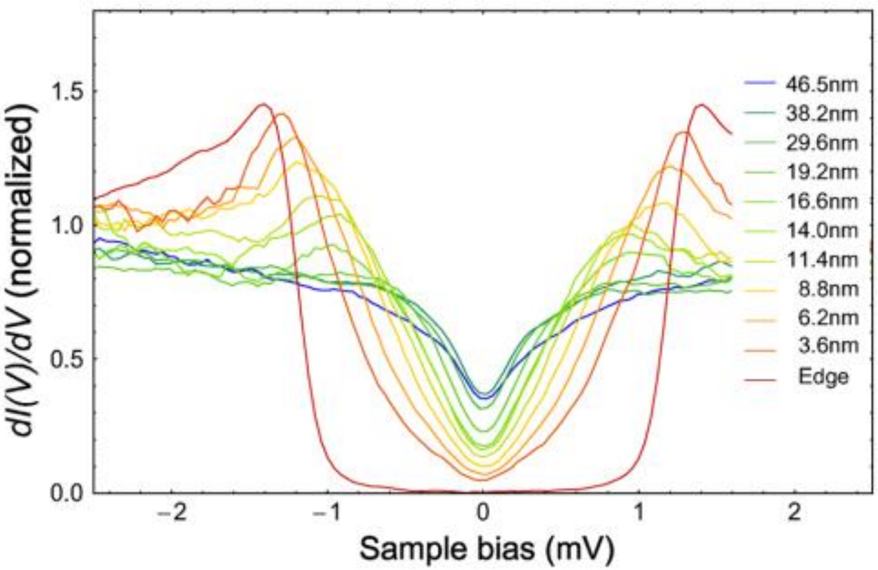
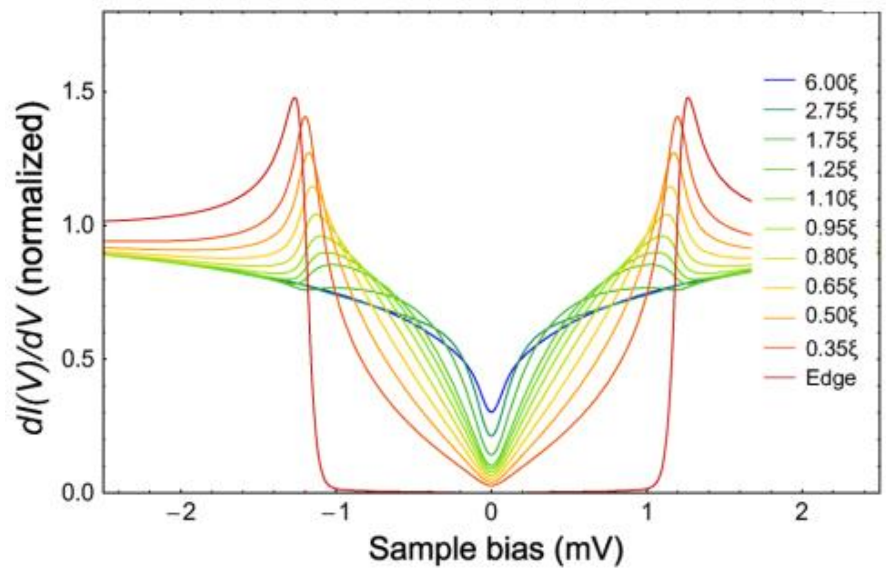
Experiment



Theory



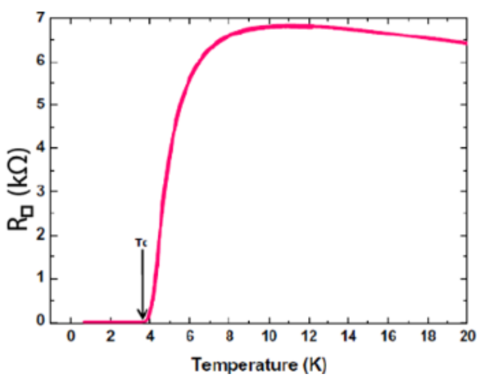
ZBA in non-uniform systems

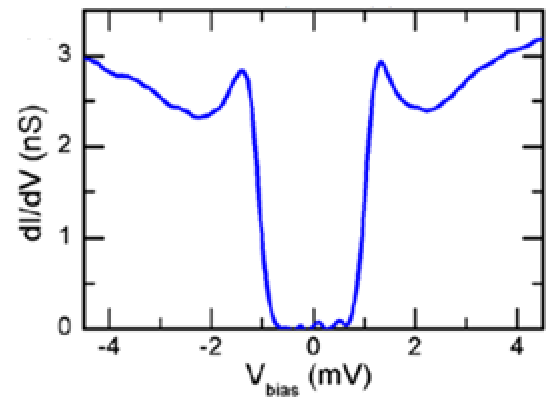
Experiment (3)

