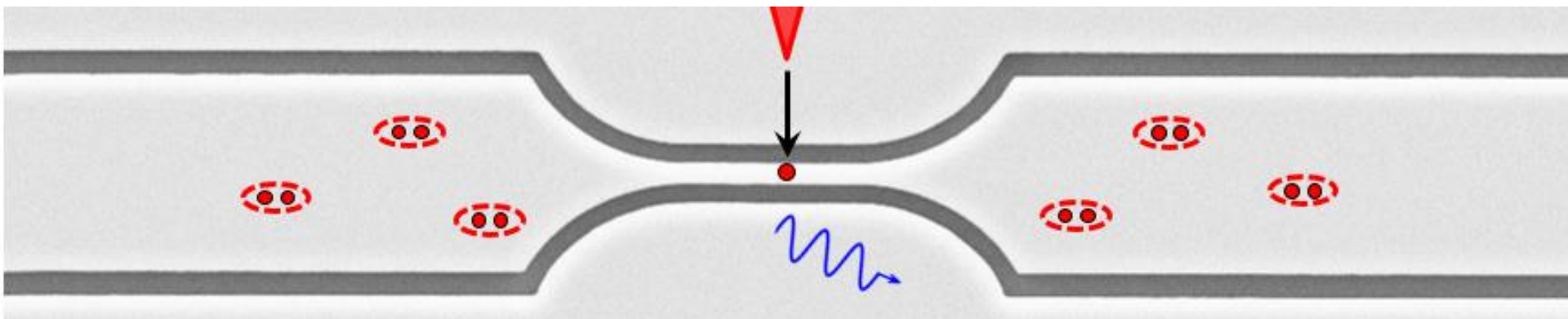


Scanning Critical Current Microscopy on an out-of-equilibrium superconducting nanowire



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Florence Levy-Bertrand
Frédéric Gustavo
Jean-Luc Thomassin
Claude Chapelier

The team



Florence Levy-Bertrand



Eduard Driessen



Thomas Jalabert



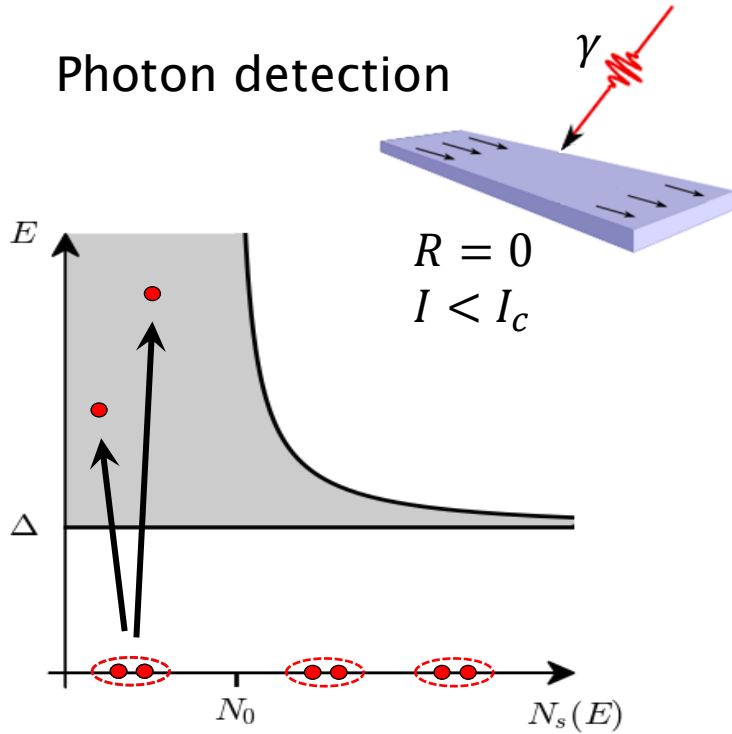
Frédéric Gustavo



Jean-Luc Thomassin

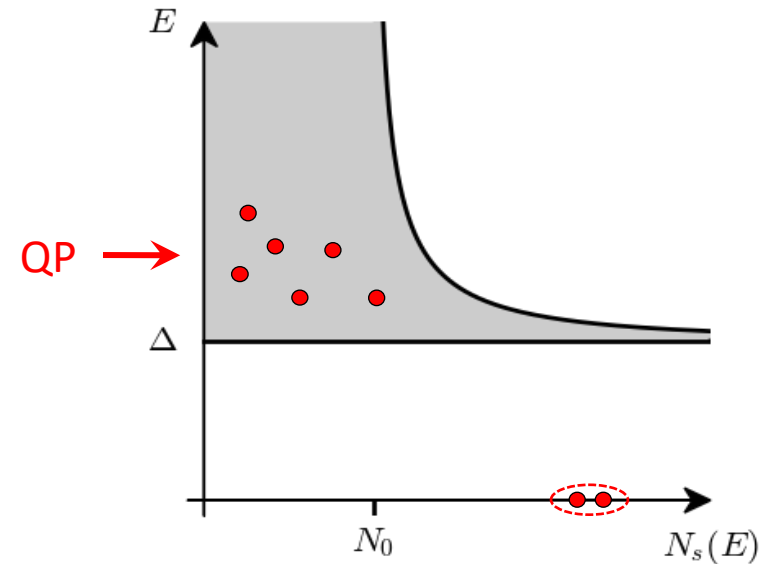
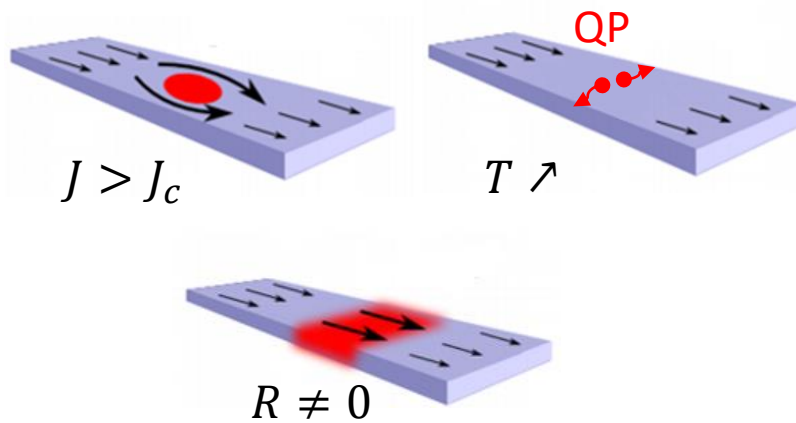
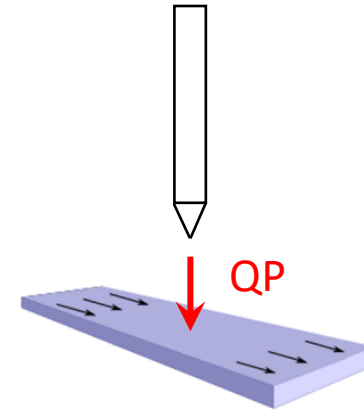
Superconducting Nanowire Single Photon Detector

