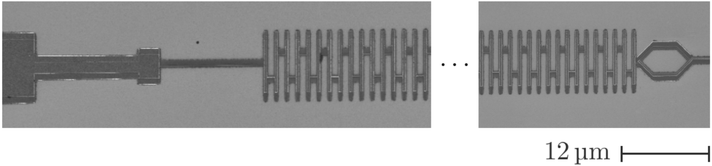
Many-body impurity physics in designed Josephson circuits

Serge Florens [Néel Institute - Grenoble]



Acknowledgments

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Work based mainly on [Léger *et al.*, *"Revealing the finite-frequency response of a bosonic quantum impurity"*, SciPost (2023)]

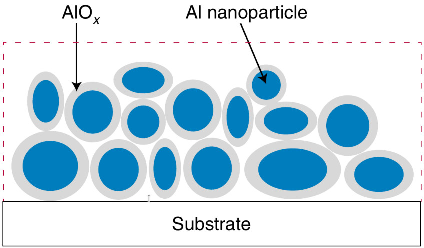
Plan of the talk

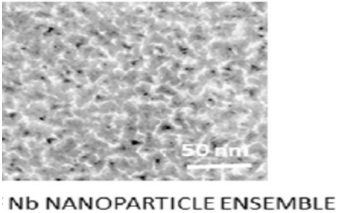
- Motivations (according to taste)
- Building circuits for quantum impurity models
- Extracting renormalized parameters from spectroscopy
- Analysis of many-body dissipation
- Theoretical modeling

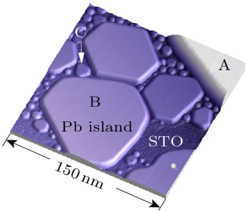
Motivation for Josephson array aficionados

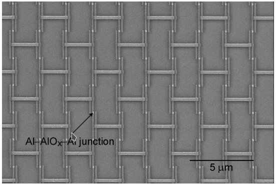
Variety of Josephson junction arrays

Diversity of geometrical structures and spatial scales:









Solving the standard model?

Josephson junction array Hamiltonian:

$$H = 2e^2 \sum_{i,j} [\hat{n}_i - n_i^g(t)] \left[\hat{C}^{-1}\right]_{ij} [\hat{n}_j - n_j^g(t)] - \sum_{ij} E_{ij}^J \cos(\hat{\varphi}_i - \hat{\varphi}_j)$$

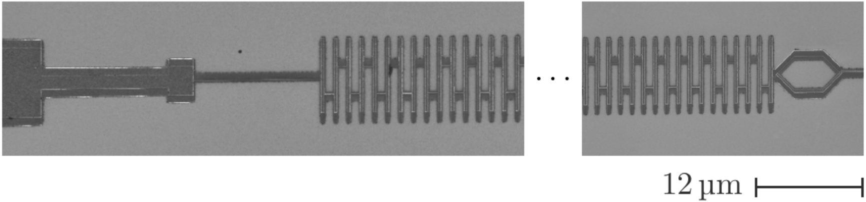
with $\hat{\varphi}_i$ and $\hat{n}_i$ the phase and charge of island i

It is a really complicated problem:

- Bulk non-linearity: $\cos(\hat{\varphi}_i - \hat{\varphi}_j)$
- Disordered couplings $\left[\hat{C}^{-1}\right]_{ij}$ and E_{ij}^J
- Charge noise $n_i^g(t)$, possibly time-dependent
- DC measurement: $I(V)$ has complex non-equilibrium dynamics
- Quantum effects: $[\hat{n}_i, \hat{\varphi}_j] = \mathrm{i}\delta_{i,j} \Rightarrow$ many-body problem

Making life simpler: single Josephson quantum "impurity"

Device: chain of identically large junctions ended by a SQUID

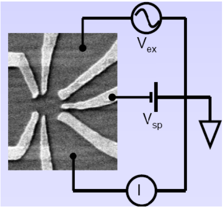


Life becomes easier:

- Bulk linearity: $1 - \cos(\hat{\varphi}_i - \hat{\varphi}_{i+1}) \simeq (\hat{\varphi}_i - \hat{\varphi}_{i+1})^2/2$
- Couplings $\left[\hat{C}^{-1}\right]_{ij}$ and $E^J_{i,i+1}$ clean and well characterized
- Charge noise: no island in this design
- AC measurement in linear response: spectroscopy
- Quantum effects: driven by a single and tunable non-linearity

Motivation for quantum dots practitioners

Fermionic vs bosonic quantum impurities

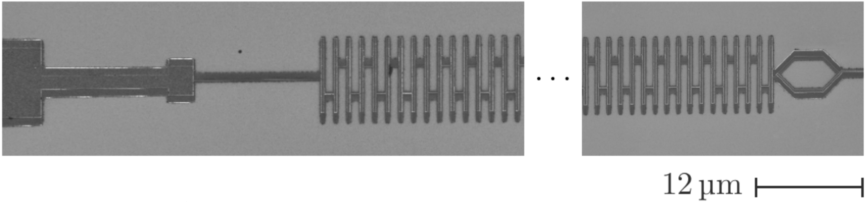


Kondo model:

$$H = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + J\vec{s}(x=0).\vec{S}$$

