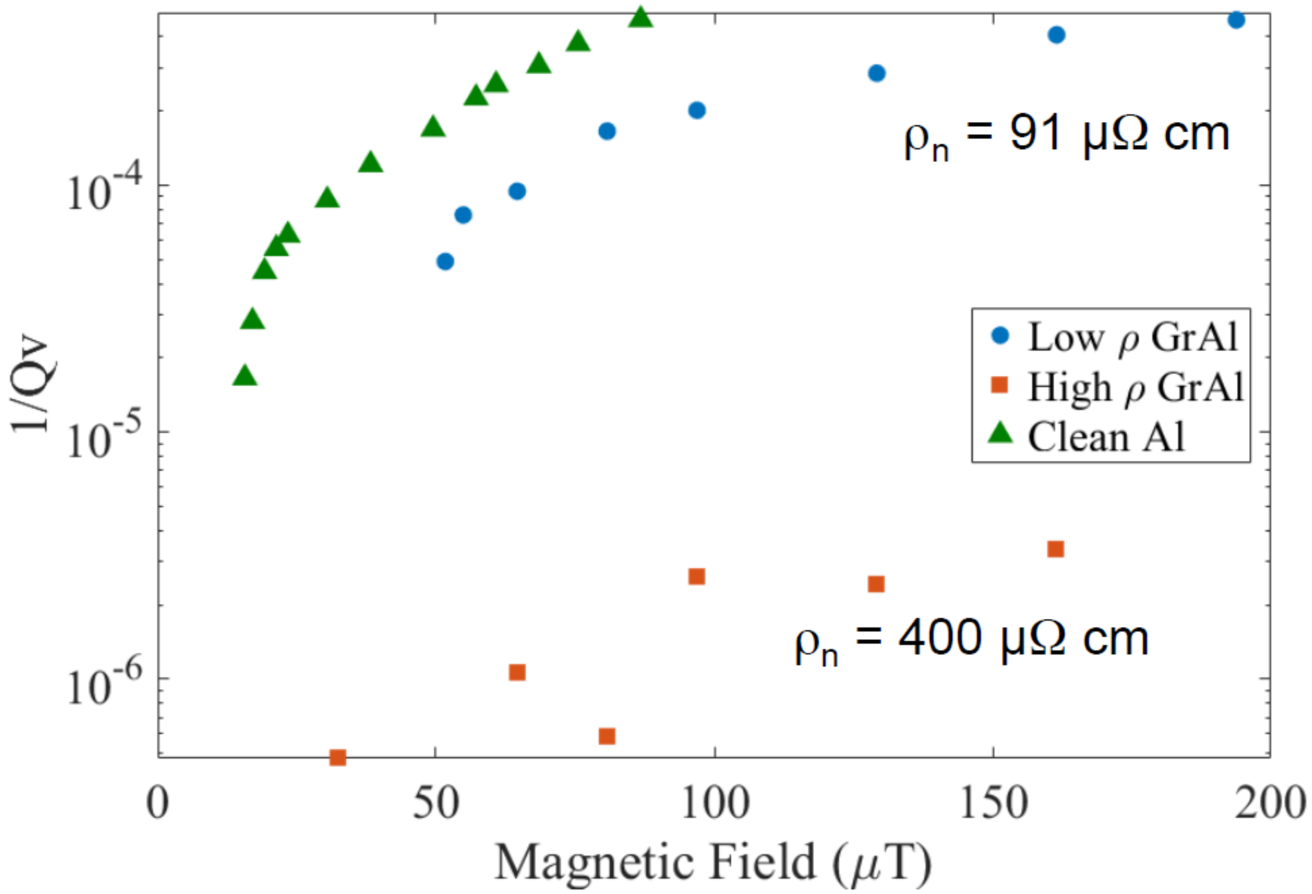
Dmitry E. Kiselov[1], Mikhail A. Skvortsov[2], Mikhail V. Feigel'man[3]

[1]Weizmann Institute of Science, Rehovot, Israel
[2]L. D. Landau Institute for Theoretical Physics, Chernogolovka, Russia
[3]Floralis & LPMMC, Universite Grenoble - Alpes, France

Why bother?

- Unexpectedly low dissipation rate for some samples at microwave frequencies near 1GHz