Photon detection



Direct injection of quasiparticles

STM tip

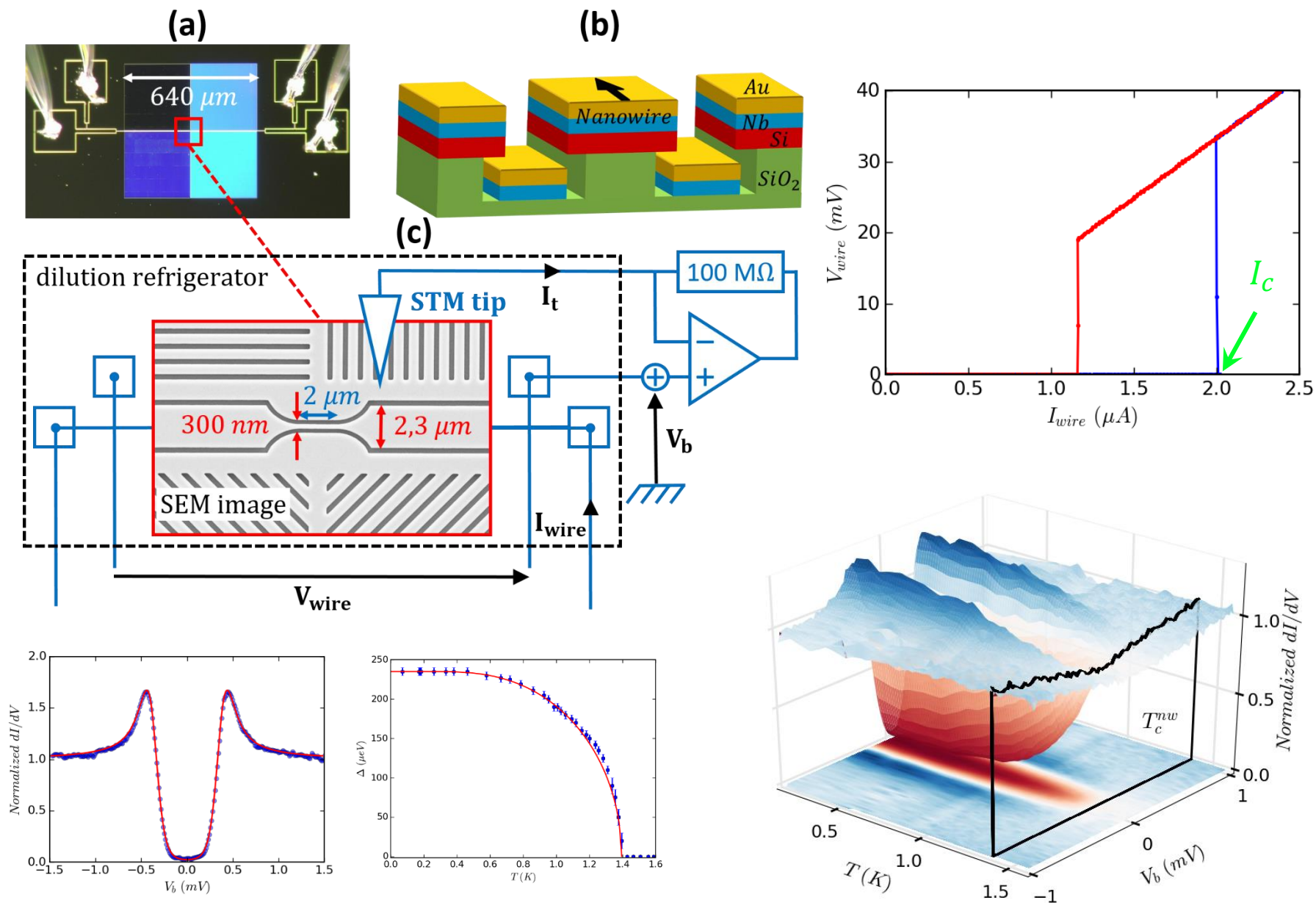


1 – High energy quasiparticles injection

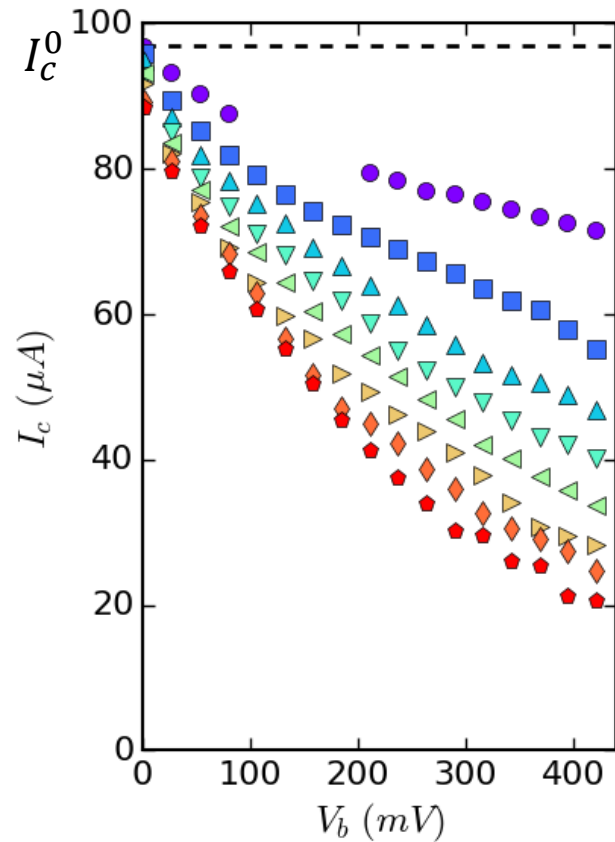
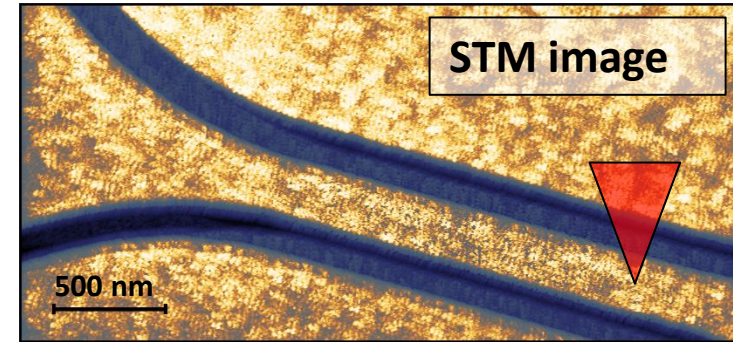
T. Jalabert, et al., Nat. Phys. (2023)

2 – Low energy quasiparticles injection

Scanning Critical Current Microscopy

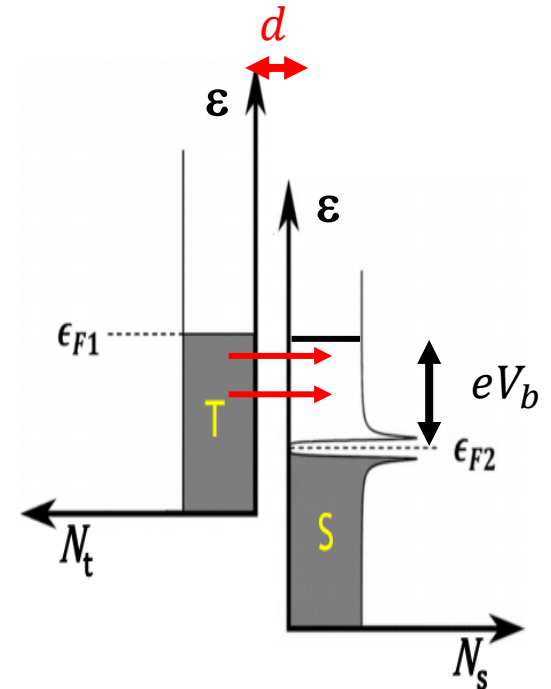


Tuning the critical current

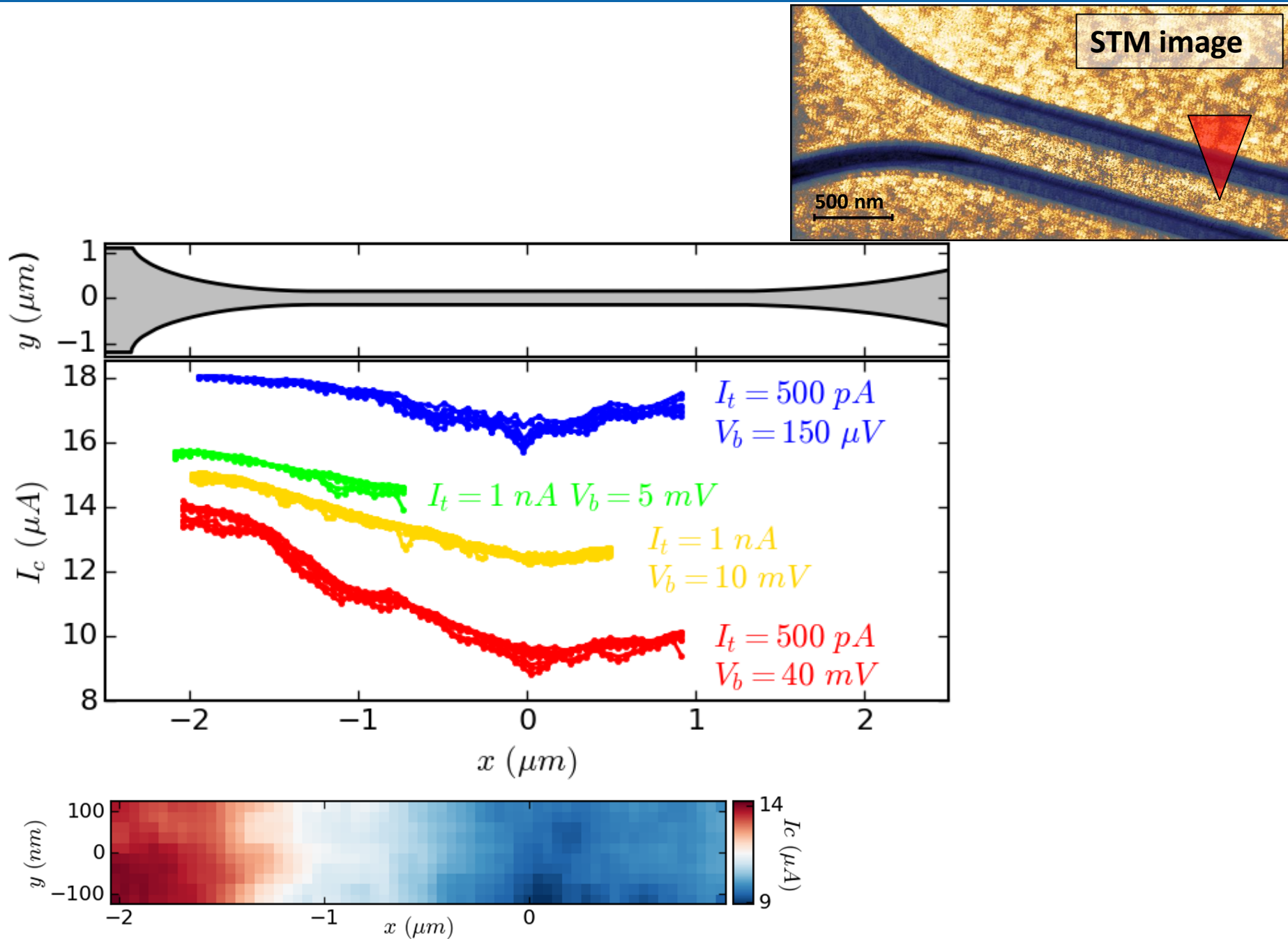


- $I_t = \dots 0 pA$
- 121 pA
 - 332 pA
 - ▲ 543 pA
 - ▼ 754 pA
 - ◀ 965 pA
 - ▶ 1177 pA
 - ◆ 1388 pA
 - ◆ 1599 pA

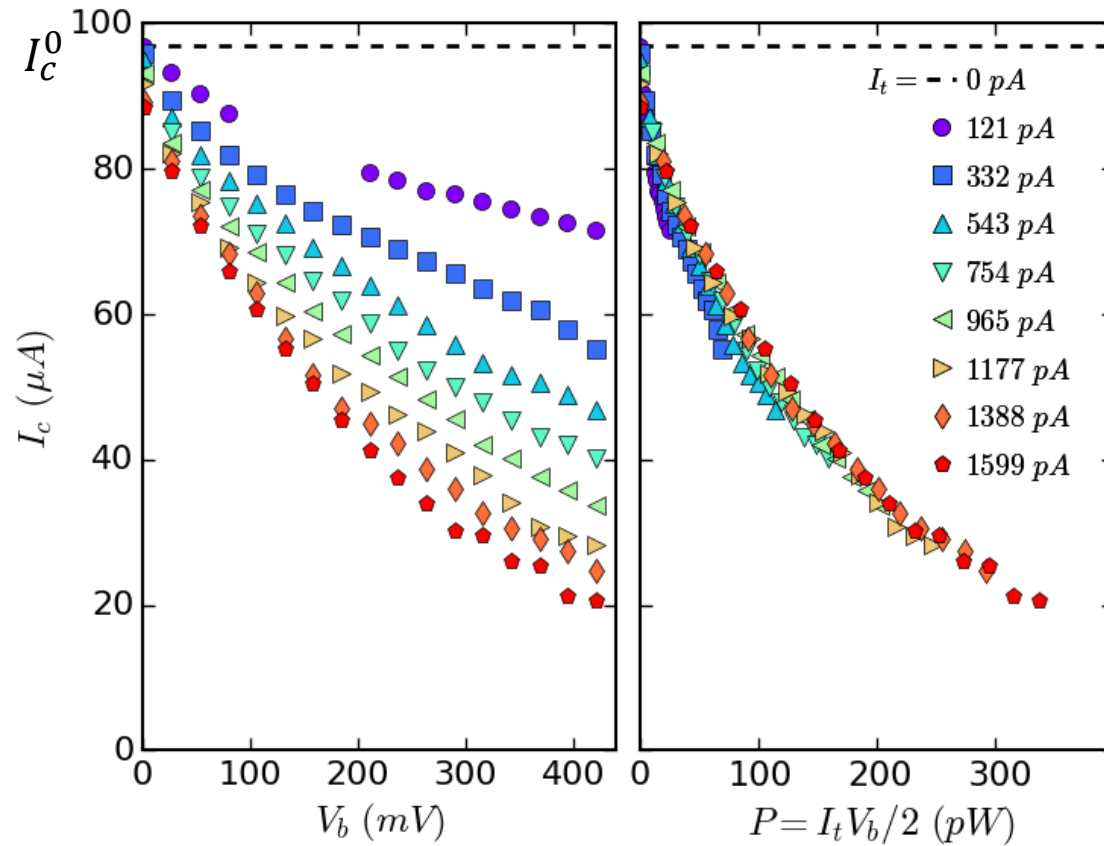
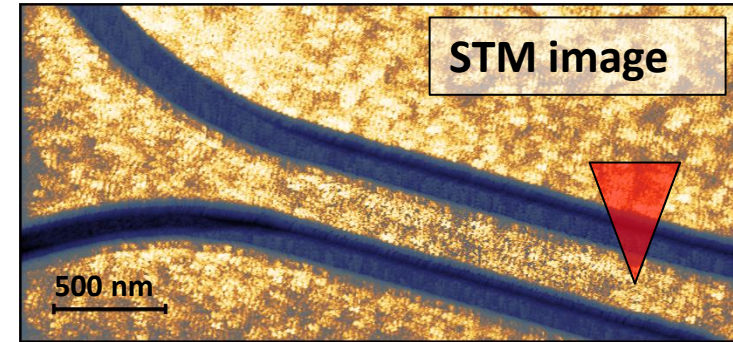
$$\triangleright \frac{I_c}{I_t} \sim 10^6$$



