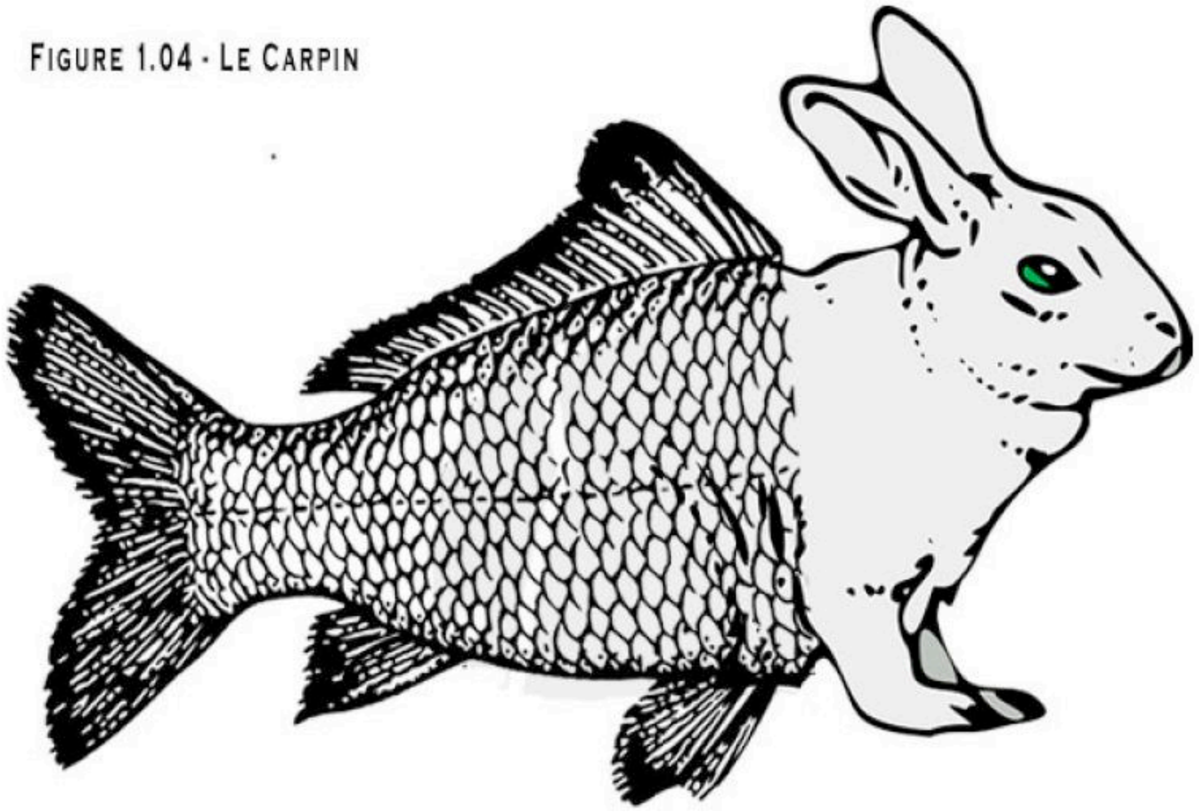
Quantum Optics Aspects of Inelastic Tunneling with a GrAl Resonator

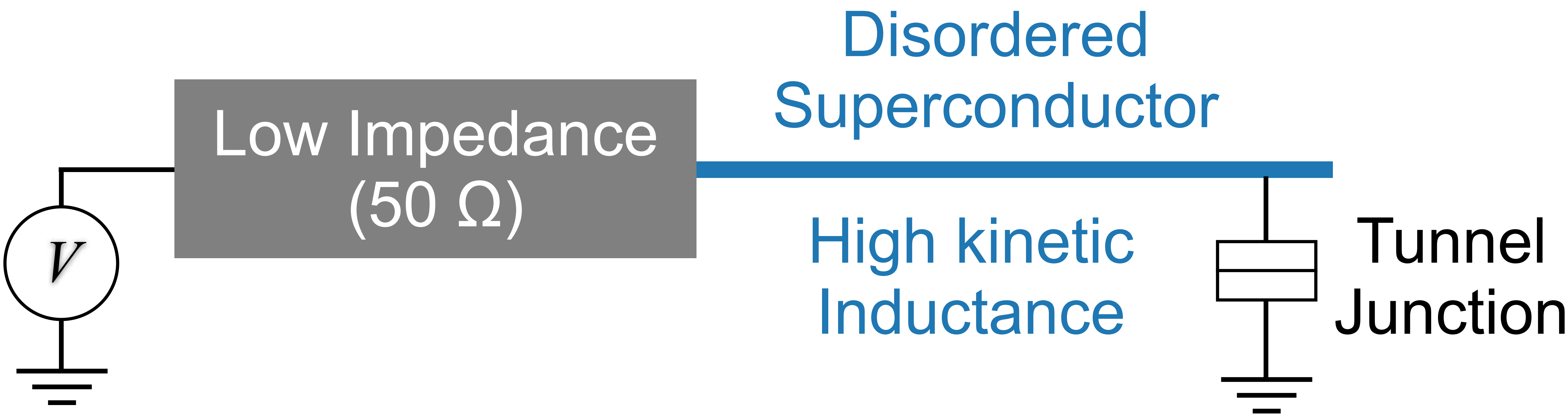


Gianluca Aiello, Ognjen Stanisavljevic, Jean-Côme Philippe, Marco Aprili, Julien Gabelli, Julien Basset & Jérôme Estève