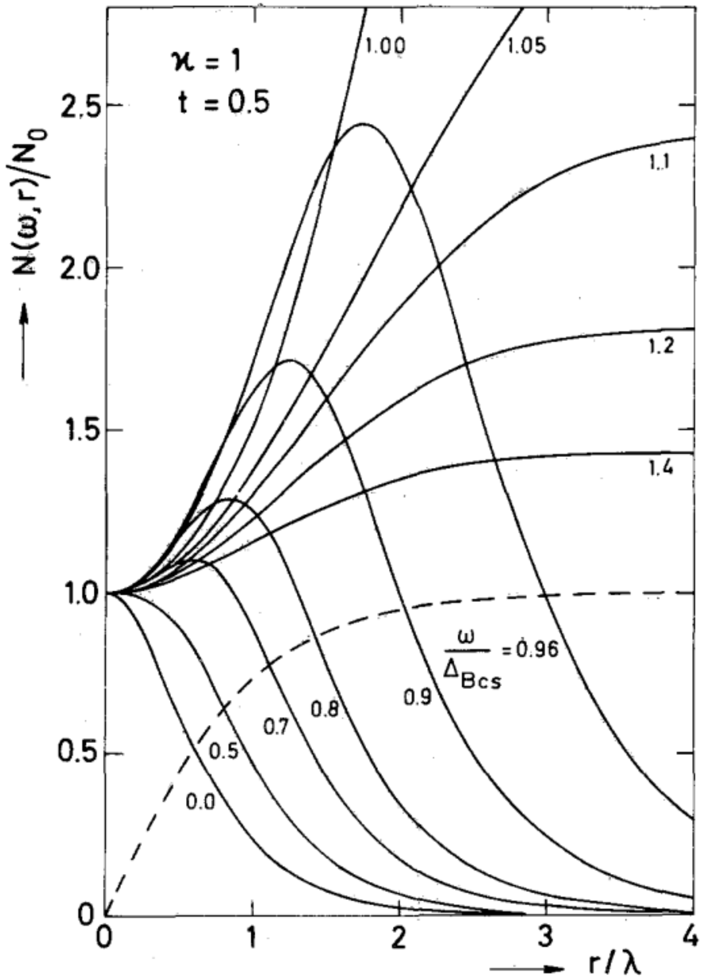
- Britton Plourde talk

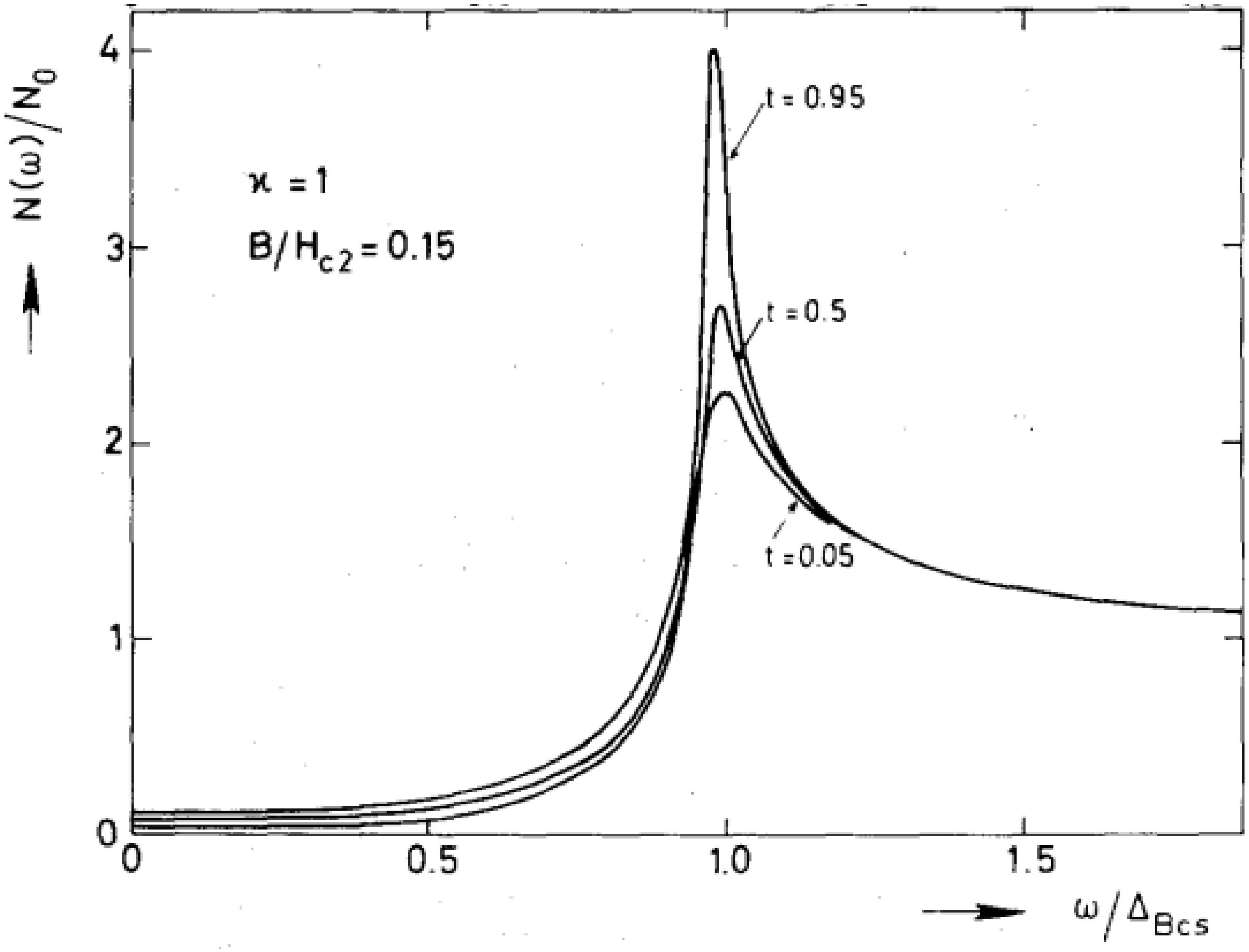


What is known?