Mapping the local critical current



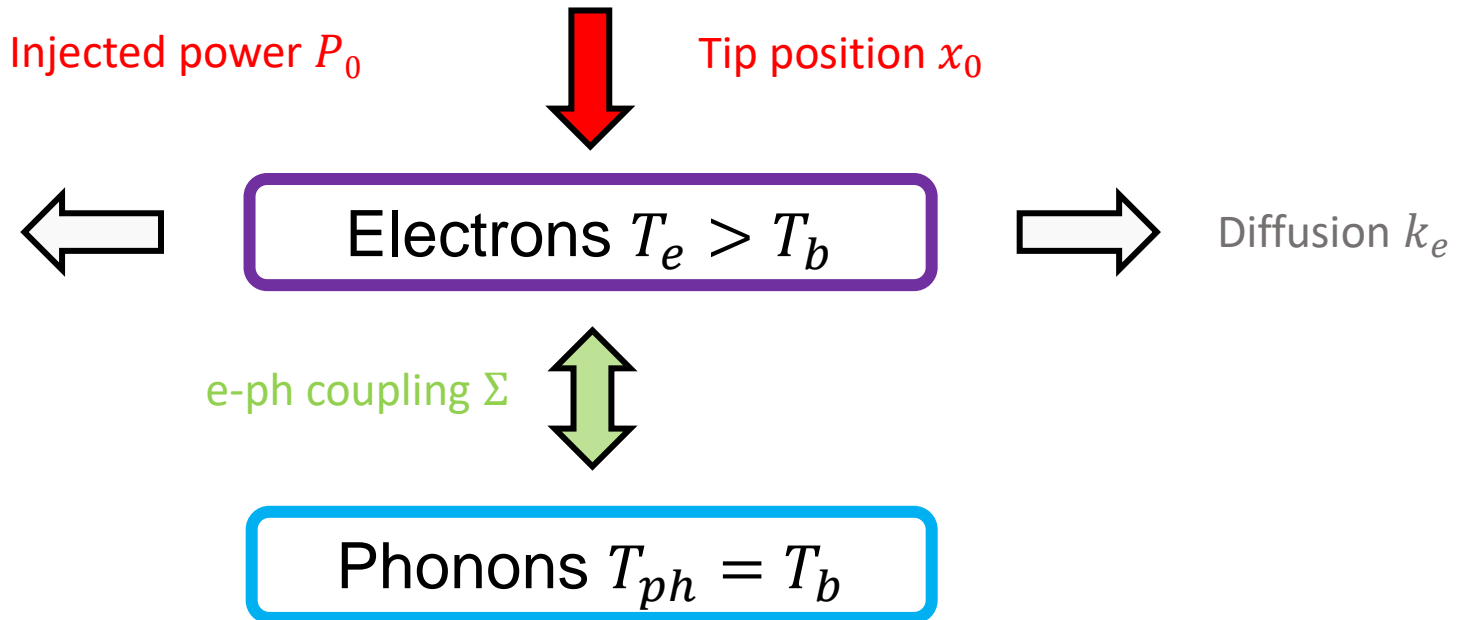
Power dependence



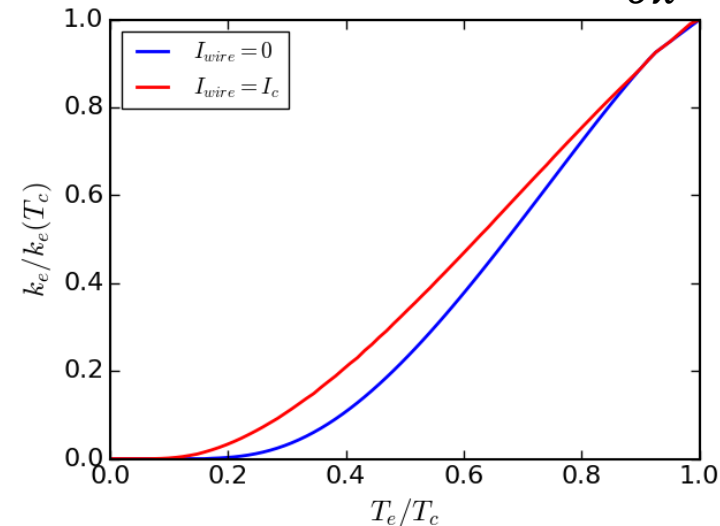
➤ I_c controlled by P

➤ $\frac{I_c}{I_t} \sim 10^6$

Heat diffusion model



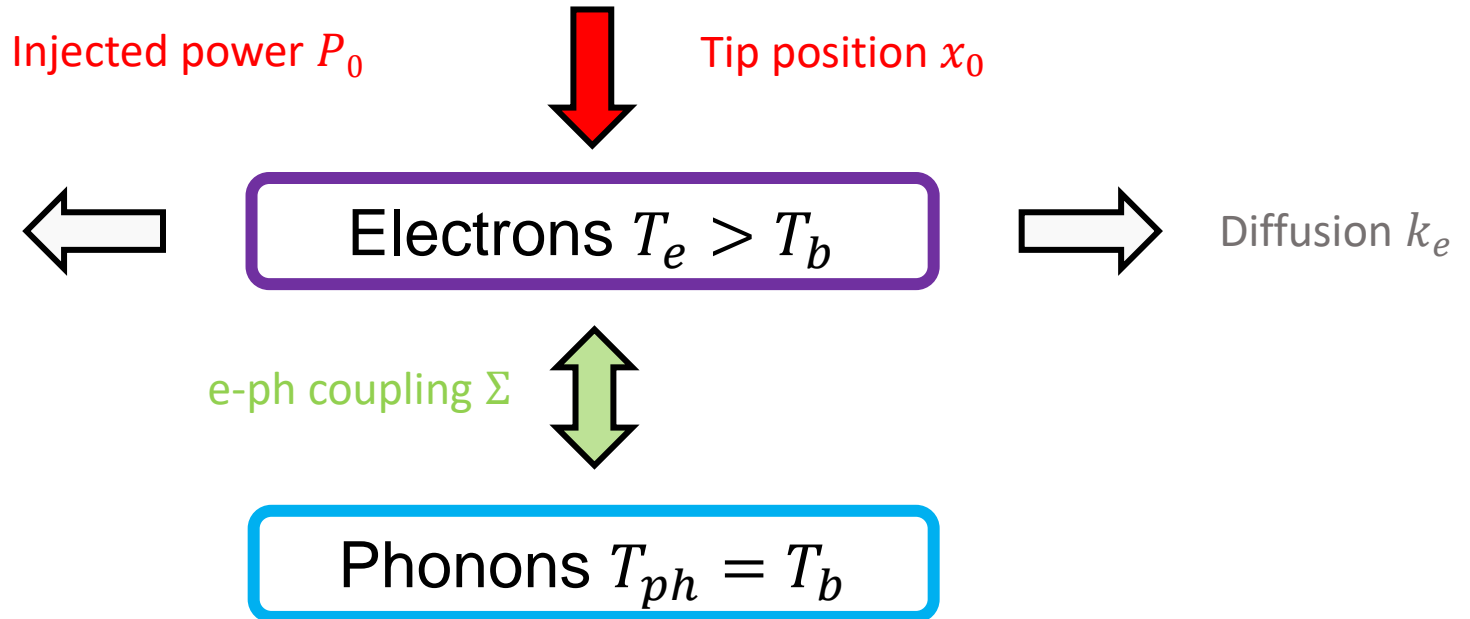
$$\nabla(twk_e \frac{\partial T_e(x)}{\partial x}) = \Sigma wt (T_e^p(x) - T_{ph}^p) - P_0 \delta(x - x_0)$$



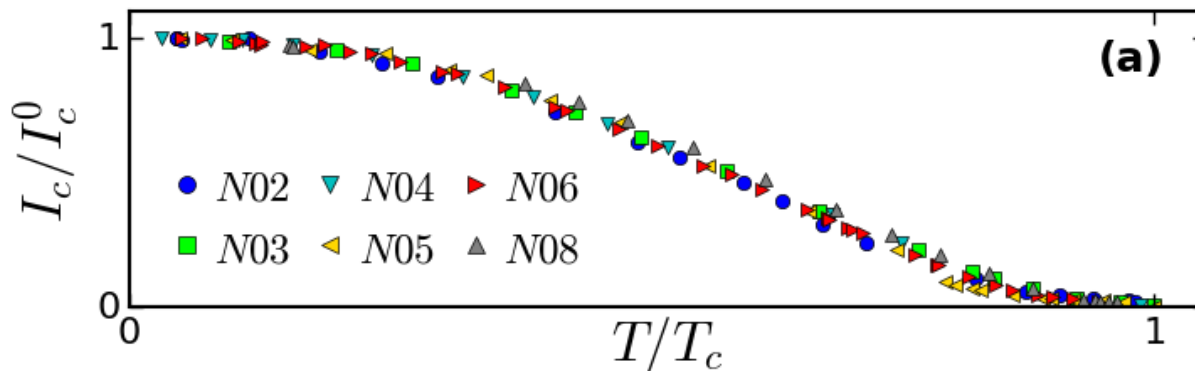
$$k_e = \frac{\sigma_N}{2e^2} \int_{-\infty}^{\infty} d\varepsilon \frac{\varepsilon^2}{2k_B T^2} (1 - \tanh^2(\frac{\varepsilon}{2k_B T})) \cos^2(\text{Re}[\theta])$$

$$\frac{k_e(T_c)}{\sigma_N T_c} = \frac{\pi^2}{3} \left(\frac{k_B}{e} \right)^2$$

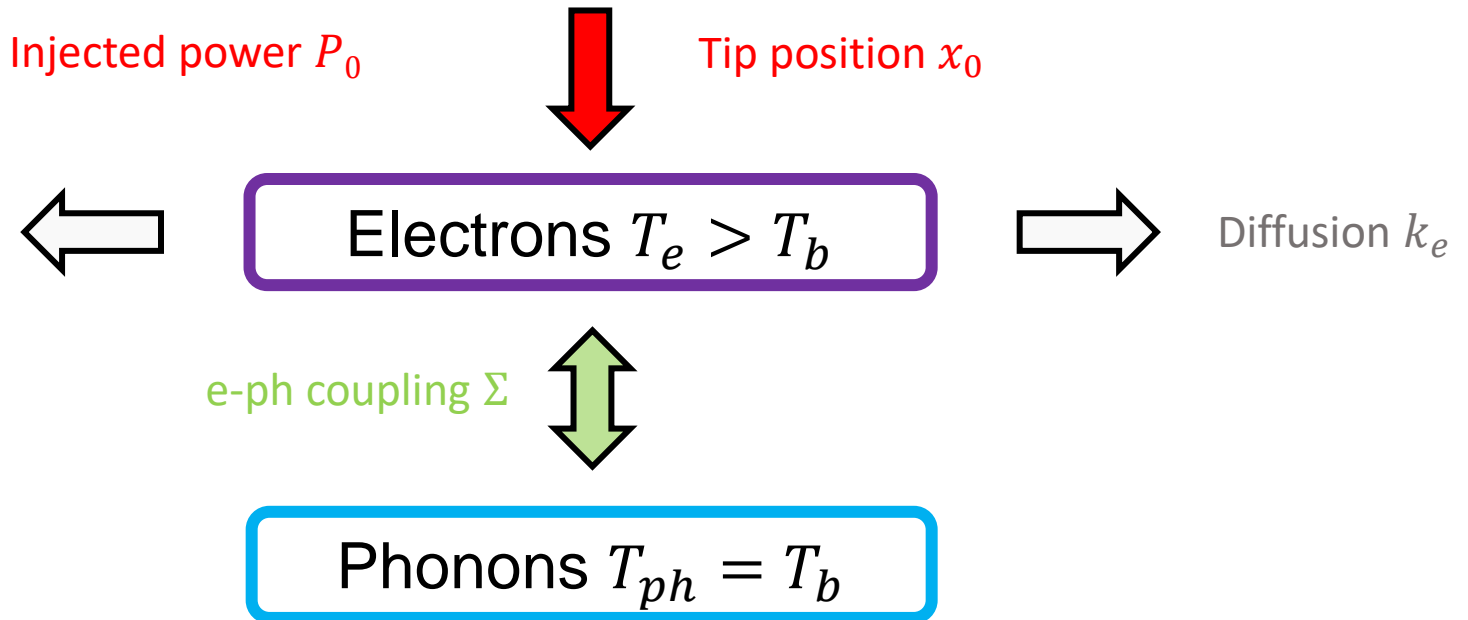
Heat diffusion model



$$\nabla(twk_e \frac{\partial T_e(x)}{\partial x}) = \Sigma wt (T_e^p(x) - T_{ph}^p) - P_0 \delta(x - x_0)$$