- Fermi liquid (some variants show quantum phase transitions)
- Tunable exchange coupling J via gates
- Renormalized Kondo scale $T_K \simeq De^{-D/J} \ll J$

Fermionic vs bosonic quantum impurities



Boundary sine-Gordon model:

$$H = \sum_k \omega_k a_k^\dagger a_k - E_J(\Phi_B)\cos[\phi(x=0)]$$

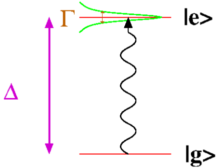
- Important QFT with algebraic correlations
- Tunable non-linearity: $E_J(\Phi_B) = E_J|\cos(\pi\Phi_B/\Phi_q)|$ with Φ_B the flux in the SQUID, and $\Phi_q = h/2e$
- Bosonic and fermionic models have mathematical connection

Motivation for quantum opticians

QED at ultra-strong coupling

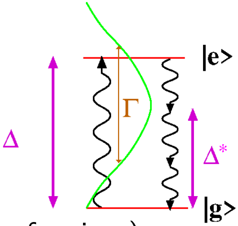
Linewidth of atomic level in 3d vacuum:

$$\frac{\Gamma}{\Delta} \simeq (\alpha_{\mathrm{QED}})^3$$



Ultra-strong coupling regime of QED:

$$\frac{\Gamma}{\Delta} \simeq 1$$



- Huge Lamb shift of Δ^* (akin to T_K for fermions)
  [Leggett *et al.*, RMP 1987]
- Higher probability for down-conversion
  [Goldstein, Devoret, Houzet & Glazman, PRL 2013]

Building circuits for bosonic impurities

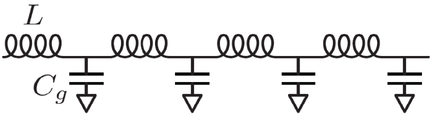
[Léger *et al.*, SciPost 2023]
[Kuzmin *et al.*, PRL 2021 and arxiv 2023]
[Murani *et al.*, PRX 2020]
[Léger *et al.*, Nat. Comm. 2019]
[Kuzmin *et al.*, npj Quantum 2019]
[Puertas-Martinez *et al.*, npj Quantum 2019]
[Magazzu *et al.*, Nat. Comm. 2018]
[Forn-Diaz *et al.*, Nat. Phys. 2017]
[Snyman & Florens, PRB 2015]
[Peropadre, Zueco, Porras, & García-Ripoll, NJP 2013]
[Goldstein, Devoret, Houzet & Glazman, PRL 2013]
[LeHur, PRB 2012]

High impedance waveguide

Alternative expression: $\alpha_{\rm QED} = \frac{Z_0}{2R_K}$

- $Z_0 = \sqrt{\mu_0/\epsilon_0} \simeq 376\Omega$: vacuum impedance
- $R_K = h/e^2 \simeq 25812\Omega$: resistance quantum

LC waveguide: $Z_{\rm chain} = \sqrt{L/C_g}$

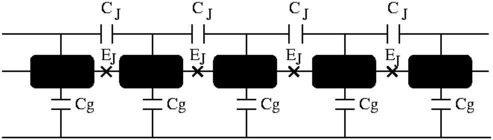


Waveguide from linear Josephson junctions:

$$V = \frac{\hbar}{2eI_c} dI/dt = L_J dI/dt$$

- Putting numbers: $L_J \simeq 1$ nH$/\mu$m $\simeq 10^4 L_{\rm geometric}$
- Effective coupling constant: $\alpha_{\rm chain} = \frac{(2e)^2}{e^2}\frac{Z_{\rm chain}}{2R_K} \simeq 0.1 \rightarrow 1$

Waveguide engineering



Harmonic regime:

- For $E_J \gg (2e)^2/(2C_J + C_g)$, weak phase fluctuations:

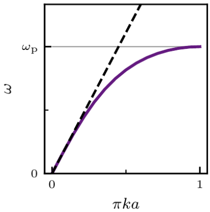
$$H_{\mathrm{chain}} \simeq \frac{1}{2}\sum_{i,j}(2e)^2 n_i \left(C^{-1}\right)_{ij} n_j + \sum_i \frac{E_J}{2}(\Phi_i - \Phi_{i+1})^2 = \sum_k \omega_k a_k^\dagger a_k$$

Spectrum:

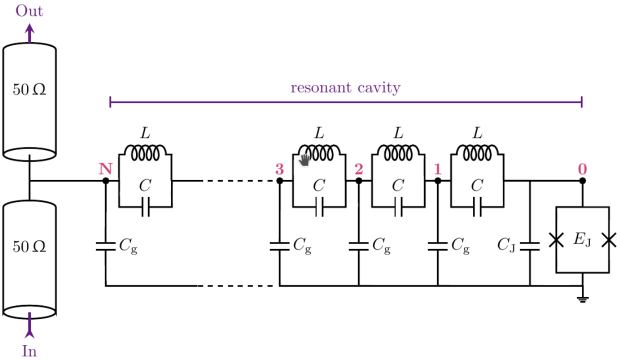
$\omega_k = 2\sin(\frac{k}{2})\sqrt{\frac{(2e)^2 E_J}{C_g + 4C_J \sin^2(k/2)}}$

$k = \frac{\pi n}{N}$ with $n = 1 \dots N$

N = number of junctions



A circuit view of the full device

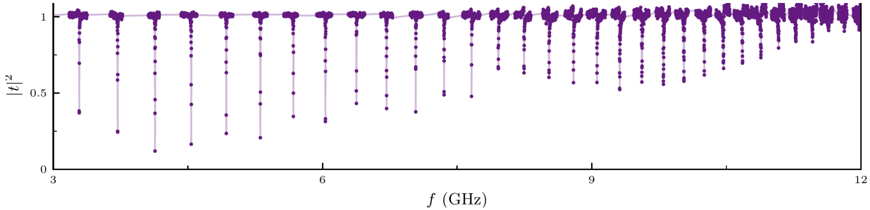


- Boundary = terminal junction (SQUID) with tunable $E_J(\Phi_B)$
- Chain of microwave resonators = resonant cavity
- AC measurement: $I_{\text{out}} e^{i\omega_{\text{out}} t}$ vs $I_{\text{in}} e^{i\omega_{\text{in}} t}$ (in GHz range)

[Léger et al., Nat. Comm. 2019; Kuzmin et al., npj 2019]

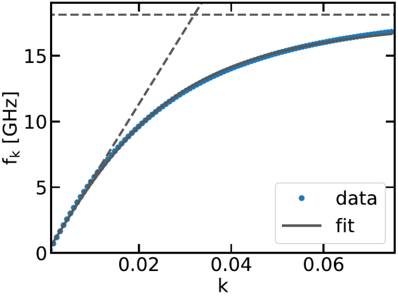
Transmission of the device

Measurement at zero flux: $E_J(\Phi_B = 0)$ is large $\rightarrow$ linear regime



- Eigenmodes are clearly resolved as sharp anti-resonances
- Very high quality factor
- Level spacing decreases at high frequency: UV cutoff ω_P

Properties of the array



Dispersion relation: $\omega_k = \sqrt{\frac{4}{LC_g} \frac{\sin^2(k/2)}{1+4(C/C_g)\sin^2(k/2)}}$

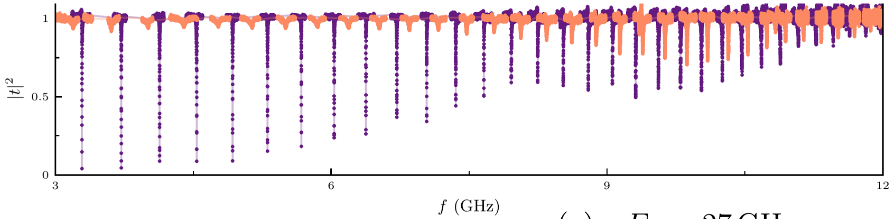
Fitted chain parameters: $L = 0.52$nH, $C_g = 0.15$fF, $C =$144fF

Chain impedance: $Z_{\text{chain}} = \sqrt{L/C_g} = 1.9k\Omega \Rightarrow \boxed{\alpha = 0.3}$

Plasma frequency: $\omega_P = 18$ GHz (UV cutoff)

Extracting renormalized parameters from spectroscopy

Impact of the boundary on the mode spectrum



(•) : $E_J \simeq 27\,\mathrm{GHz}$,
(•) : $E_J \simeq 7.7\,\mathrm{GHz}$.

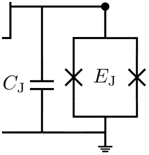
Two clear effects by decreasing E_J:

- Peaks shift $\rightarrow$ Re$[\Sigma(\omega)]$ = dispersive response
- Peaks broaden $\rightarrow$ Im$[\Sigma(\omega)]$ = dissipative response

Question: where hides the terminal junction?

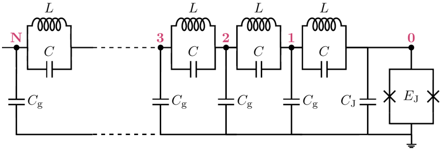
Terminal junction frequency?