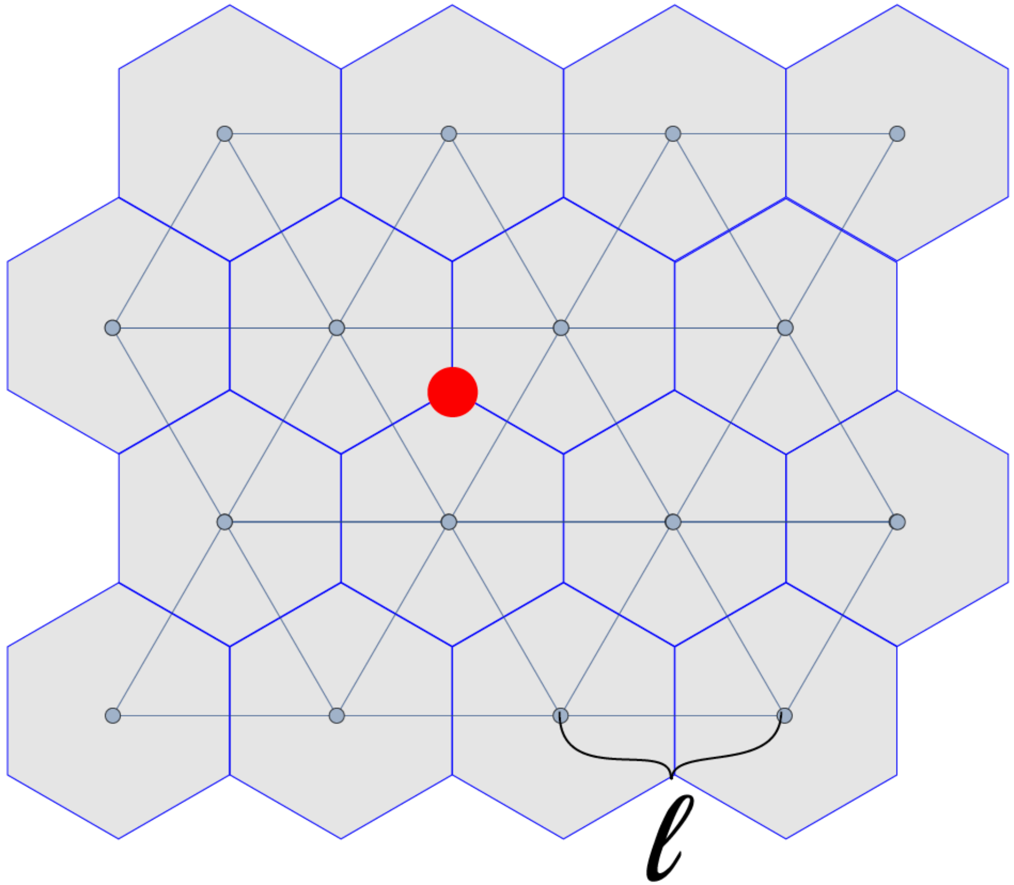
[1]Watts-Tobin, R., Kramer, L. & Pesch, W. "Density of states, entropy, and specific heat for dirty type II superconductors at arbitrary temperature."J Low Temp Phys 17, 71–86 (1974).

$$\xi^2\left[\frac{d^2\phi}{dr^2}+\frac{1}{r}\frac{d\phi}{dr}\right]=4\xi^2\frac{e^2A^2}{\hbar^2c^2}\sin\phi\cos\phi-\frac{\Delta}{\Delta_0}\cos\phi+\frac{\hbar\omega_l}{\Delta_0}\sin\phi$$





Motivation



SIS → No low-frequency dissipation as Josephson vortices are coreless and there is no sub-gup excitations

C. Beenakker 1991

S-N-S → $E_n = \Delta\sqrt{1 - \mathcal{T}_n \sin^2(\varphi/2)}$

$\max \mathcal{T}_n \sim 1$ (O.Dorokhov 1984)