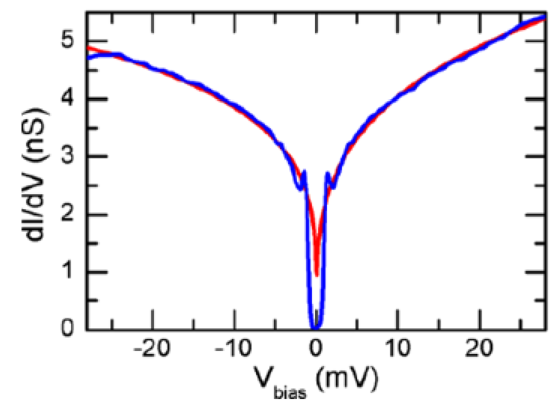
Spectroscopic evidence for strong correlations between local superconducting gap and local Altshuler-Aronov density of states suppression in ultrathin NbN films

C. Carbillet,[1] V. Cherkez,[1] M. A. Skvortsov,[2,3,*] M. V. Feigel'man,[3,2] F. Debontridder,[1] L. B. Ioffe,[4,3] V. S. Stolyarov,[1,5,6] K. Ilin,[7] M. Siegel,[7] C. Noûs,[8] D. Roditchev,[1,9] T. Cren,[1] and C. Brun[1,†]

Study of nominally homogenous, but strongly disordered NbN films







The film superconducts at $T_c \approx 3.8K$

Low-energy STS explores the local gap which fluctuates around $1.2meV$

At higher energy, STS probes Altshuler-Aronov anomaly

(fitted by a red curve, $dI/dV \propto V^\alpha$)

Experimental data: NbN films

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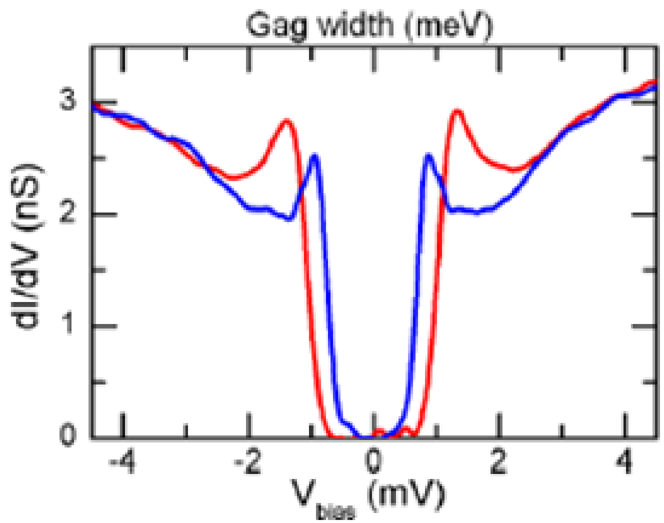
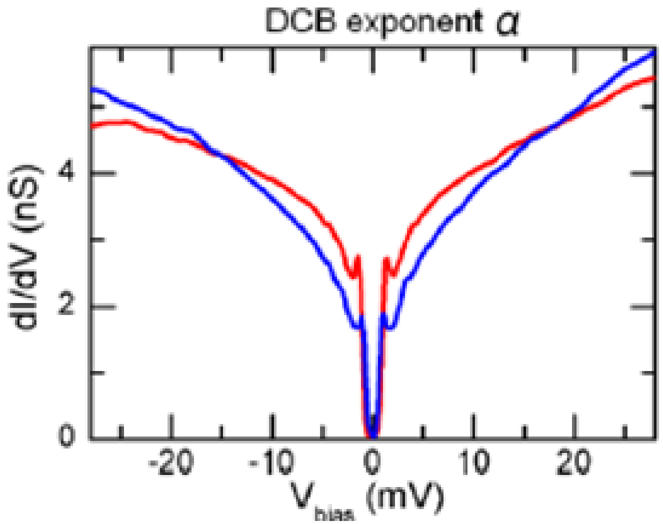
Spatial fluctuations due to fluctuating local resistivity $\rho(r) = R_\square + \delta\rho(r)$