Isolated junction:



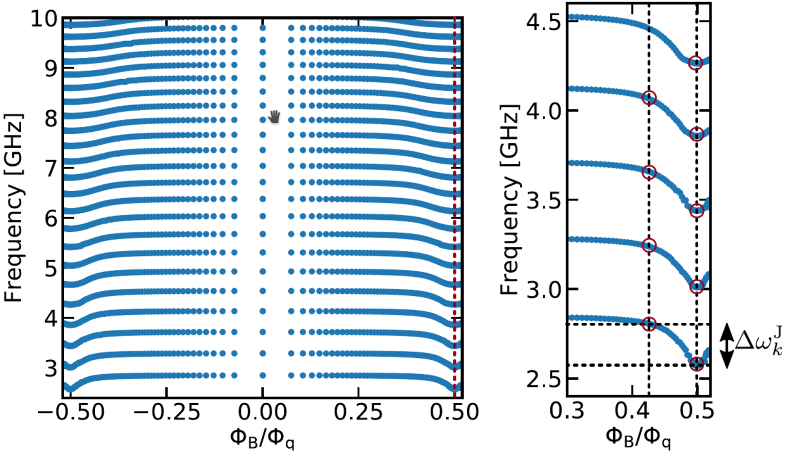
$$\omega_J = \frac{1}{\sqrt{L_J C_J}} = \sqrt{2E_c E_J(\Phi_B)}$$

Boundary junction + chain:



$E_J(\Phi_B)$ changes the boundary condition of the waveguide
$\Rightarrow$ affects all eigenmodes via phase-shift

Dispersive response: phase shift spectroscopy



Phase shift: $\delta\theta_k = \pi \frac{\Delta\omega_k}{\omega_{k+1} - \omega_k}$

[DeWitt, Phys. Rev. (1956); Puertas et al., npj Quantum Inf. (2019)]

Dispersive response: phase shift spectroscopy

PHYSICAL REVIEW VOLUME 103, NUMBER 5 SEPTEMBER 1, 1956

Transition from Discrete to Continuous Spectra*

BRYCE S. DEWITT
Department of Physics, University of North Carolina, Chapel Hill, North Carolina
(Received April 17, 1956)

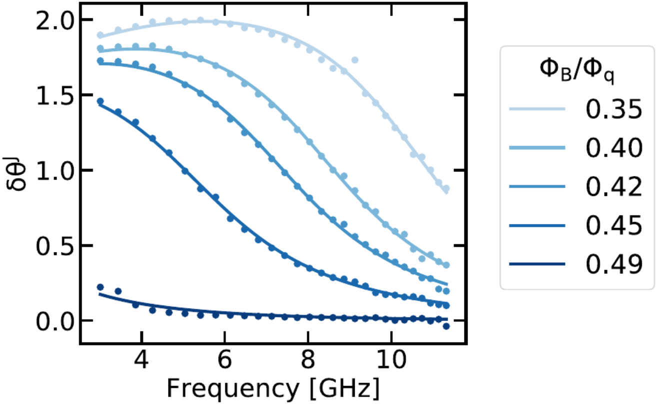
The stationary states of a system bound in a spherical box and additionally subjected to a perturbation of finite range are studied in the limit as the box radius becomes infinite. The transition from formal discrete-spectrum theory to formal scattering theory is carried out explicitly by two different methods. It is shown quite generally (i.e., even when the total Hamiltonian is not separable) that the level shift produced by the perturbation is proportional to the corresponding scattering phase shift.

I. INTRODUCTION

AN attempt has recently been made by Reifman and Newton, in collaboration with the author,[1] to justify a procedure of Brueckner[2] which attempts to deal with nuclear many-body bound-state problems in the language of scattering theory, by imagining that the nuclear radius is sufficiently large so that the stationary two-body states are quasi-continuous. In particular, the attempt was made to justify Brueckner's cases that the level shift produced on a quasi-continuous state by a perturbation of finite range becomes, in the limit as boundary walls recede to infinity, proportional simply to the corresponding phase shift, *not* to its tangent.

It is curious that this result, which seems to have been known more or less privately for some time by various individuals, has not previously achieved the dignity of a special statement in the literature.† In the

Dispersive response: phase shift spectroscopy

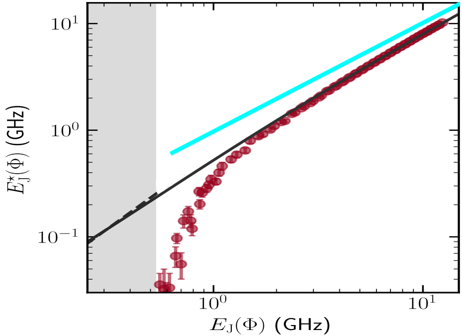


Lines are predictions from linear model

- Inflexion point of $\delta\theta_k \Longleftrightarrow$ terminal junction frequency ω_J^*
- Clearly $\omega_J^*(\Phi_B) = \sqrt{2E_c E_J^*(\Phi_B)}$ decreases with increasing flux

  Is it just due to $E_J^*(\Phi_B) = E_J^* |\cos(\pi\Phi_B/\Phi_q)|$?