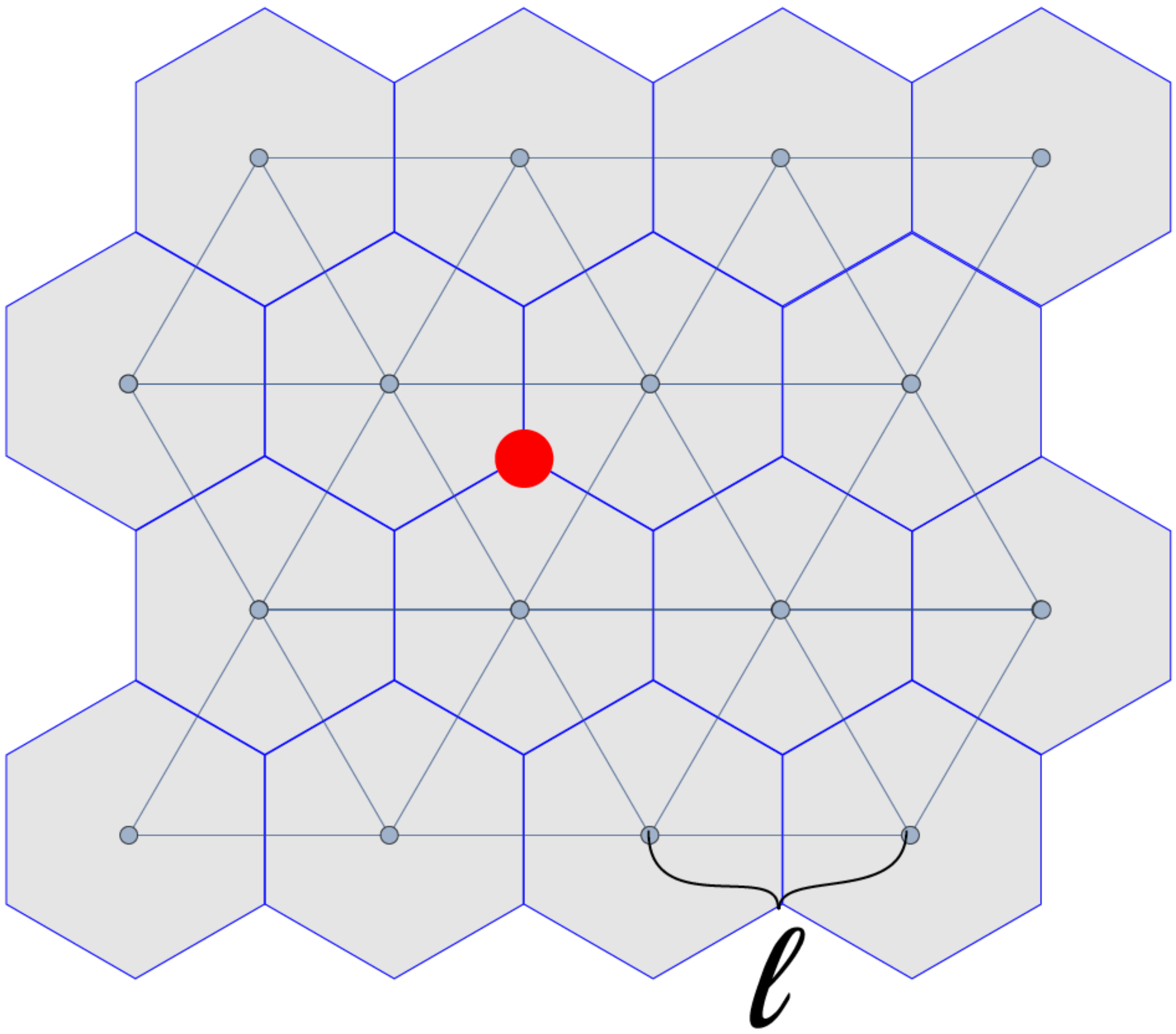
So energy gap is absent

SINS → $\max \mathcal{T}_n < 1$

(Yu. Nazarov 1994)

In this case energy gap is nonzero

The model and how to think about it?



$$\frac{\tau_{\text{tunnel}}}{\tau_{\text{spread}}} \gg 1$$

Thus zero-mode approximation is reasonable.

Each grain is characterized by single-valued Green function

$$\frac{D}{2}\frac{d^2\theta}{dx^2} + |\Delta|\cos(\chi - \varphi)\cos\theta - \omega\sin\theta - \frac{D}{2}\sin\theta\cos\theta\left(\frac{d\chi}{dx}\right)^2 = 0$$

What have been done?

$$S[Q]=\frac{\pi}{\delta}\left[-\sum_i \mathrm{Tr}\left(\varepsilon\hat{\tau}_3+\hat{\Delta}_i\right)\hat{Q}_i-\gamma\sum_{\langle ij\rangle}\mathrm{Tr}\hat{Q}_i\hat{Q}_j+\sum_i\frac{|\Delta_i|^2}{\pi\lambda T}\right]\qquad \hat{Q}_i=\begin{pmatrix}\cos\theta_i & e^{i\chi_i}\sin\theta_i\\ e^{-i\chi_i}\sin\theta_i & -\cos\theta_i\end{pmatrix}\quad (1)$$

$$\begin{cases}\sum_{j:\langle ij\rangle}\sin\theta_j\sin\left(\chi_i-\chi_j\right)=-\sin\left(\chi_i-\varphi_i\right)\frac{|\Delta_i|}{\gamma}\\ -\epsilon\sin\theta_i+\cos\theta_i\left|\Delta_i\right|\cos\left(\varphi_i-\chi_i\right)+\gamma\sum_{j:\langle ij\rangle}\left[\cos\left(\chi_j-\chi_i\right)\cos(\theta_i)\sin(\theta_j)-\sin(\theta_i)\cos(\theta_j)\right]=0\\ \left|\Delta_i\right|e^{i\phi_i}=\pi\lambda T\sum_{\epsilon=-\omega_D}^{\omega_D}e^{i\chi_i}\sin\theta_i\end{cases}\quad (2)$$

$$\varphi_i=\arctan(y_i/x_i)$$

$$\begin{cases}\left|\Delta_i\right|=2\pi\lambda T\sum_{\epsilon=0}^{\omega_D}\sin\theta_i\\ -\epsilon\sin\theta_i+\cos\theta_i\left|\Delta_i\right|+\gamma\sum_{j:\langle ij\rangle}\left[\cos\left(\chi_j-\chi_i\right)\cos(\theta_i)\sin(\theta_j)-\sin(\theta_i)\cos(\theta_j)\right]=0\end{cases}$$

$$A\to 0$$

Ansatz:

$$\Delta=\tanh\left(\frac{r}{\xi}\right)\Delta_0$$

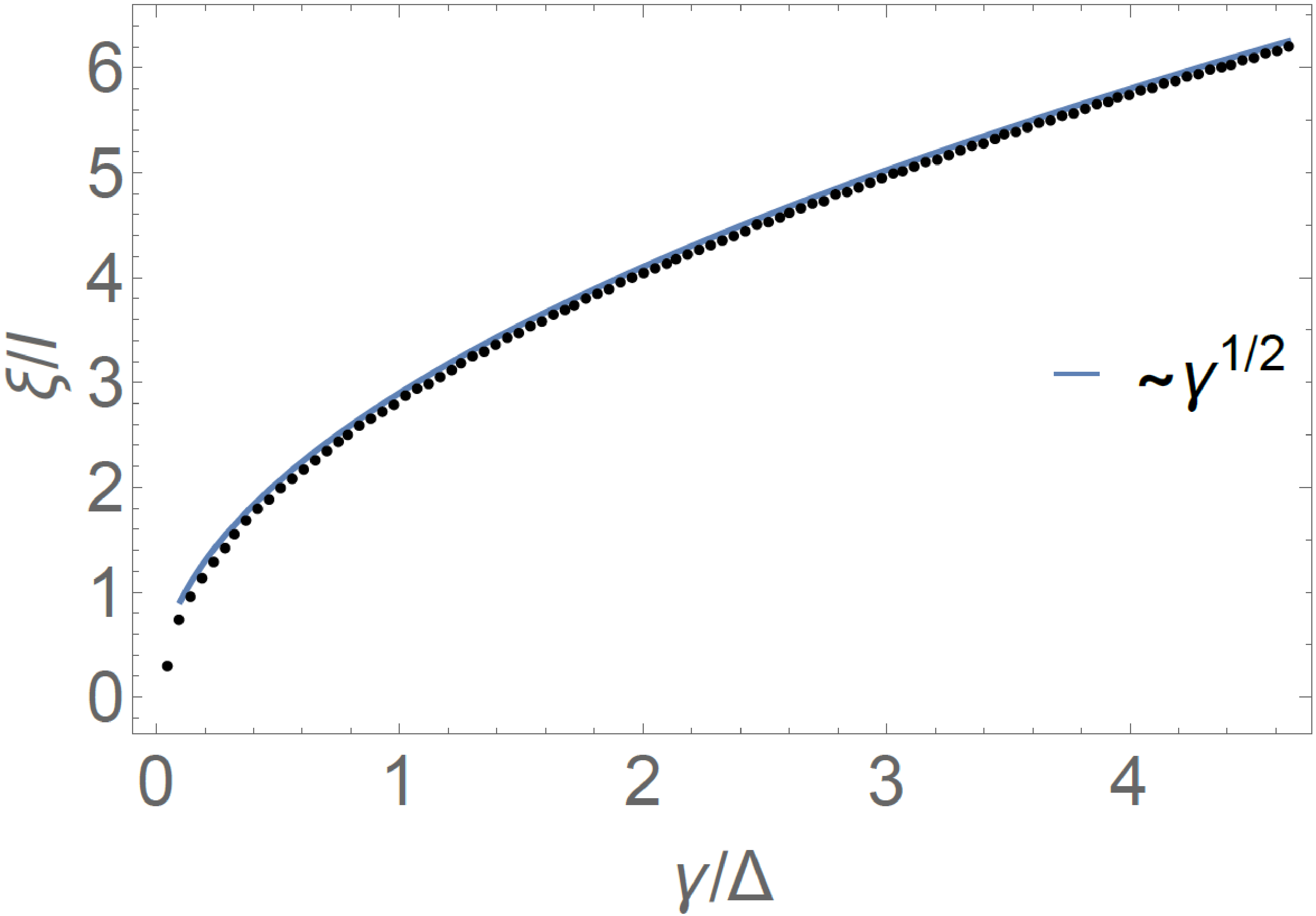
$$\theta_\epsilon=\arctan\left(\tanh\left(\frac{r}{\xi_\epsilon}\right)\frac{\Delta_0}{\epsilon}\right)$$

A.S. Osin, Ya.V. Fominov, *Superconducting phases and the second Josephson harmonic in tunnel junctions between diffusive superconductors*, Phys. Rev. B 104, 064514 (2021)