Heat diffusion model



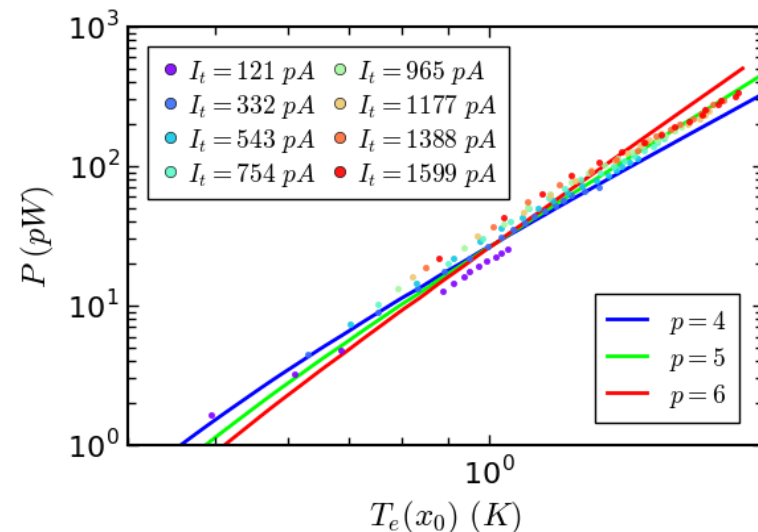
$$\nabla(twk_e \frac{\partial T_e(x)}{\partial x}) = \Sigma wt (T_e^p(x) - T_{ph}^p) - P_0 \delta(x - x_0)$$

$p = 4$ $l < \lambda_{ph}$ Static disorder

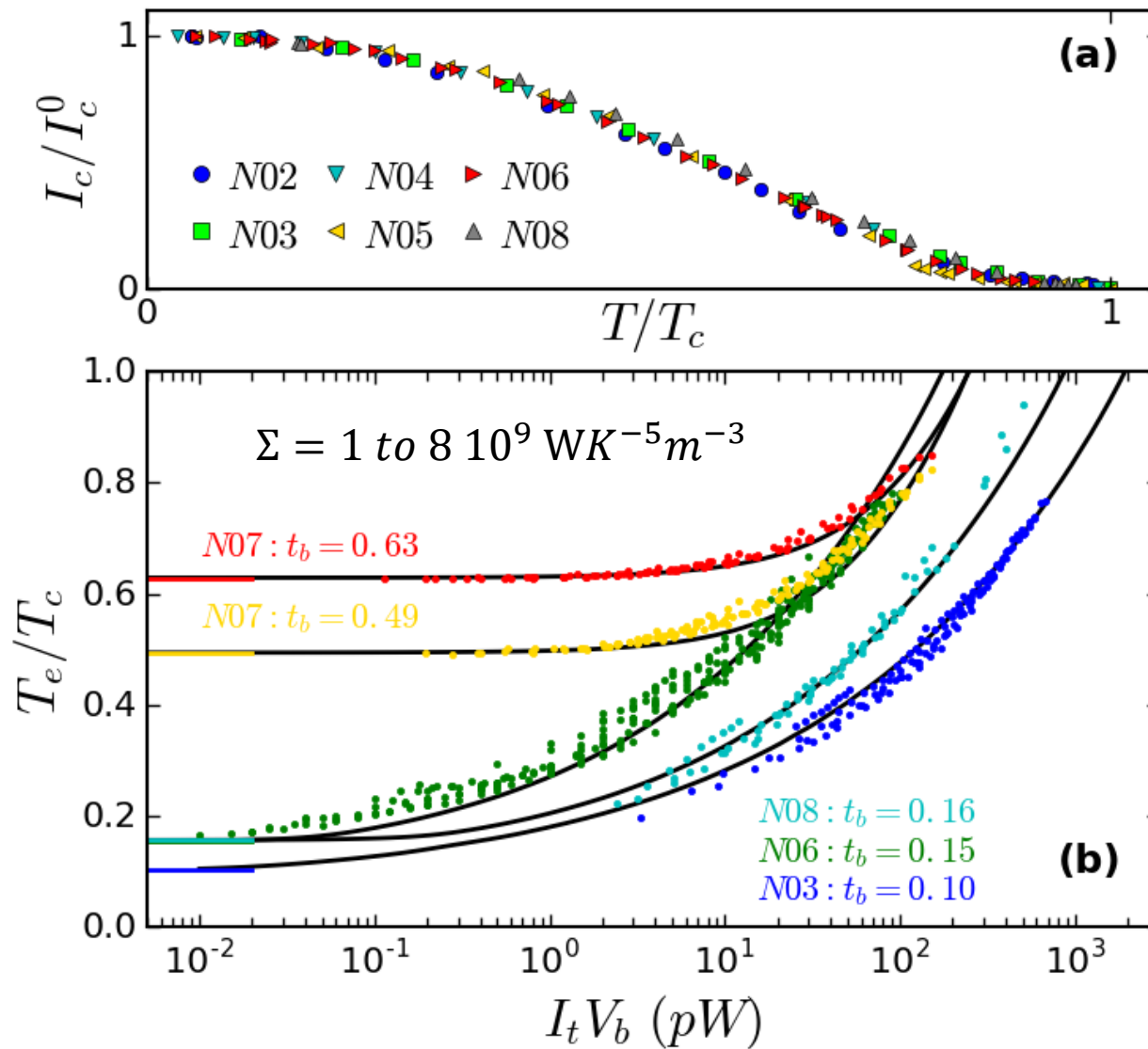
$p = 6$ $l < \lambda_{ph}$ Vibrating disorder