Renormalization of the Josephson energy



Red dots = measured E_J^*
Blue line = bare E_J
Black line = SCHA E_J^*

SCHA = microscopic self-consistent harmonic approximation
$E_J^*(\Phi_B) = E_J(\Phi_B)e^{-\langle\hat{\varphi}_0^2\rangle/2}$ **[Schön & Zaikin, Phys. Rep. (1990)]**

Fit of unknown parameters: $E_J(0) = 27\text{GHz}$
$E_J(\Phi_q/2)/E_J(0) = 3\%$ (SQUID asym.)

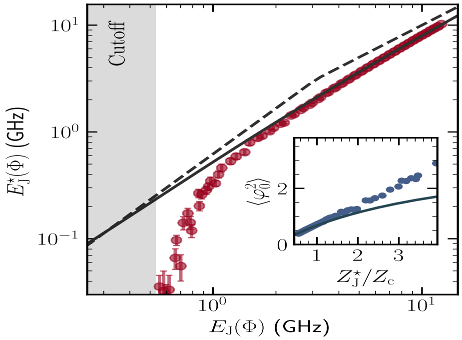
Note: Ambegaokar-Baratoff gives $E_J(0) = 26\text{GHz}$, OK!

Can we see scaling law of E_J^*? (almost)

Known result: $E_J^* = E_J^{\text{scaling}} \propto E_J(E_J/\omega_P)^{\alpha/(1-\alpha)}$ for $E_J \ll \omega_P$

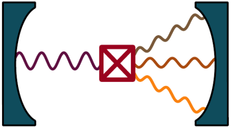
[Panyukov & Zaikin Physica B 1988, Hekking & Glazman PRB 1997]

Warning: due to shunting capacitance, true UV cutoff is $E_C < \omega_P$
$\Rightarrow$ scaling regime is not accessible



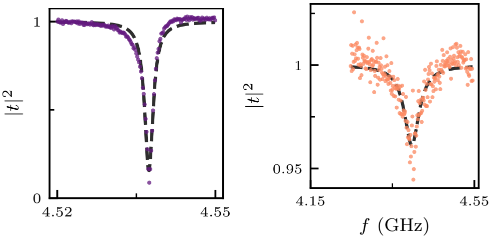
Dots = E_J^* measurement
Dashed = Min$(E_J, E_J^{\text{scaling}})$
Black line = E_J^* from SCHA

Analysis of many-body dissipation



Dissipative response: quality factor spectroscopy

Analysis of two given resonances at small and large Φ_B:

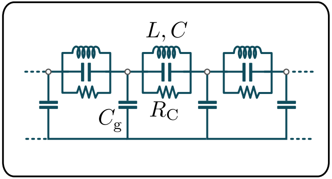


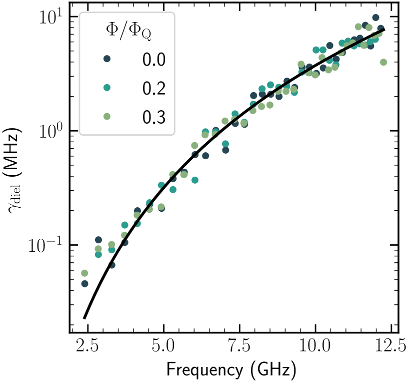
- Nearly Lorentzian lineshape
- Peak depth $= \frac{1}{1+Q_e/Q_i}$
- Peak total width $= \gamma_k = \left[\frac{1}{Q_e} + \frac{1}{Q_i}\right]\omega_k = \gamma_k^{\text{external}} + \gamma_k^{\text{internal}}$

$\rightarrow$ extract internal quality factor Q_i

$Q_i < \infty \Rightarrow$ photons are lost somewhere inside the circuit

Dielectric losses in the chain

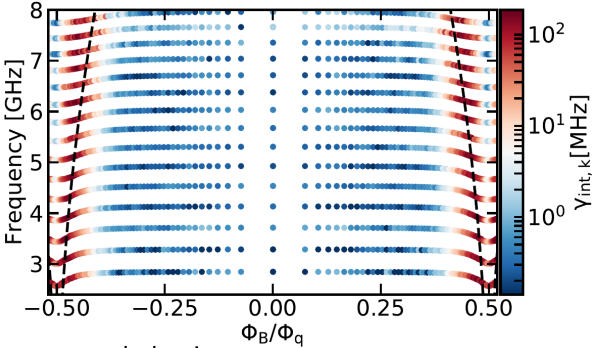




- Dielectric losses dominate at low flux
- Phenomenological fit [Nguyen *et al.* PRX 2019] is subtracted to obtain the intrinsic internal losses

Internal losses for all modes

Peak linewidth due to internal losses: $\gamma_k^{\text{internal}} = \omega_k / Q_i(\omega_k)$



Striking re-entrant behavior:

$\Phi_B = 0 : -E_J(\Phi_B)\cos(\hat{\varphi}_0) \simeq E_J(0)\hat{\varphi}_0^2/2 \rightarrow$ low-loss linear regime