In first order in gamma

$$\varphi_i=\chi_i$$

Sanity check

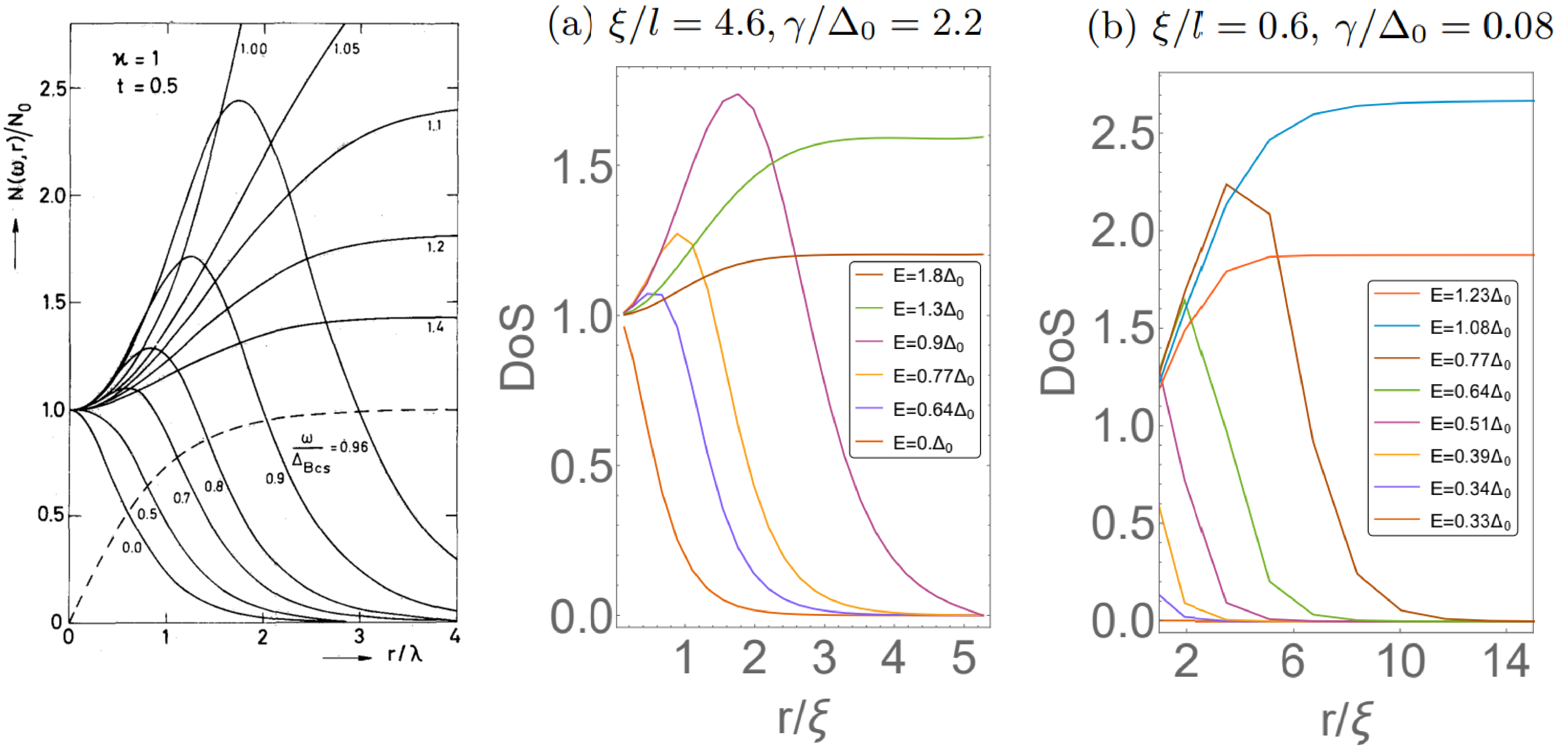


In continuous limit $l/\xi \to 0$

$$D = 3\gamma l^2$$

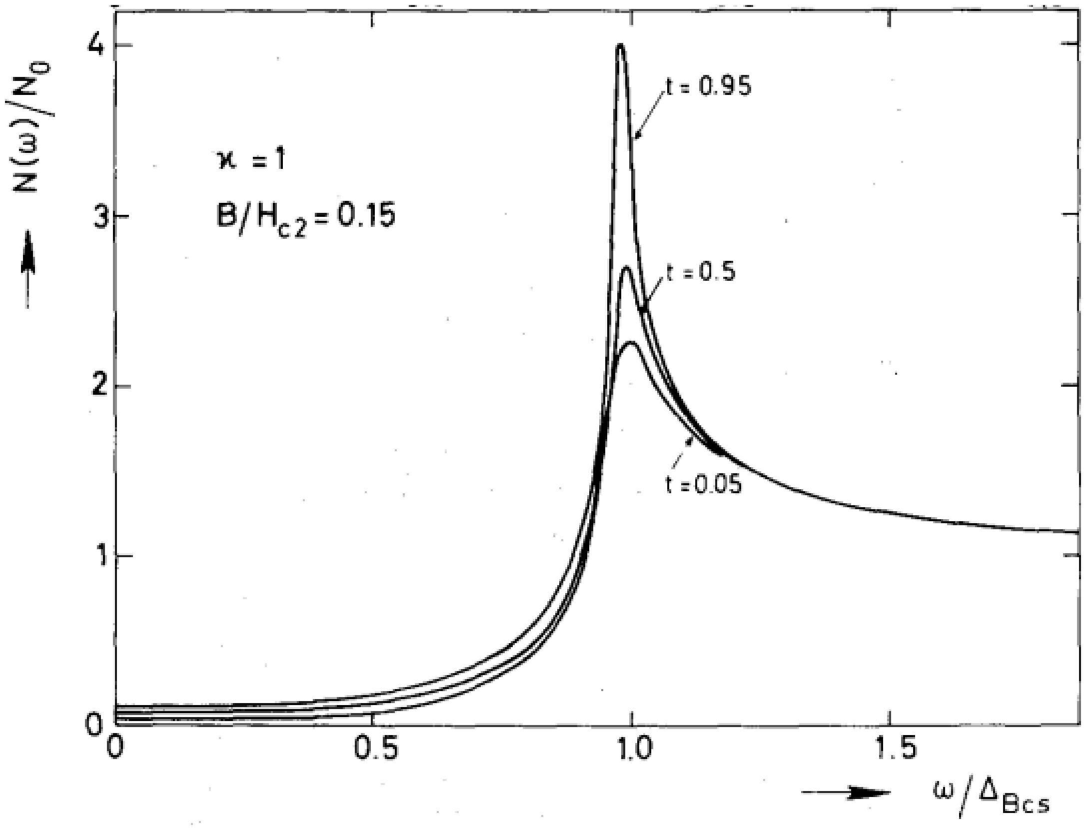
$$\xi = (D/2\Delta)^{1/2}$$

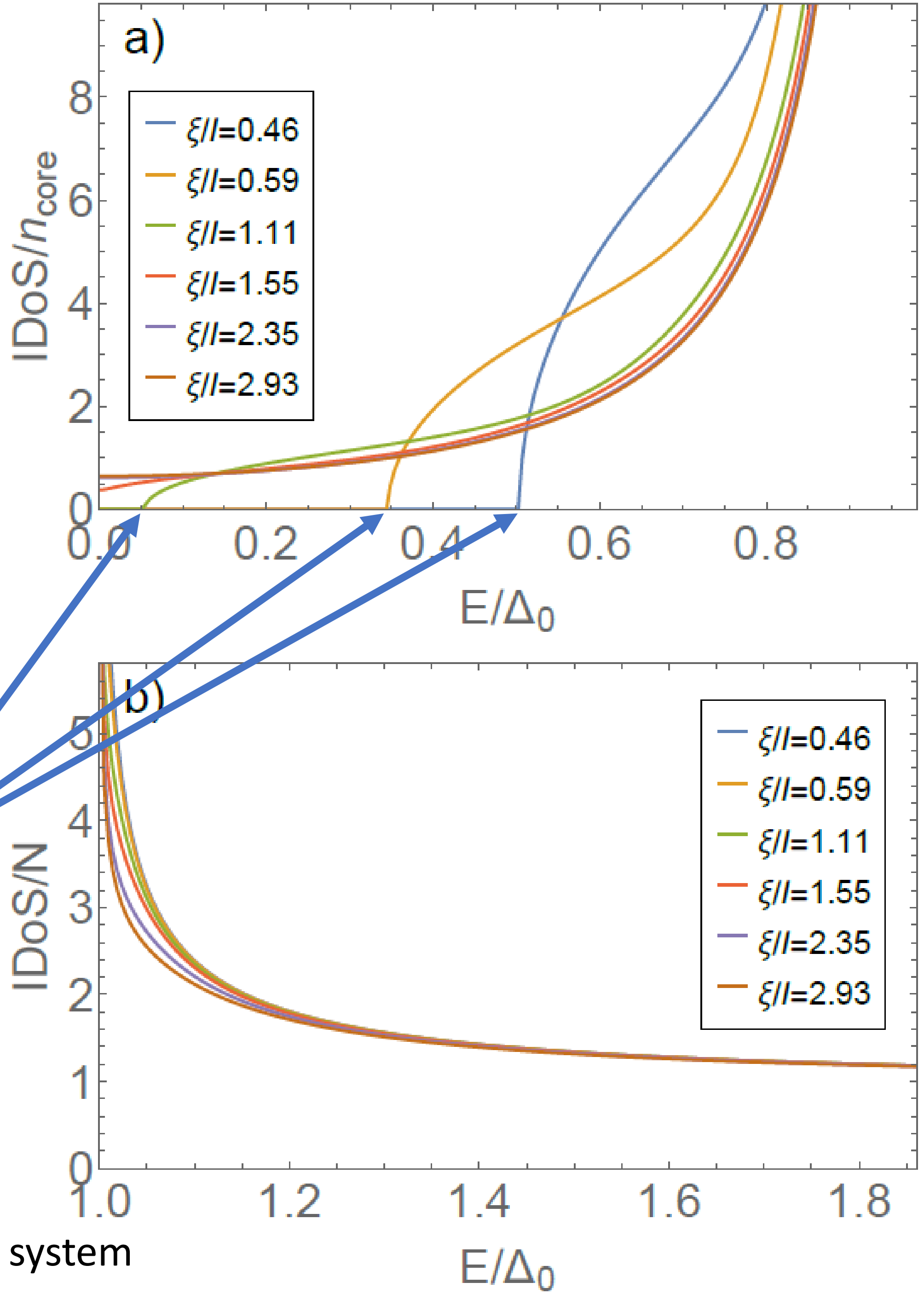
Result


κ = 1
t = 0.5
N(ω,r)/N₀
r/λ
1.00
1.05
1.1
1.2
1.4
ω/Δ_Bcs = 0.96
0.9
0.8
0.7
0.5
0.0
(a) ξ/l = 4.6, γ/Δ₀ = 2.2
DoS
r/ξ
E=1.8Δ₀
E=1.3Δ₀
E=0.9Δ₀
E=0.77Δ₀
E=0.64Δ₀
E=0.Δ₀
(b) ξ/l = 0.6, γ/Δ₀ = 0.08
DoS
r/ξ
E=1.23Δ₀
E=1.08Δ₀
E=0.77Δ₀
E=0.64Δ₀
E=0.51Δ₀
E=0.39Δ₀
E=0.34Δ₀
E=0.33Δ₀