$p = 5$ $l > \lambda_{ph}$

$l \approx \lambda_{ph}$ for $V_b = 20$ meV

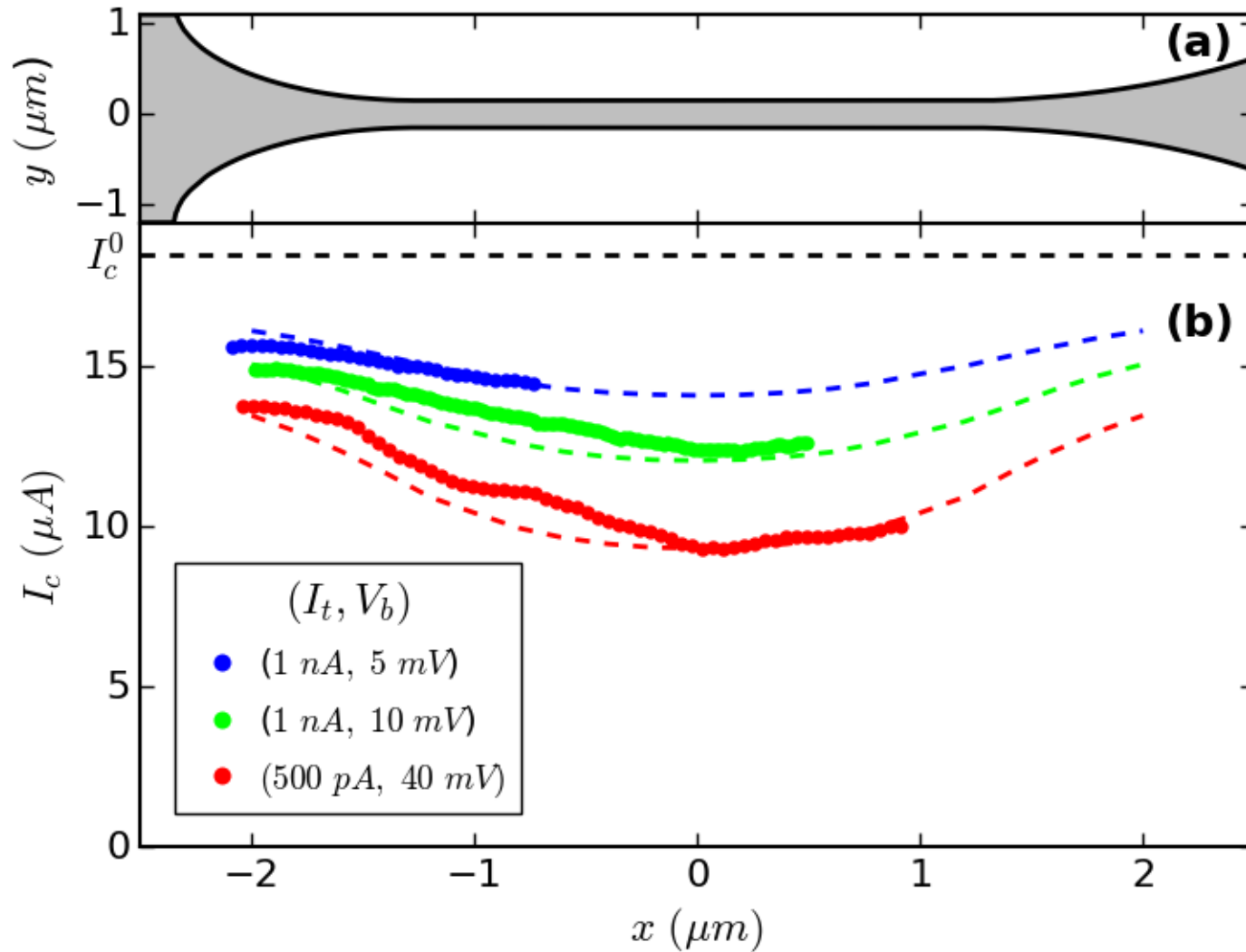


Electronic temperature



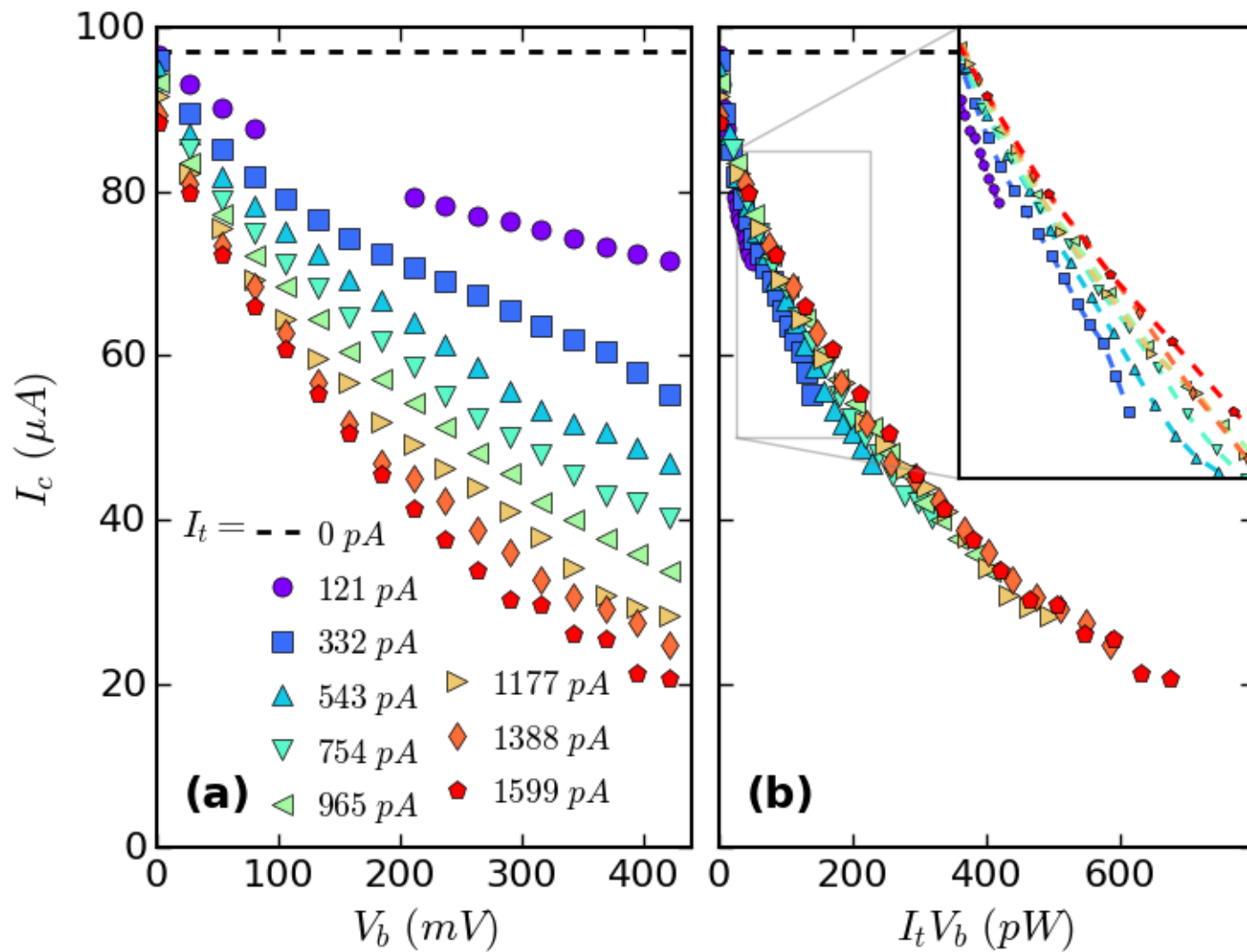
$$\nabla \left(t w k_e \frac{\partial T_e(x)}{\partial x} \right) = \Sigma w t (T_e^5(x) - T_{ph}^5) - P_0 \delta(x - x_0)$$

Spatial dependence

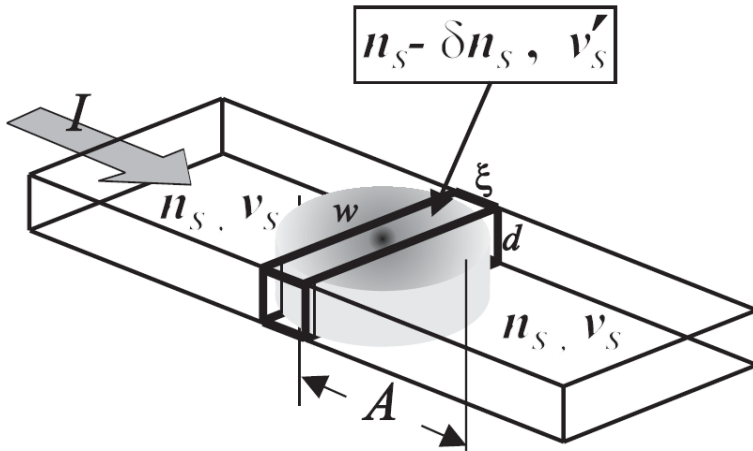


$$\nabla \left(t w k_e \frac{\partial T_e(x)}{\partial x} \right) = \Sigma w t (T_e^5(x) - T_{ph}^5) - P_0 \delta(x - x_0)$$

Beyond the stationary thermal model



Hot spot model



$$n_s \approx N_0 \Delta$$

$$j_s = n_s v_s e \quad j'_s = (n_s - \Delta n_s) v'_s e$$

$$I_s = j_s d w = n_s v_s e d w$$

$$I_s = I'_s \Rightarrow v'_s = \frac{n_s}{n_s - \Delta n_s} v_s$$

$$I_c = n_s v_c e d w$$

A. Semenov, et al., Eur. Phys. J. B 47 (2005)

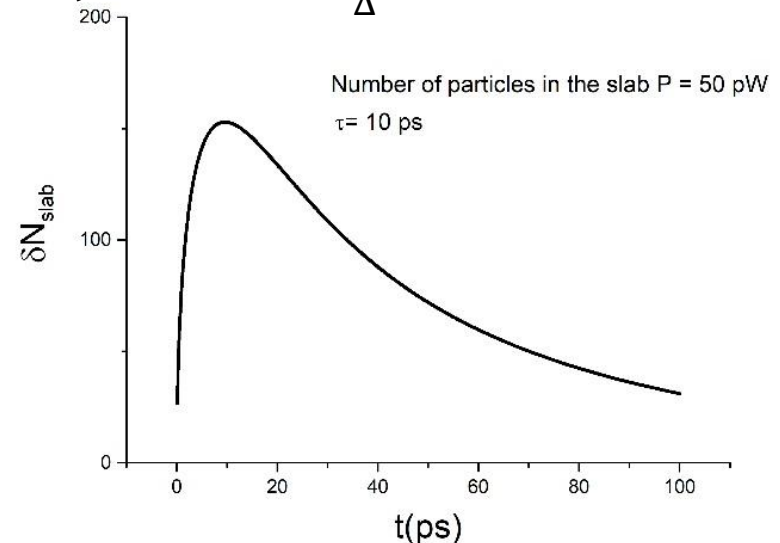
Superconducting transition : $v'_s = v_c \Rightarrow \Delta n_s = n_s \left(1 - \frac{I_c}{I_s}\right)$

Out-of-equilibrium quasiparticles in the slab at the transition : $\Delta N_c = N_0 \Delta \xi w d \left(1 - \frac{I_c}{I_s}\right)$

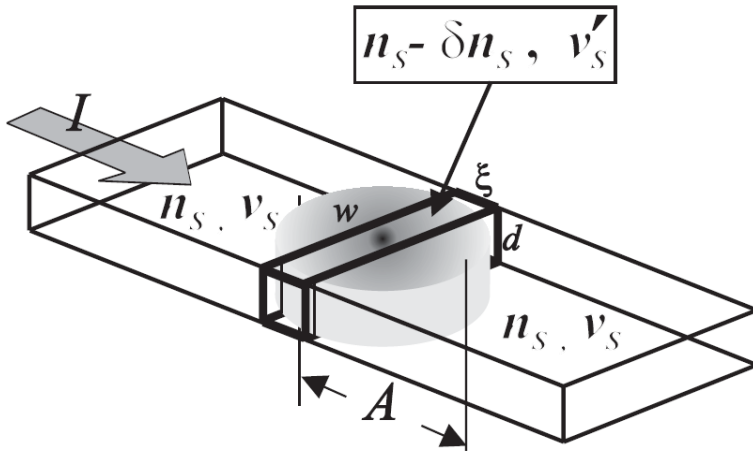
Generation of quasiparticles : $M(t) = K (1 - e^{-\frac{t}{\tau}})$ with $K \approx \frac{e V_i}{\Delta}$

Hot spot radius : $A(t) = 4 \sqrt{D t \ln(M(t))}$

$$\Delta N_{slab} = \frac{K (1 - e^{-\frac{t}{\tau}}) \xi}{\sqrt{\pi D t}}$$



Hot spot model



$$n_s \approx N_0 \Delta$$

$$j_s = n_s v_s e \quad j'_s = (n_s - \delta n_s) v'_s e$$

$$I_s = j_s d w = n_s v_s e d w$$

$$I_s = I'_s \Rightarrow v'_s = \frac{n_s}{n_s - \delta n_s} v_s$$

$$I_c = n_s v_c e d w$$

A. Semenov, et al., Eur. Phys. J. B 47 (2005)

Superconducting transition : $v'_s = v_c \Rightarrow \delta n_s = n_s \left(1 - \frac{I_c}{I_s}\right)$

Out-of-equilibrium quasiparticles in the slab at the transition : $\delta N_c = N_0 \Delta \xi w d \left(1 - \frac{I_c}{I_s}\right)$

Generation of quasiparticles : $M(t) = K (1 - e^{-\frac{t}{\tau}})$ with $K \approx \frac{e V_i}{\Delta}$

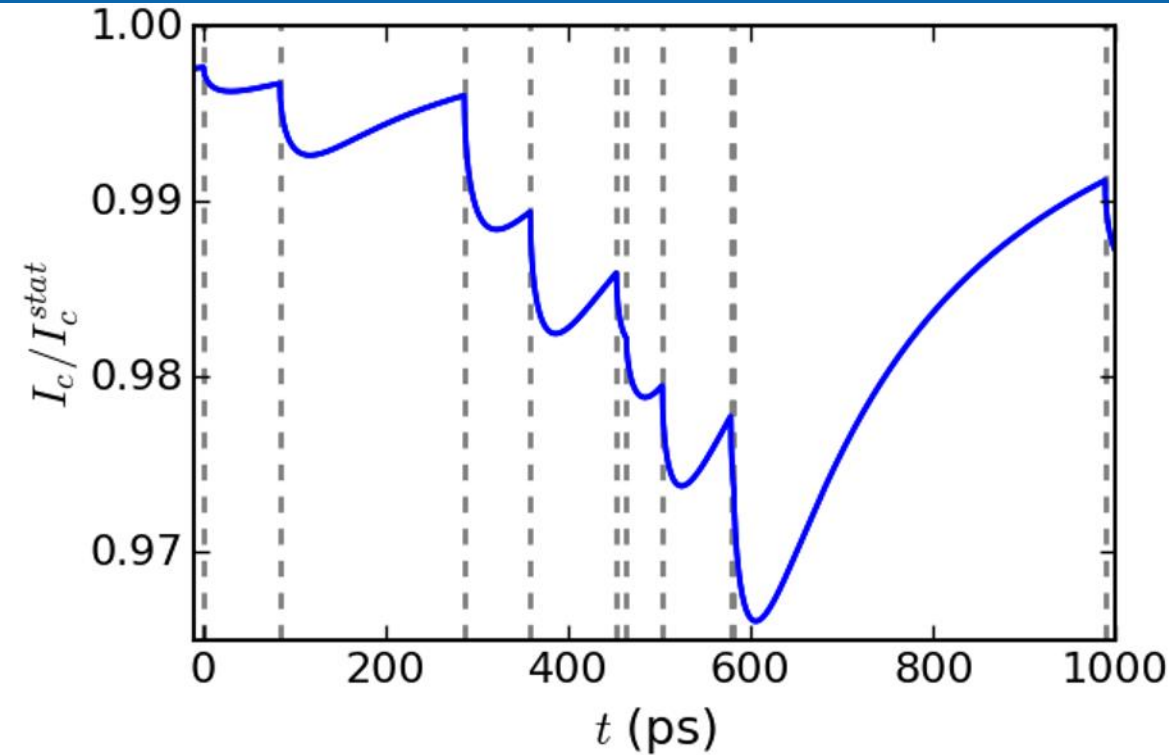
Hot spot radius : $A(t) = 4 \sqrt{D t \ln(M(t))}$

$$\delta N_{slab} = \delta N_c \Rightarrow I_c = I_{c0} \left(1 - \frac{\tau_{inj} P_i (1 - e^{-\frac{t}{\tau}})}{N_0 \Delta^2 w d \sqrt{\pi D t}}\right)$$

$$P_i = I_t V_i \quad \tau_{inj} = \frac{e}{I_t}$$

$$N_0 = 31 \cdot 10^{46} \text{ J}^{-1} \text{ m}^{-3} \quad \Delta = 370 \text{ } \mu\text{eV} \quad \xi = 35 \text{ nm} \quad w = 300 \text{ nm} \quad d = 10 \text{ nm} \quad D = 6.8 \cdot 10^{-4} \text{ m}^2 \text{ s}^{-1}$$