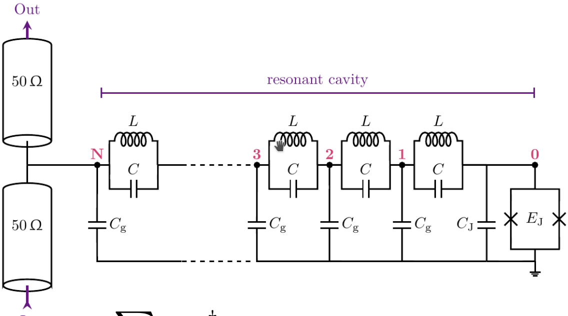
$\Phi_B = \Phi_q/2 : -E_J(\Phi_B)\cos(\hat{\varphi}_0) \simeq 0 \rightarrow$ back to linear again?

But $E_J(\Phi_q/2) \neq 0$, due to SQUID asymmetry $\Rightarrow$ losses persist!

[Kuzmin et al. PRL 2021]

Theoretical modeling

Full modeling in the non-linear case



$$H = \sum_k \omega_k a_k^\dagger a_k - E_J(\Phi_B)\cos[\phi(x=0)]$$

$$\text{Dyson equation :}\quad \left[\omega^2 \hat{C} - 1/\hat{L} + \frac{i2\omega}{Z_{\rm tl}}\hat{\delta}^{(N)} - \Sigma(\omega)\hat{\delta}^{(0)}\right]\hat{G} = 1$$

$Z_{\rm tl} = 50\Omega$: external broadening from transmission line

Transmission from Kubo: $t(\omega) = 2i\omega G_{N,N}(\omega)/Z_{\rm tl}$

Josephson-Keldysh-Feynman diagrams

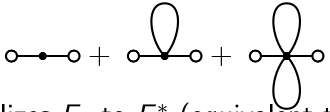
Hamiltonian:

$$H = \int \mathrm{d}k\, \omega(k) a_k^\dagger a_k - E_J \cos(\phi_0)$$

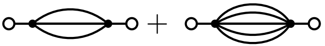
$$\phi_0 = \int \mathrm{d}k\, g(k) \left(a_k^\dagger + a_k \right)$$

$\cos(\phi_0)$ contains $a_{k1}^\dagger a_{k2}^\dagger a_{k3}^\dagger a_{k4} + \ldots \Rightarrow$ frequency conversion

Expansion: Φ_B/Φ_q close to $1/2 \Longleftrightarrow E_J$ small

$\Sigma(t) =$  $+ \ldots = E_\mathrm{J} \delta(t) e^{-\frac{1}{2} G_F(0)}$

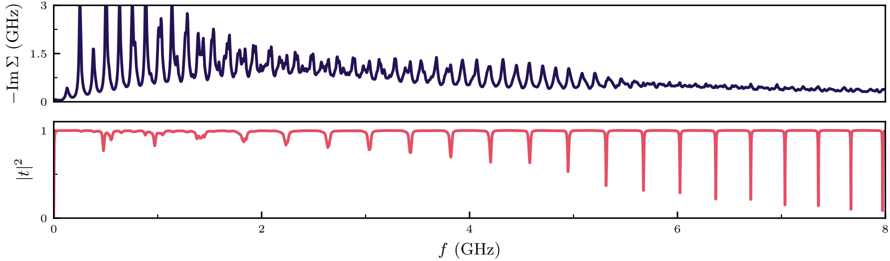
$\rightarrow$ renormalizes E_J to E_J^* (equivalent to "SCHA")

$\Sigma(t) =$  $+ \ldots = E_\mathrm{J}^2 \left[\sin(G(t)) - G(t) \right]$

$\rightarrow$ provides dissipative response $\mathrm{Im}[\Sigma(\omega)]$

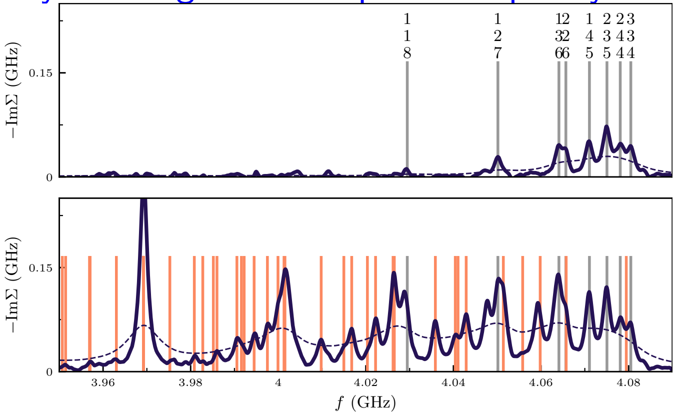
Frequency response from lowest order diagrams

Full microscopic model:



- OK: qualitatively similar to the experimental internal width γ_k
- Not OK: incorrect multiplet structure (not seen experimentally)
  This is due to sharp resonances in the self-energy