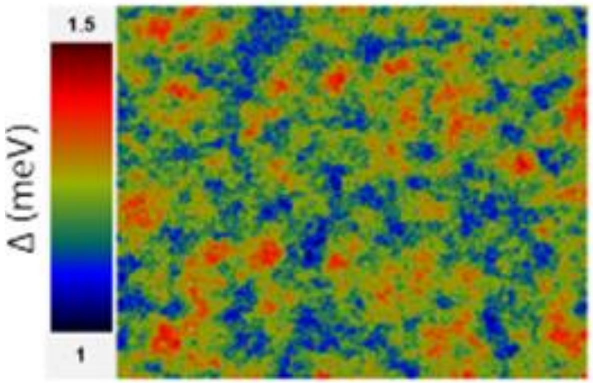


$$\delta\Delta(r) \approx -\langle\Delta\rangle \frac{\delta\rho(r)}{6\pi R_Q} \ln^3 \frac{\omega_D}{\Delta(0)}$$

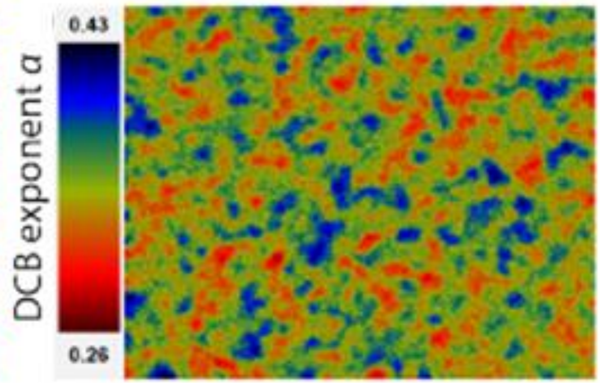
$$\delta\alpha(r) = \frac{\delta\rho(r)}{2\pi R_Q} \ln \frac{V}{D/a^2}$$

Experimental data: NbN films

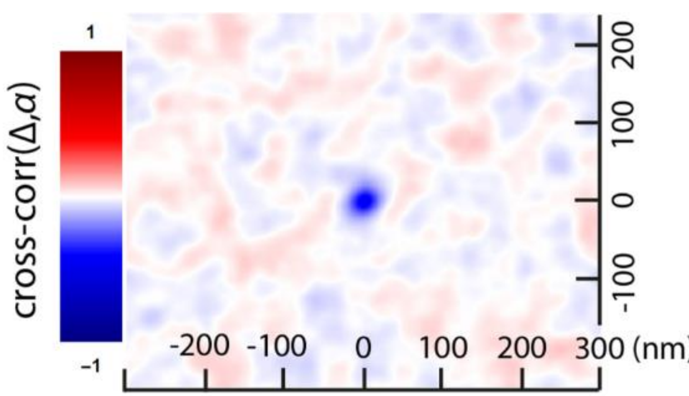
Map of the local SC gap



The of the local α



Cross-correlation

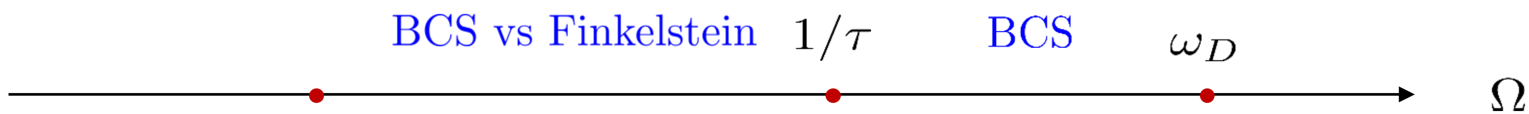


In this experiment, local ZBA is a good probe for local resistivity, relevant for gap suppression.