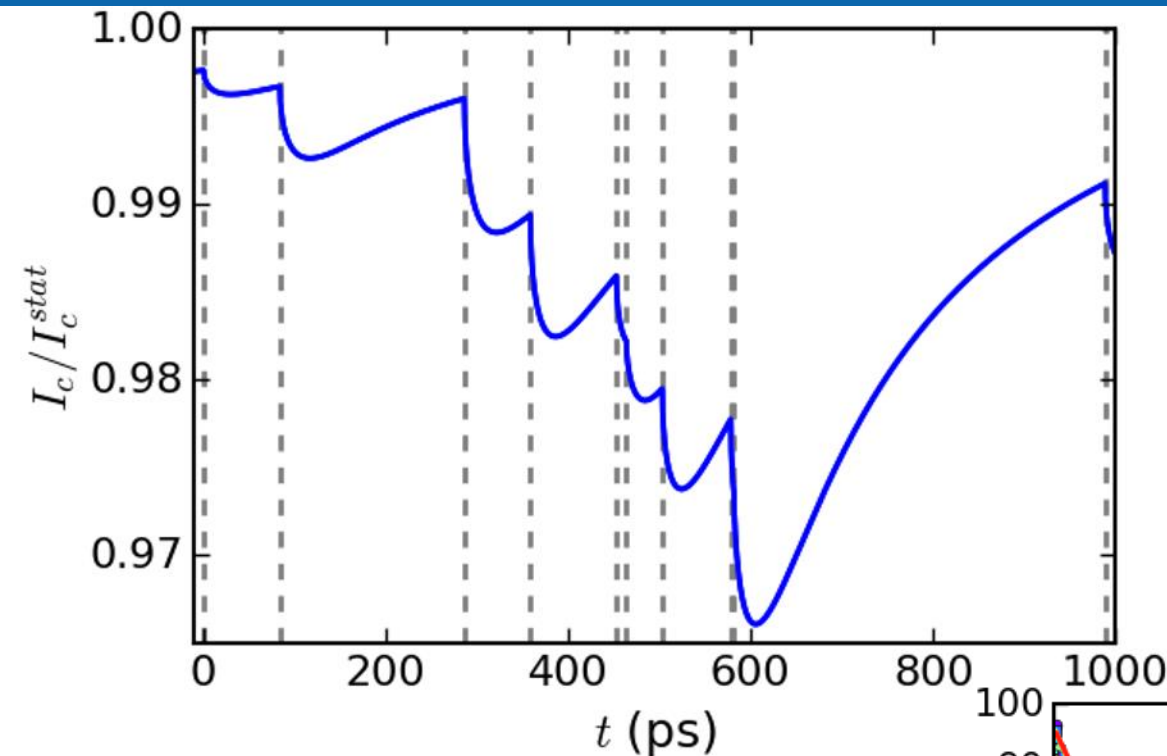
Numerical simulation of the hot spot dynamics



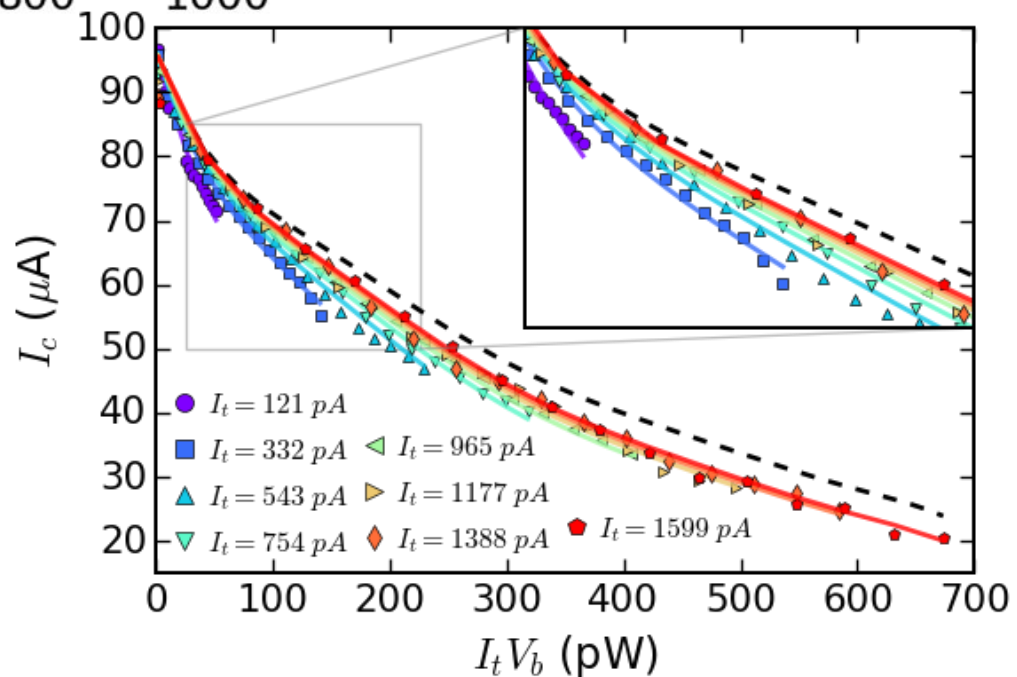
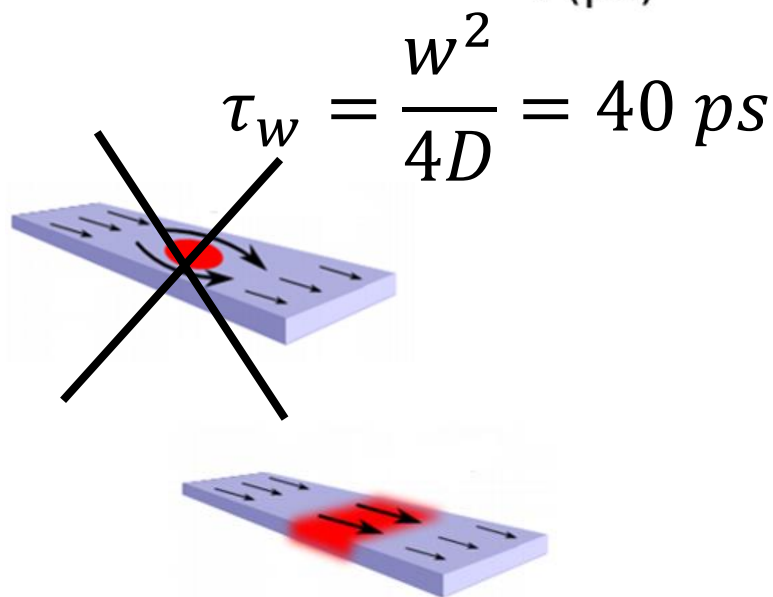
$$\delta N_{slab} = \delta N_c \Rightarrow I_c = I_{c0} \left(1 - \frac{\tau_{inj} P_i (1 - e^{-\frac{t}{\tau}})}{N_0 \Delta^2 w d \sqrt{\pi D t}} \right)$$

$$N_0 = 31 \cdot 10^{46} \text{ J}^{-1} \text{ m}^{-3} \quad \Delta = 370 \text{ } \mu\text{eV} \quad \xi = 35 \text{ nm} \quad w = 300 \text{ nm} \quad d = 10 \text{ nm} \quad D = 6.8 \cdot 10^{-4} \text{ m}^2 \text{ s}^{-1}$$

Numerical simulation of the hot spot dynamics



$$\tau = 40 \text{ ps}$$



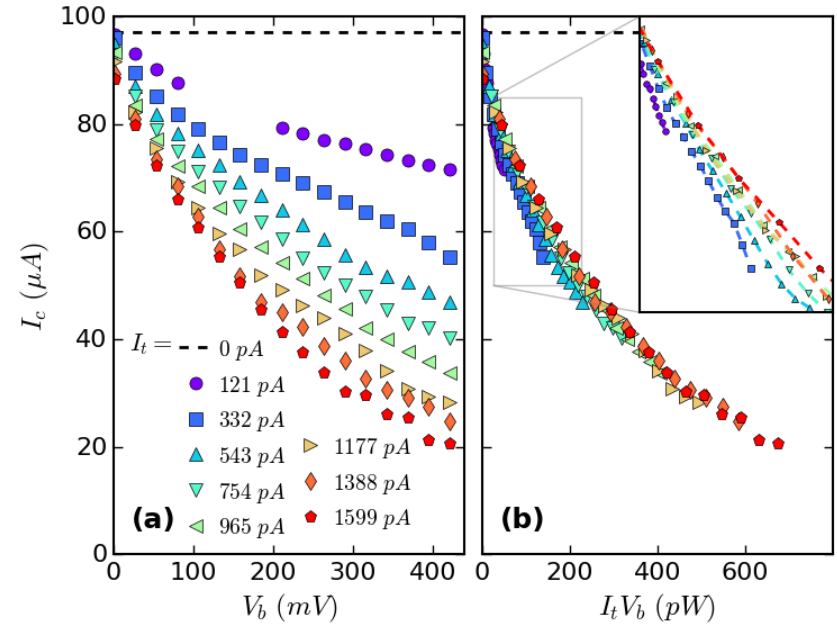
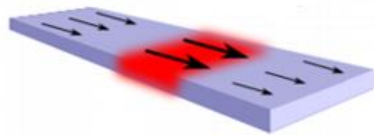
High bias injection conclusion