Result

$n_{core} = \pi \xi^2 / S_{gr}$

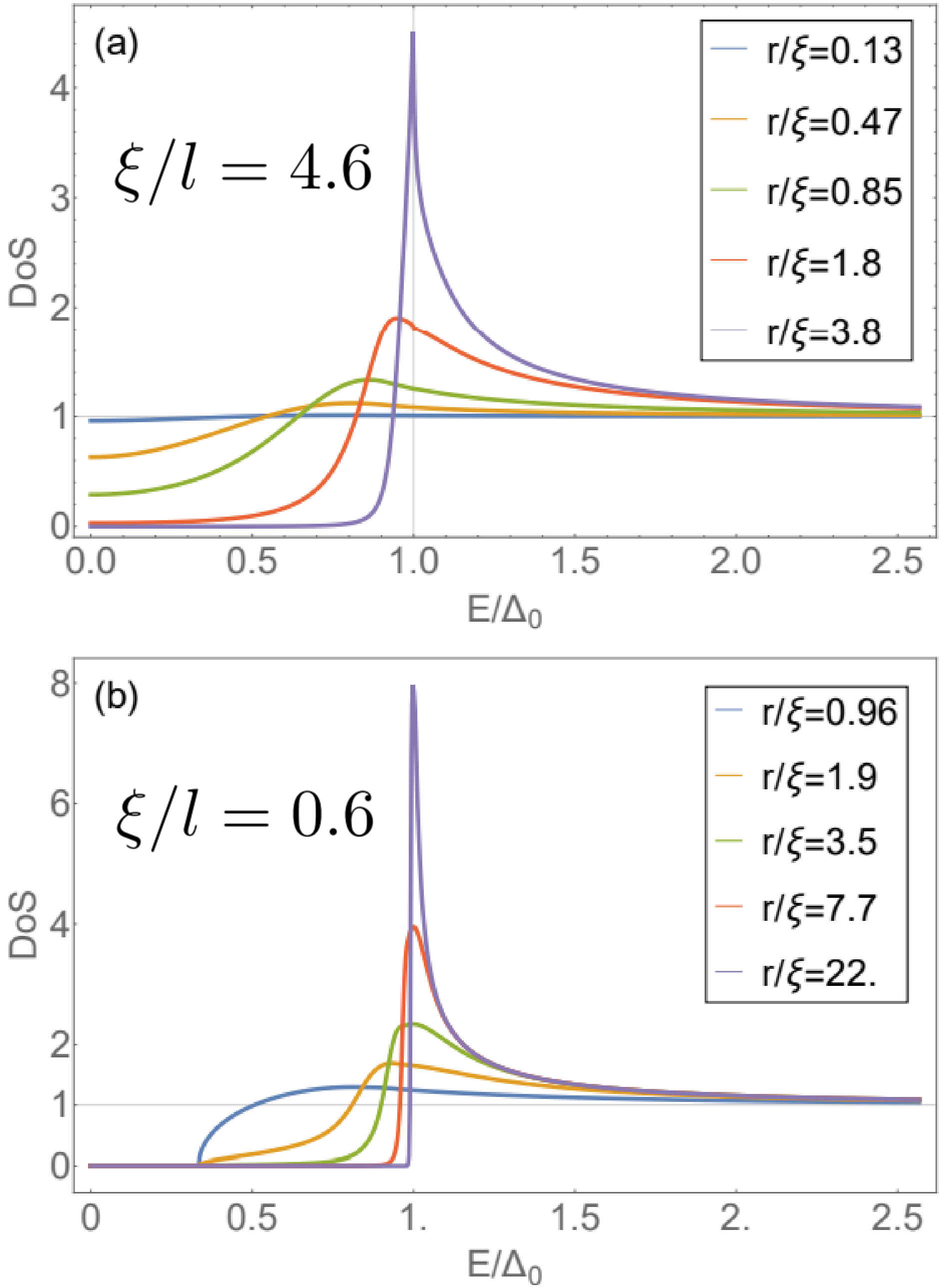




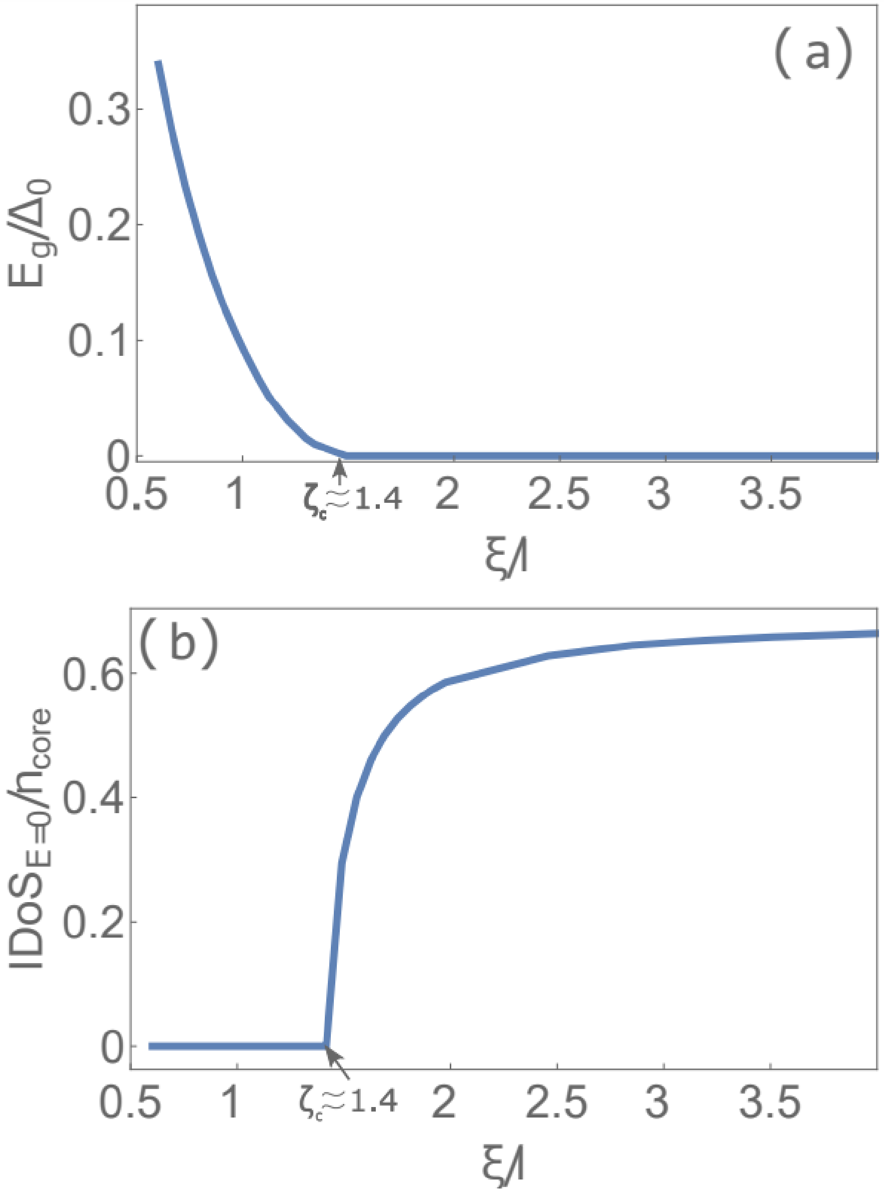
Gap exists at smallest ξ/l

N – size of the system

Space-resolved DOS



Minigap as a function of ξ



Estimations for resistivity

$$\xi/l < \zeta_c$$ → $$\frac{e^2}{\hbar}R_\square > 0.2\frac{l}{d}\frac{\delta}{\Delta_0}$$

$$\sigma = 2\nu_0 e^2 D$$

$$\xi^2 = \hbar D/2\Delta_0$$

$$\delta = 1/(2\nu_0 S_{\rm gr} l)$$

$$\rho = R_\square \cdot d.$$

$$D = 3\gamma l^2$$

$$0.2\frac{\delta}{\Delta_0}\frac{l}{d} < \frac{e^2 R_\square}{\hbar} < \frac{l}{d}.$$

$$\delta < 6\gamma$$ → $$\frac{e^2 R_\square}{\hbar} < \frac{l}{d},$$

Conclusions

- A minigap in the spectrum of single-particle states near the core of a vortex in granular superconducting system was found
- Such a situation occurs only in a specific window of tunneling parameter. Estimations were made in terms of normal resistivity per square.
- This provides qualitative explanation for surprisingly low dissipation rate in such a system observed in recent experiments.