Repulsion in the Cooper channel

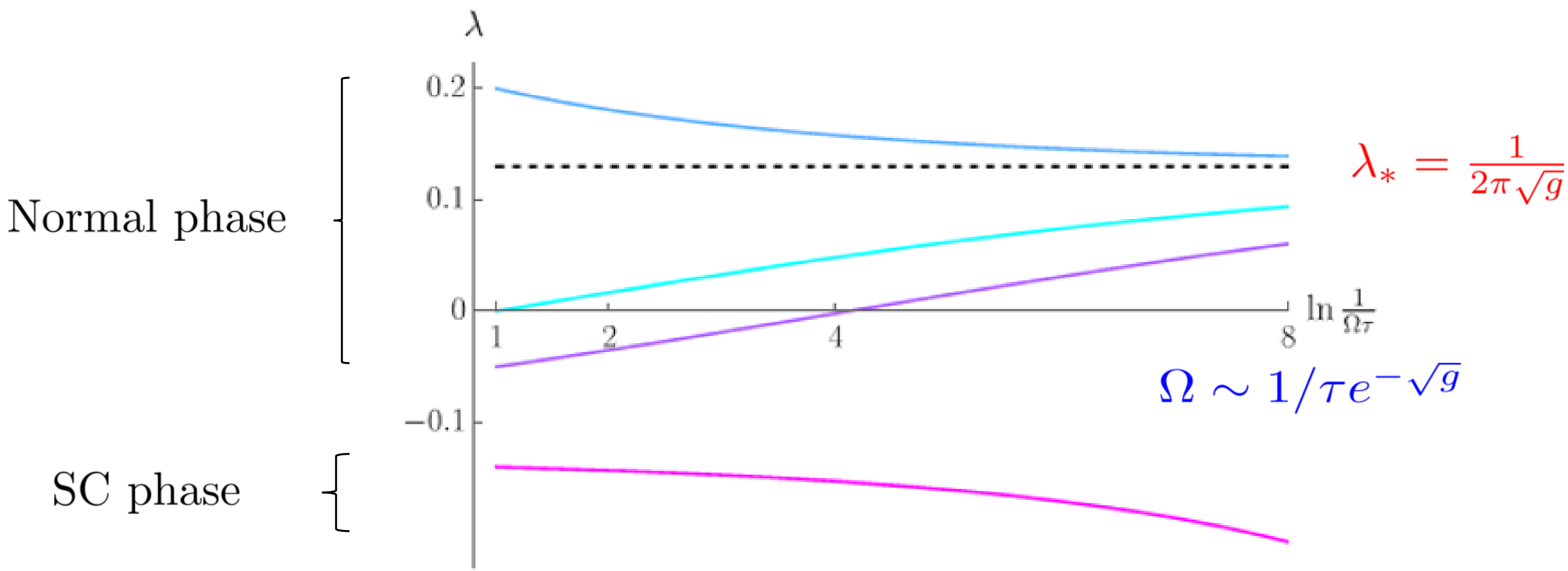
Repulsion in the Cooper channel

Renormalization of interaction in the Cooper channel $\quad \zeta = \ln \frac{1}{\Omega\tau}$



$$\frac{d\lambda}{d\zeta} = -\lambda^2 + \frac{1}{4\pi^2 g} \qquad \frac{d\lambda}{d\zeta} = -\lambda^2$$