Physical origin of multiplets: frequency conversion



Mode = 10
$Z_{\mathrm{tl}} = 10\Omega$
$T = 0\mathrm{K}$
Emission

Mode = 10
$Z_{\mathrm{tl}} = 10\Omega$
$T = 20\mathrm{mK}$
Emission+
Absorption

At $T = 0\mathrm{K}$: 3-photons (or more) resonances at $\omega = \omega_{k_1} + \omega_{k_2} + \omega_{k_3}$

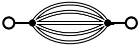
[Goldstein et al. PRL 2013; Gheeraert et al. PRA 2018; Houzet and Glazman PRL 2020; Burshtein et al., PRL 2021; Kuzmin et al. PRL 2021]

At finite T: also absorption resonances at $\omega = \omega_{k_1} \pm \omega_{k_2} \pm \omega_{k_3}$

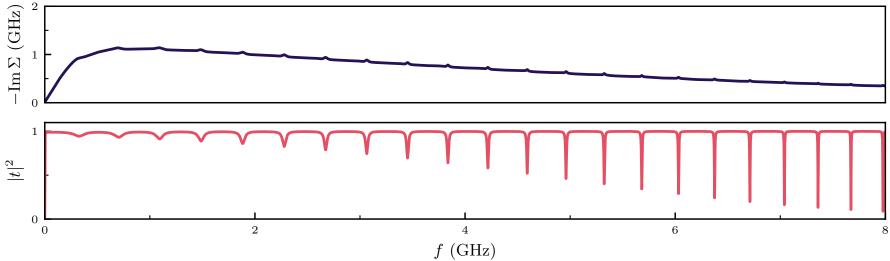
[Léger et al. SciPost 2023]

Improved diagrammatics

Resum bold (skeleton) diagrams:

$\Sigma(t) =$  $+$  $+ \ldots$

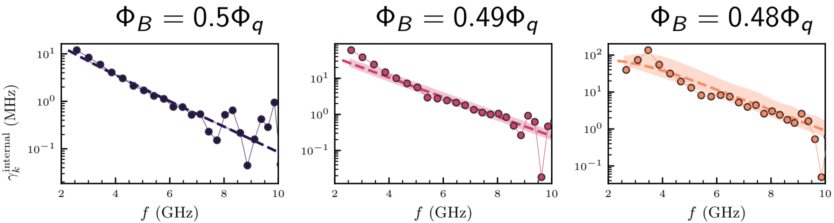
Small $\omega_k \simeq v.k \Rightarrow$ many near degeneracies in many-body spectrum
$\Rightarrow$ self-consistency provides level repulsion



- Smooth $\mathrm{Im}[\Sigma(\omega)] \Rightarrow$ true dissipation
- Leads to Lorentzian transmission peaks as seen experimentally

Many-body losses: theory vs experiment

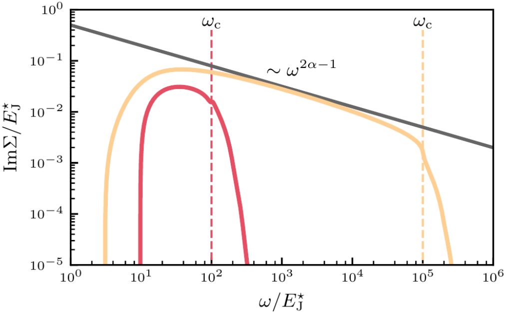
Losses at high flux: perturbative regime $E_J^* \ll \omega$



- Fit for $\Phi_B = 0.5\Phi_q$ and $0.49\Phi_q$:
  $\Rightarrow E_J(0) = 25$GHz and SQUID asym. 2.5%
  Agreement with theory at small flux
- Losses at $\Phi_B = 0.48\Phi_q$ well described (no fitting)
- Losses are a smooth function: many-body dissipation

Can one see scaling laws of losses? (no)

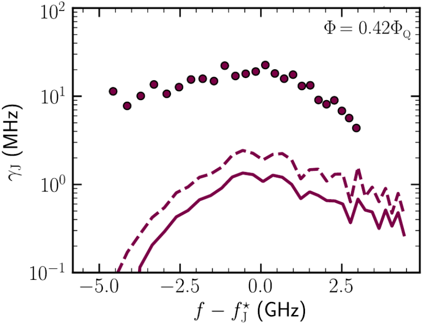
Known result: $\Sigma(\omega) \simeq \omega^{2\alpha-1}$ (Luttinger liquid analogy)



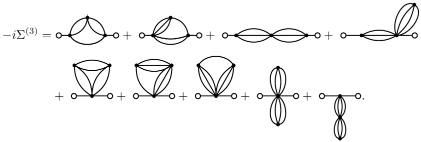
- Diagrammatics does reproduce the scaling laws
- Scaling is only found if $E_J^* \lll$ UV cutoff
  Limitation of Josephson platforms w.r.t. electronic circuits