➤ $\frac{I_c}{I_t} \sim 10^6$

➤ Thermal effect

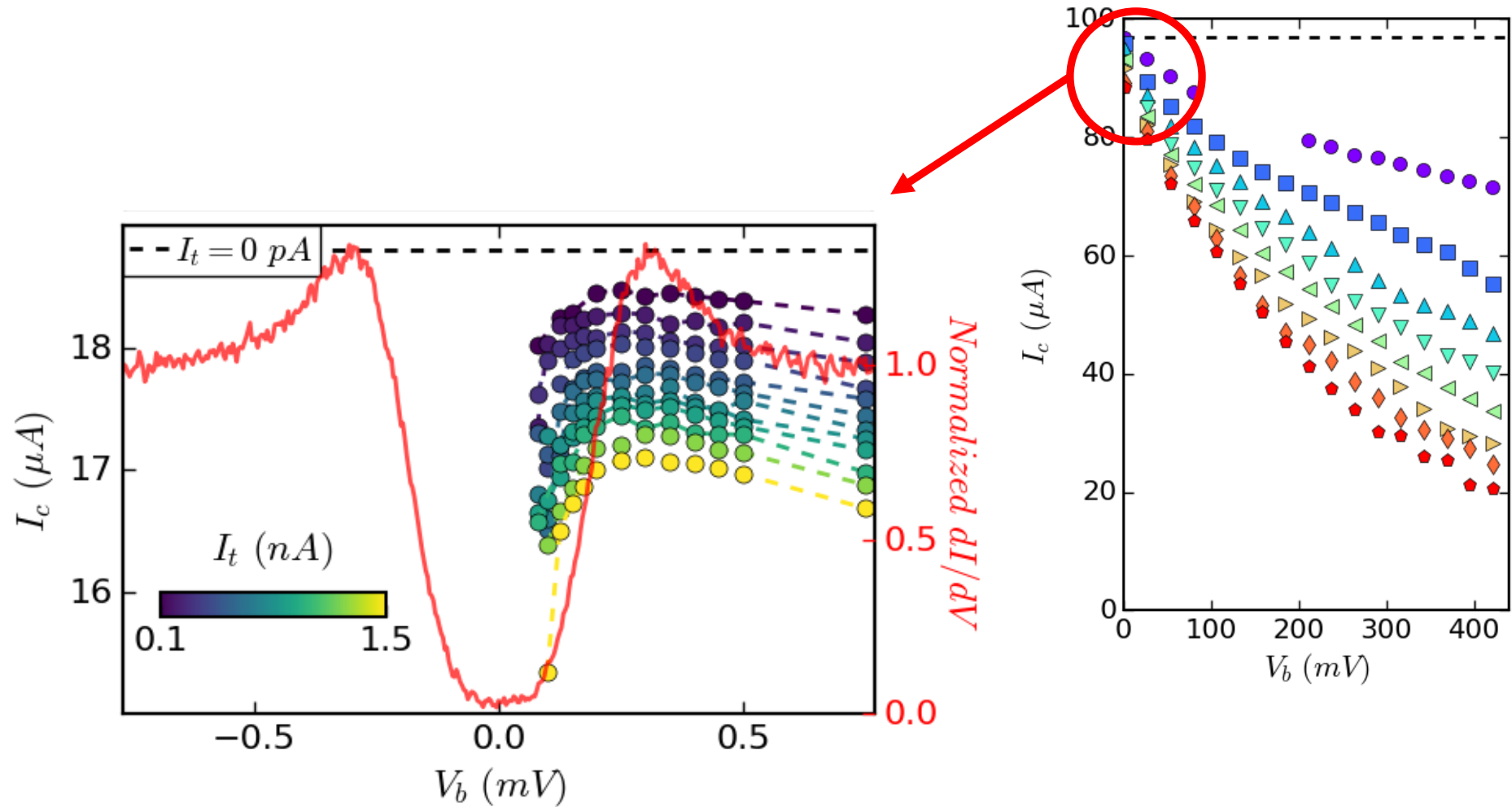
➤ Quasiparticle dynamics

➤ $\tau \sim 40 \text{ ps}$



T. Jalabert, et al., Nat. Phys. (2023)

Low bias injection



- Non-thermalized quasiparticles :
- No electronic temperature
 - Non-Fermi-Dirac distribution function

$$\frac{1}{N_0 V_{eff}} = \int_0^{\omega_D} dE \frac{1 - 2f(E)}{\sqrt{E^2 - \Delta^2}}$$

Low bias injection

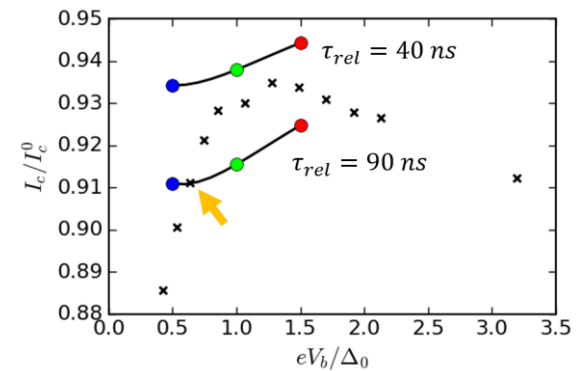
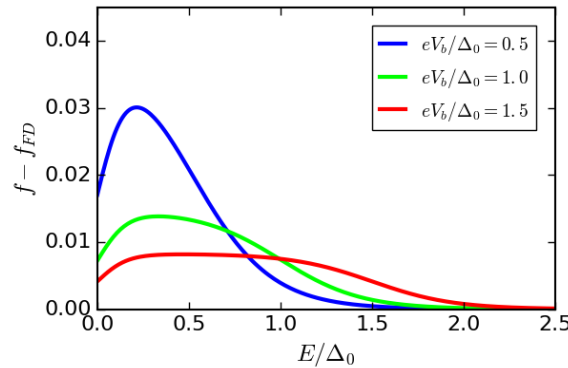
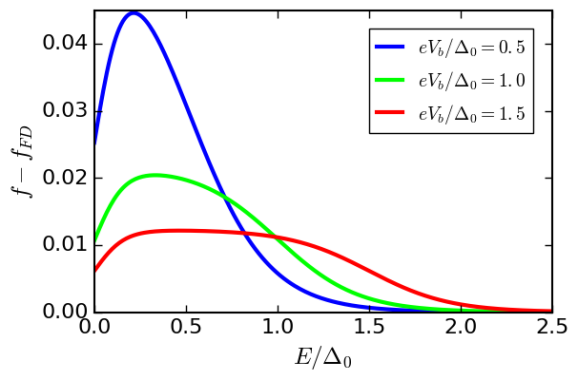
$$I_t(V_b) = \frac{1}{eR_T} \int dE N_S(E) (f_S(E) - f_{tip}(E - eV_b))$$

$$\frac{1}{eR_T} N_S(E) (f_S(E) - f_{tip}(E - eV_b)) = -e\Omega N_0 N_S(E) \frac{f_S(E) - f_{FD}(E)}{\tau_{rec}}$$

$$f_S(E) = \frac{\Gamma f_{tip}(E - eV_b) + f_{FD}(E)}{\Gamma + 1}$$

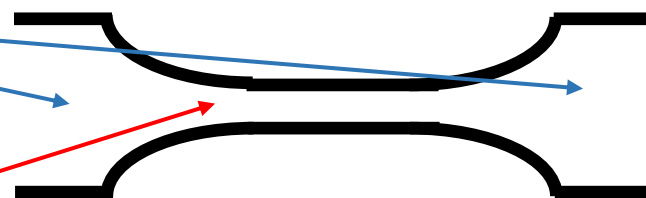
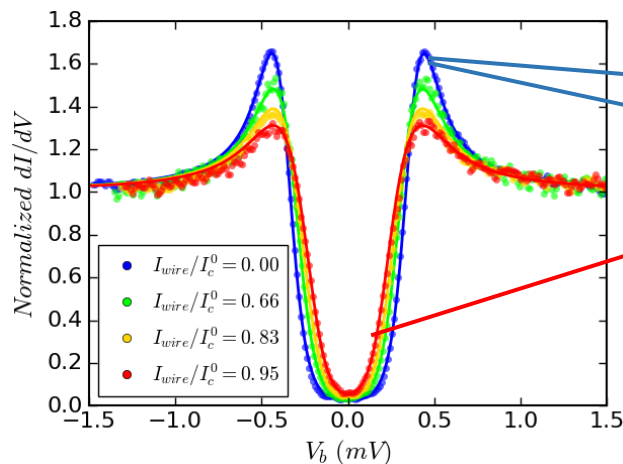
$$\Gamma = \frac{\tau_{rec}}{e^2 R_T \Omega N_0}$$

$$\Omega = \sqrt{D\tau_{rec}} wd$$



At the spectral gap $eV_b \sim 0.45 \Delta_0$

$\tau_{rec} = 100 - 600$ ns

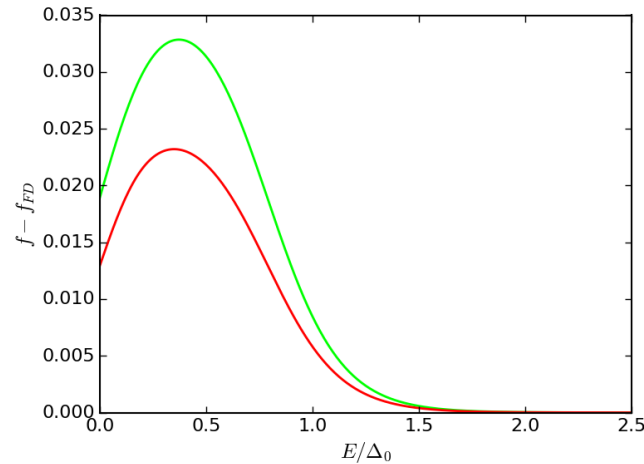
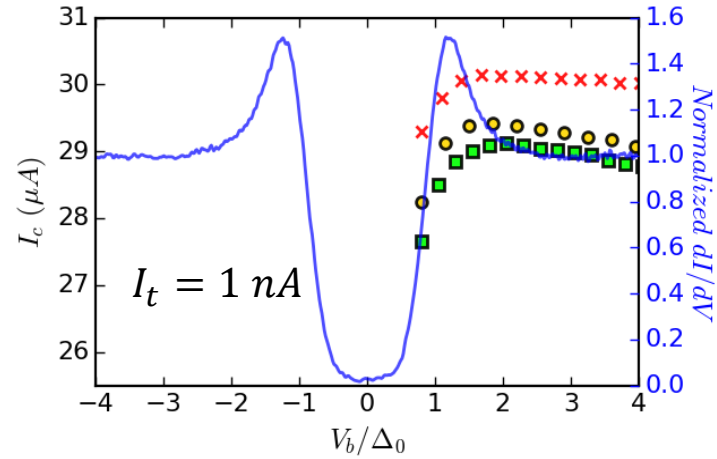
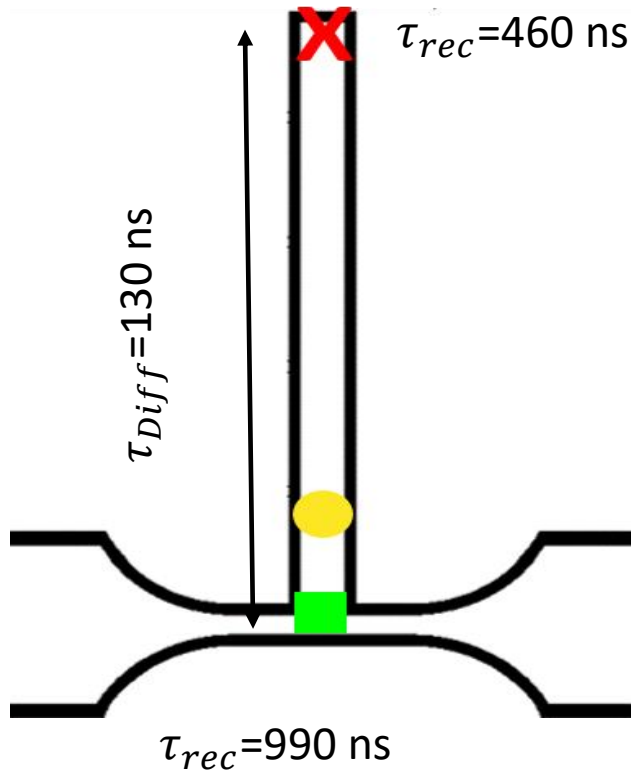