Depending on the value $\lambda(\Omega = 1/\tau)$ one ends up normal or SC

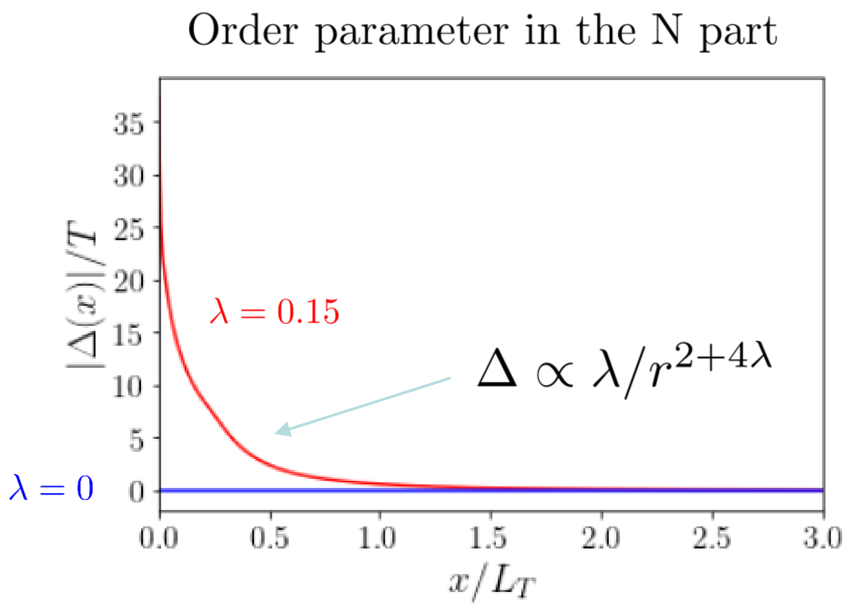


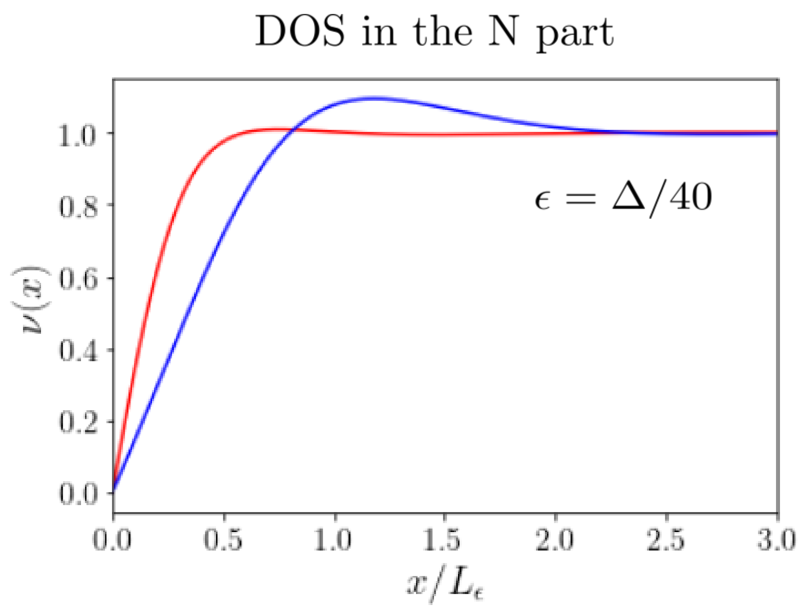
On a normal side, a natural value is $\lambda_* = \frac{1}{2\pi\sqrt{g}}$

Repulsion in the Cooper channel

With account for interaction in the N part, Usadel equation reads:

$$\frac{D}{2}\nabla^2\theta + i\epsilon\sin\theta + |\Delta|\cos\theta = 0 \qquad |\Delta(x)| = -\lambda\int_0^{1/\tau} d\epsilon\,\tanh\left(\frac{\epsilon}{2T}\right)\mathrm{Im}\left[\sin\theta(\epsilon,x)\right]$$

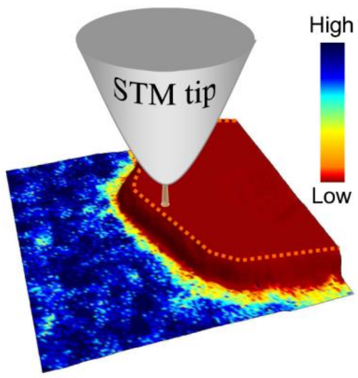


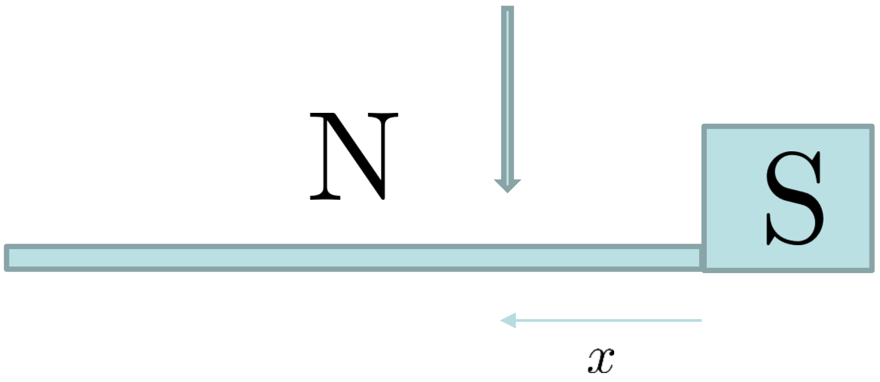


Main effect: faster decay of superconducting correlations in the N part

Bringing everything together…

Setup





1D from the point of view of superconductivity and 2D for charge spreading

Out goal is to systematically account for all effects: repulsion in the Cooper channel and position-dependent ZBA

Superconductor in its normal state is much cleaner (and thicker) than the attached 2D metallic film

$$R(\omega; x) = \frac{R_\square}{\pi} \int_{1/r_{out}(\omega)}^{1/r_{in}(\omega)} \frac{dq_x dq_y}{q_x^2 + q_y^2} \sin^2(q_x x),$$

at large x, $\sin^2 q_x x \longrightarrow 1/2$ and uniform-film result is reproduced

at $x \to 0$, spreading resistance vanishes (no ZBA)