Non perturbative regime of the experiment

Losses at intermediate flux: $\Phi_B = 0.42\Phi_q$



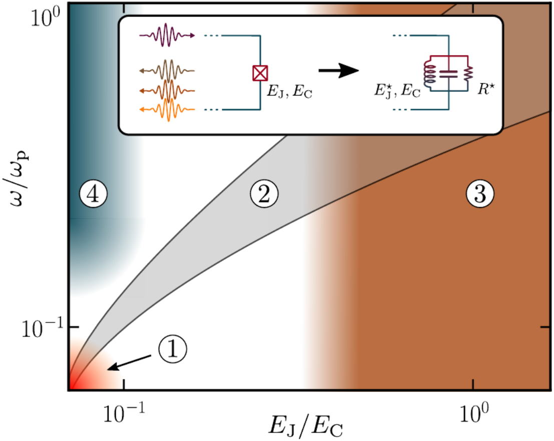
Compares experiment to $O(E_J^2)$ and $O(E_J^3)$ diagrams



- Losses have a peak at $\omega = \omega_J^*$
- Diagrammatic theory underestimates the magnitude of losses $\Rightarrow$ requires a truly non perturbative approach

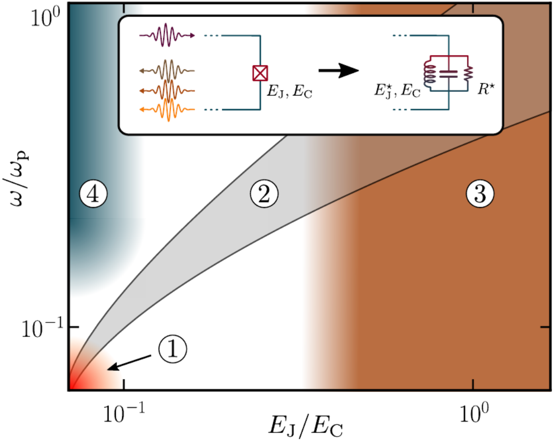
Summary

- Superconducting circuits can reach truly many-body regimes



Summary

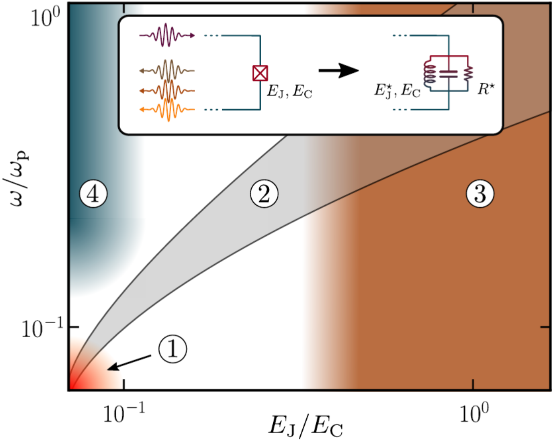
- Superconducting circuits can reach truly many-body regimes



Renormalization of the junction modeled in large E_J regime ③

Summary

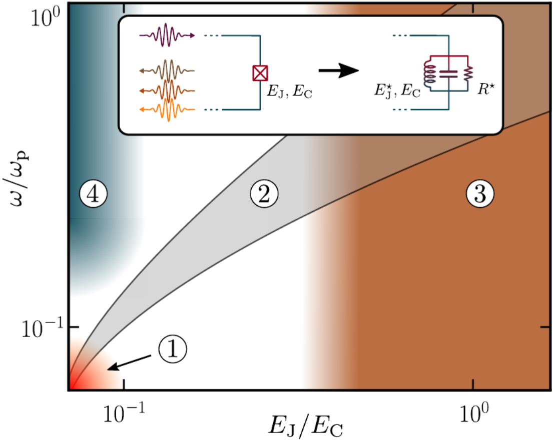
- Superconducting circuits can reach truly many-body regimes



Many-body losses modeled in small E_J regime ④

Summary

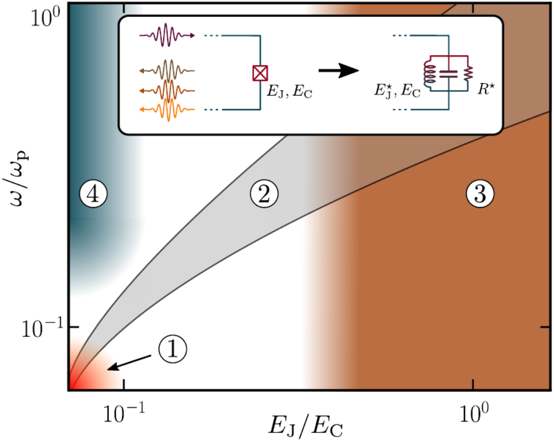
- Superconducting circuits can reach truly many-body regimes



The experiment is non-perturbative in a large accessible domain ②

Summary

- Superconducting circuits can reach truly many-body regimes



However reaching universal scaling domain ① is still hard