-Quasiparticles trapping

$$\Omega = Lwd$$

$\tau_{rec} = 60 - 90$ ns

Low bias injection

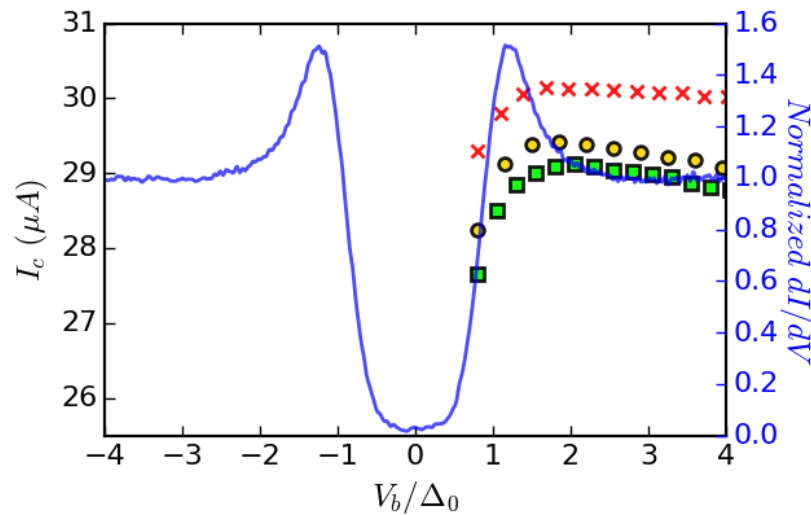


- Phonon trapping

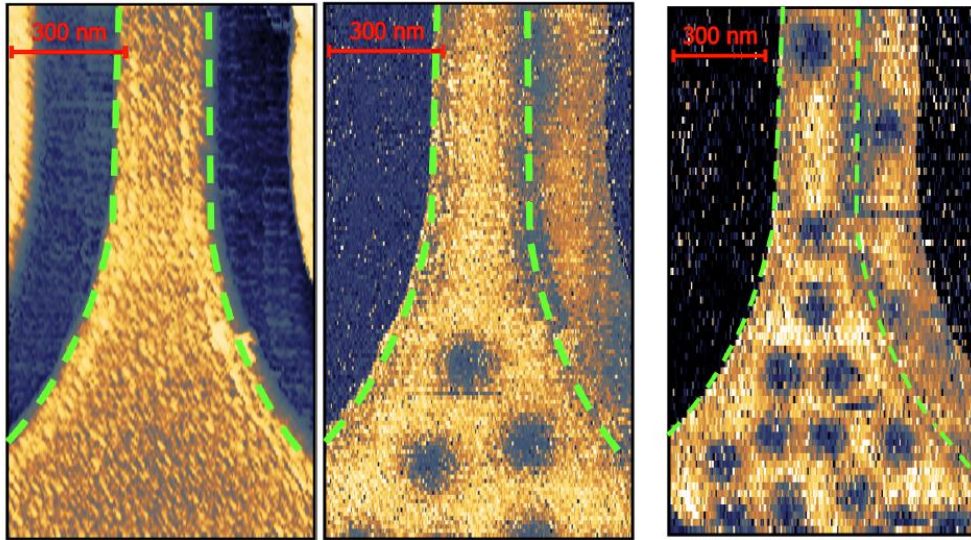
$$\tau'_{rec} = \tau_{rec} \left(1 + \frac{\tau_{esc}}{\tau_{break}} \right)^2$$

A. Rothwarf and N.B. Taylor, Phys. Rev. Lett. (1967)

- Non-thermal quasi-particles
- Distribution function effect
- $\tau_{rec} \sim 100 - 500 \text{ ns}$



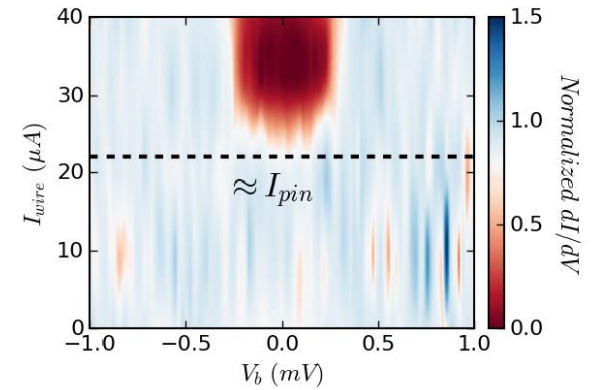
Scanning Critical Current Microscopy Outlook



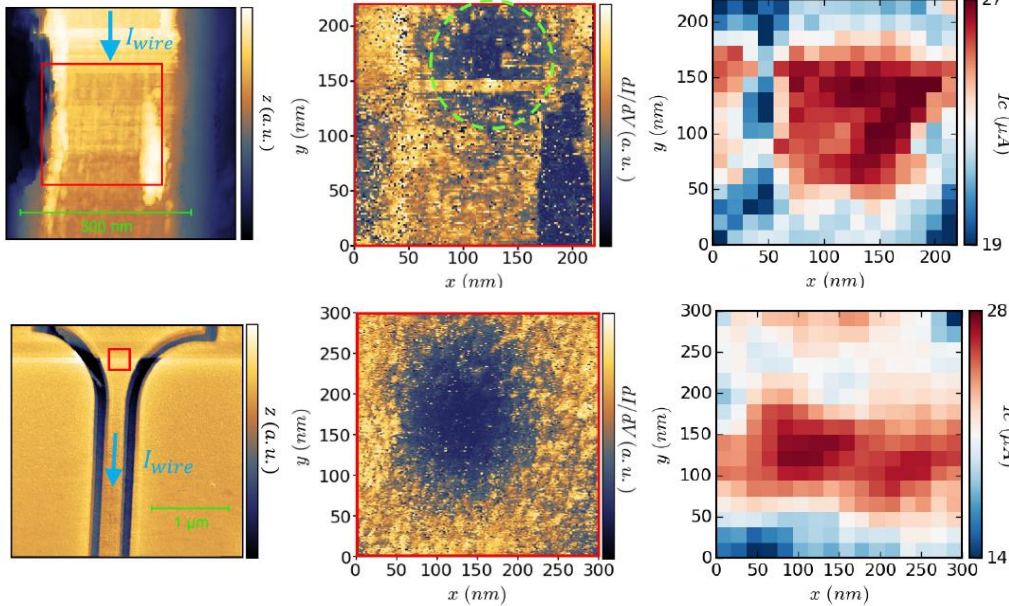
(a) B=23 mT

(b) B=23 mT

(c) B=34 mT



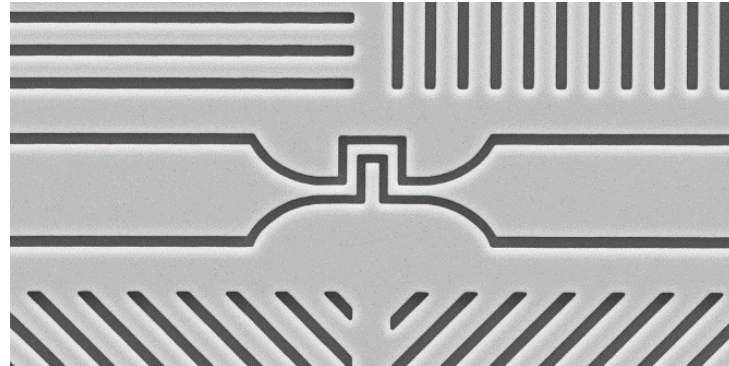
Vortex pinning



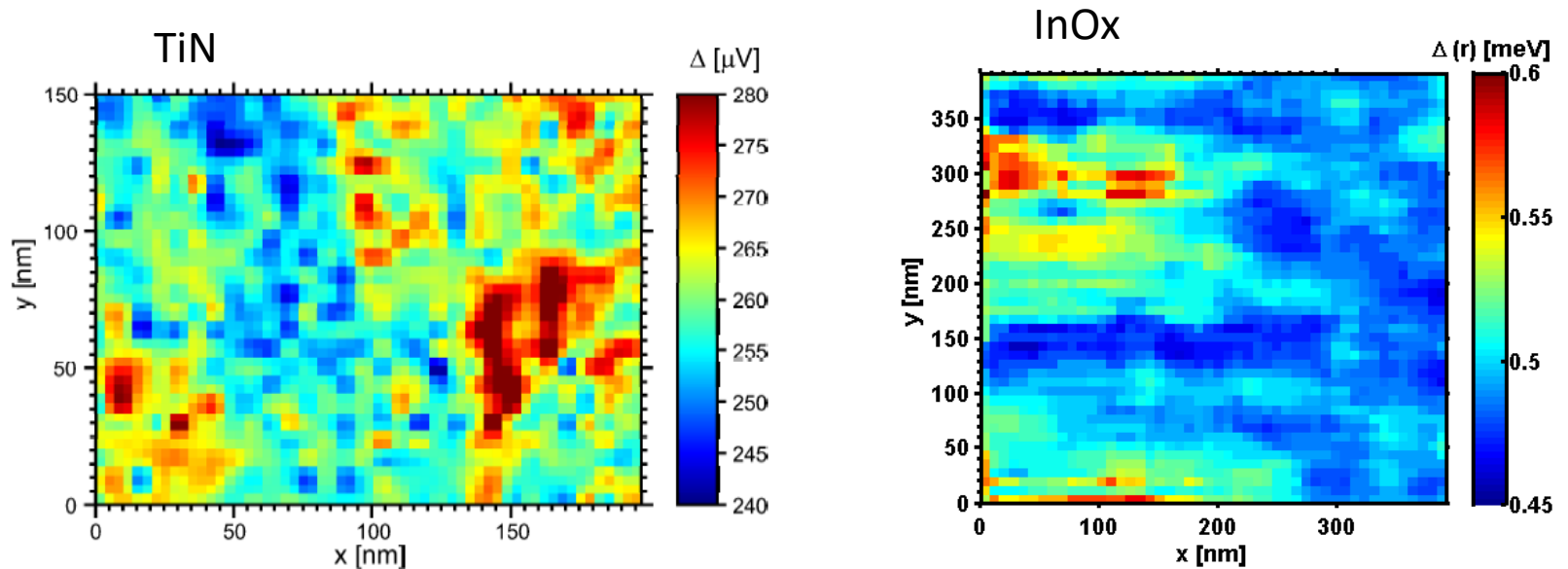
Quasi-particles trapping in a vortex

Scanning Critical Current Microscopy Outlook

Current crowding



Highly disordered superconductors



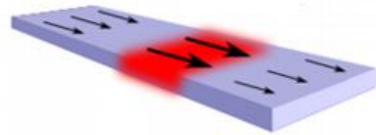
Conclusion

➤ $\frac{I_c}{I_t} \sim 10^6$

➤ Thermal effect

➤ Quasiparticle dynamics

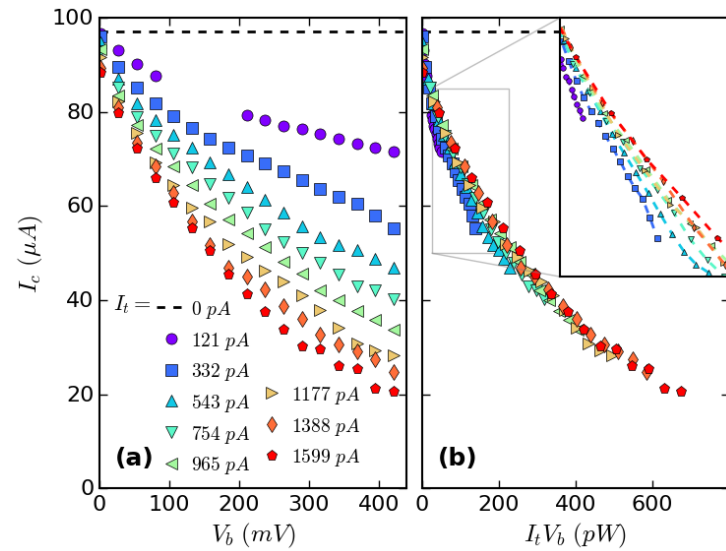
➤ $\tau \sim 40 \text{ ps}$



➤ Non-thermal quasi-particles

➤ Distribution function effect

➤ $\tau_{rec} \sim 100 - 500 \text{ ns}$



T. Jalabert, et al., Nat. Phys. (2023)