Results

$$I(x) = \frac{1}{eR_T}\int dE\nu(E,x)f(E)(f(E-V)-f(E+V))P(E,x)$$

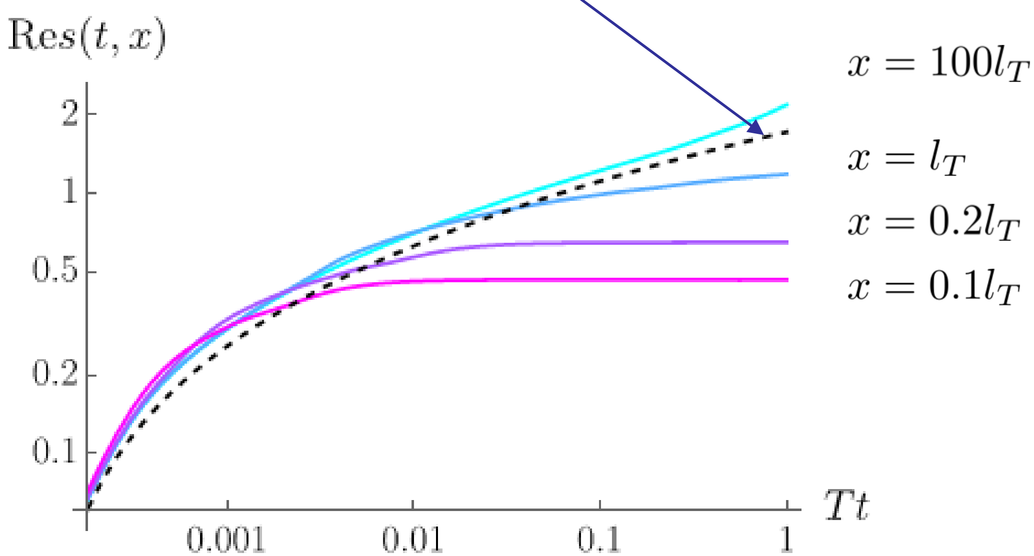
$$P(E,x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dte^{-s(t,x)+iEt}, \qquad s(x,t) = -\langle[\phi(x,t)-\phi(x,0)]\,\phi(x,0)\rangle$$

ϕ is integrated $V(t)$ over the junction

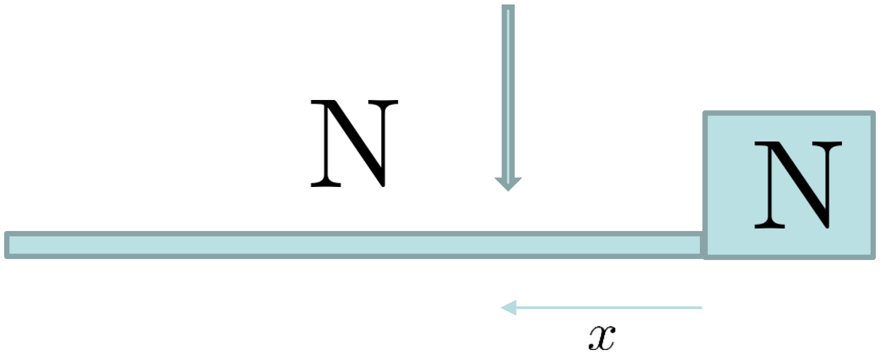
$$s(x,t) = \frac{2e^2}{2\pi\hbar}\int_0^{1/\tau_{\rm imp}} \frac{d\omega}{\omega} B(\omega,t)R(\omega;x) \qquad B(\omega,t) = \coth\left(\frac{\omega}{2T}\right)(1-\cos(\omega t)) + i\sin(\omega t)$$

Far from the boundary, $s(x\to\infty,t) = \frac{1}{8\pi^2 g}\left[\pi i\ln(t\omega_0) + \ln(t/\tau_{\rm imp})\ln(t\tau_{\rm imp}\omega_0^2)\right]$

(real-time version of the instanton action)

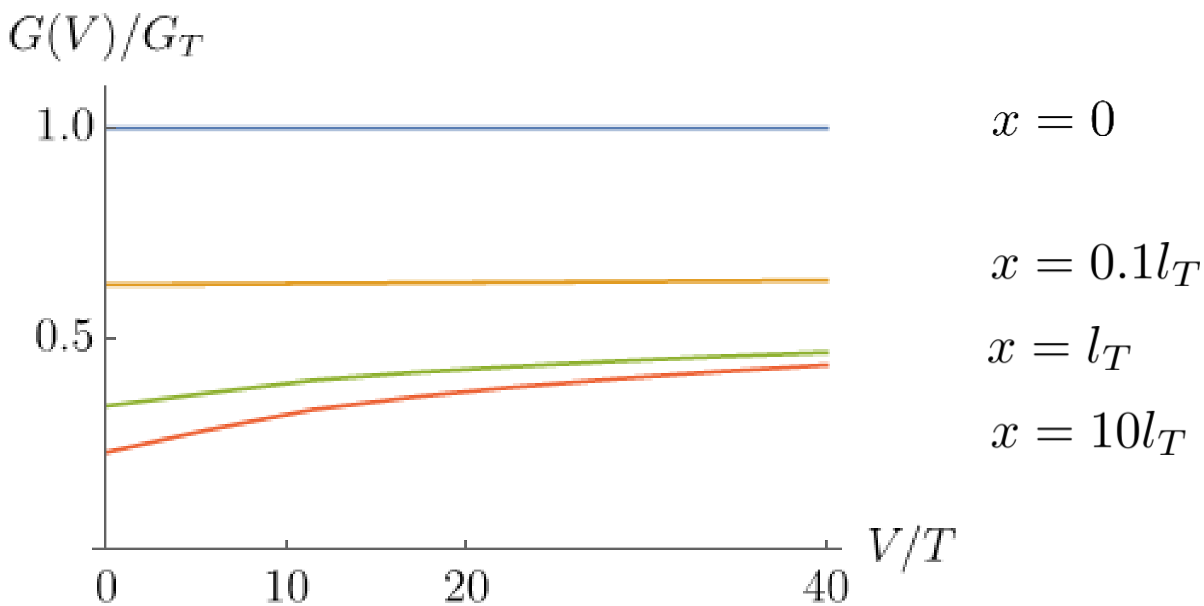


"ZBA" proximity effect



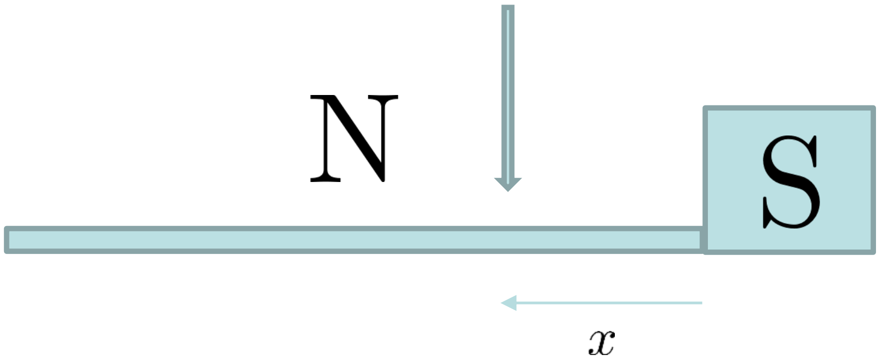
$T\tau = 10^{-4}$ $\frac{1}{\kappa L_T} = 1.5 \cdot 10^{-3}$

(κ – inverse screening length)



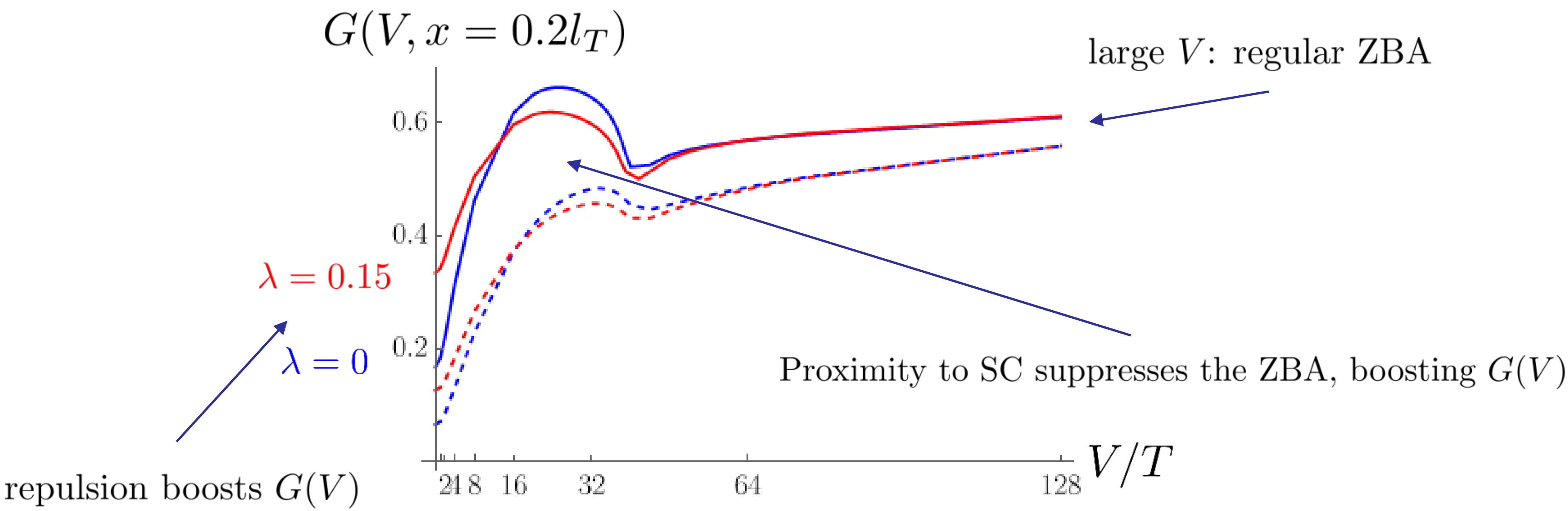
Under realistic conditions, the ZBA itself leads to quite sizeable poisiton dependence of tunneling conductance!

"ZBA" + superconducting proximity effect



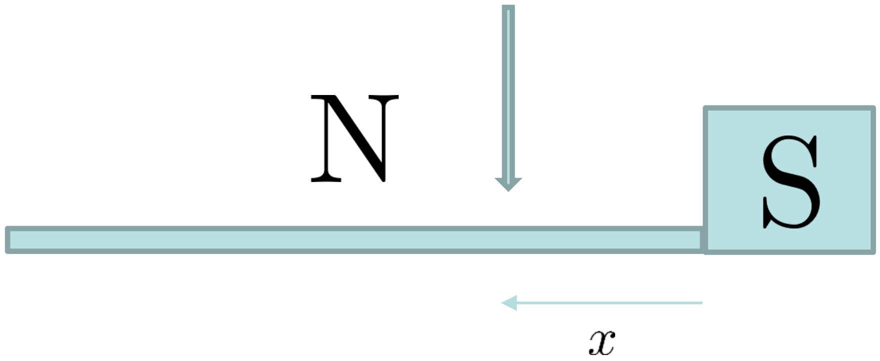
$$T\tau = 10^{-4} \quad \frac{1}{\kappa L_T} = 1.5 \cdot 10^{-3}$$

(κ – inverse screening length)



dashed lines: fictitous conductance
(in the neglect of suppression of ZBA by SC)

Non-monotonous tunneling conductance



$T\tau = 10^{-4}$ $\frac{1}{\kappa L_T} = 1.5 \cdot 10^{-3}$

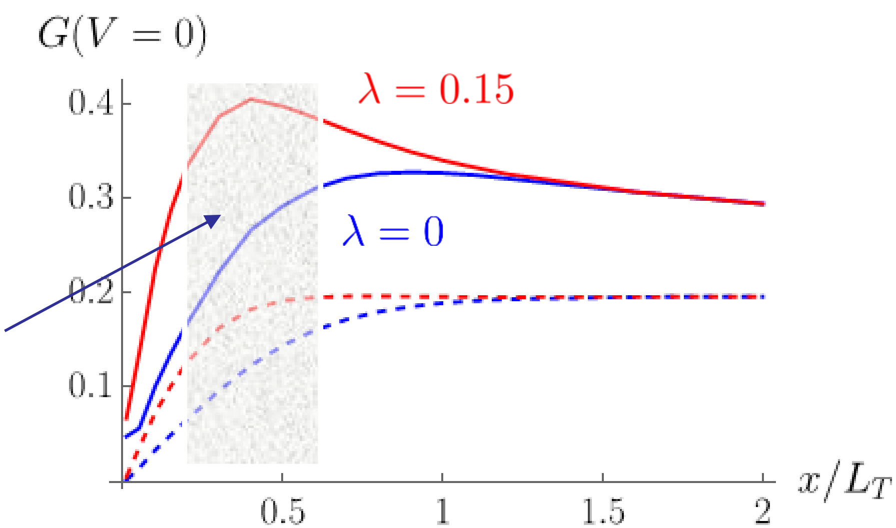
(κ – inverse screening length)

Main effect: non-monotonous $G(x)$

Interplay of repulsion and ZBA supression:

a domain where proximity is already weak

and ZBA is not active yet!



dashed lines: fictitous conductance

(in the neglect of suppression of ZBA by SC)

Summary

- In the proximity STS experiments for the dirsty 2D films $g \lesssim 2$ the effects of ZBA and Cooper repulsion are all relevant and should be taken into account together

- The bulk superconductor induces a sizeable ZBA proximity effect

- Joint effect of ZBA and superconductor proximity leads to non-monotous $G(x)$ dependence

Fitting the data of Christophe: work in progress