NS2 Group, Laboratoire de Physique des Solides, Orsay

Quandi Workshop - Les Houches - 06/06/2023

Experimental Situation

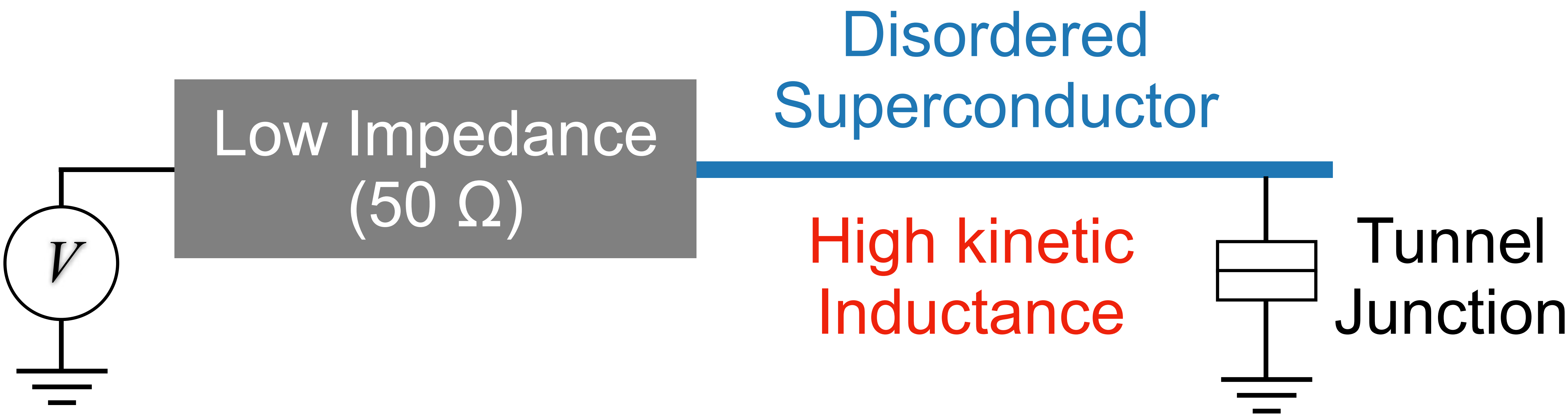


How is charge tunneling modified when:
- fine structure constant becomes large
- speed of light becomes low

1. Quasiparticle tunneling
2. Cooper pair tunneling
3. Large disorder limit

What about the photons?

Experimental Situation



- Large Fine structure constant:

$$Z_c = \sqrt{L/C} \approx 5\,\mathrm{k\Omega} \qquad \alpha = Z/2R_K \approx 0.1$$

- Light is slow:

$$v = 1/\sqrt{LC} \approx c/60$$

Experimental Situation

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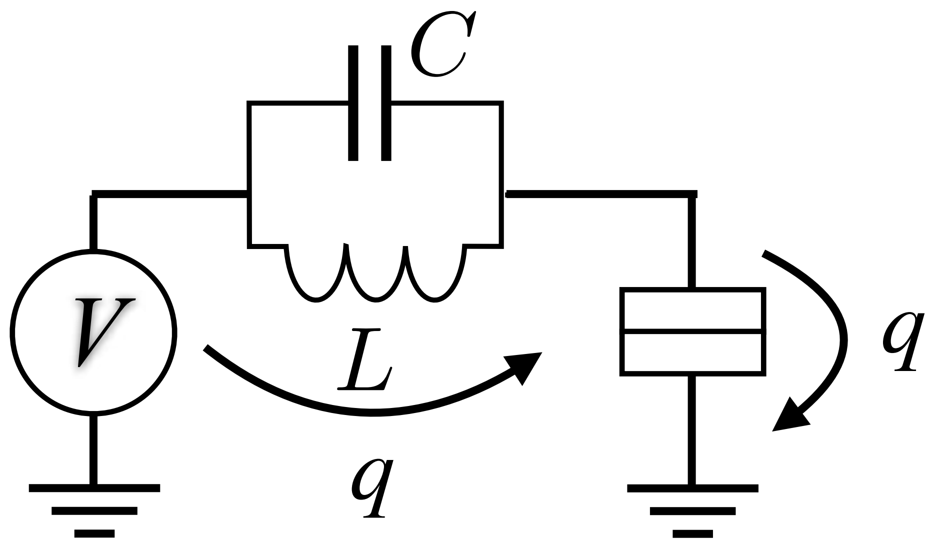
$$v = 1/\sqrt{LC} \approx c/60$$

Many modes highly coupled to the junction

Inelastic Tunneling

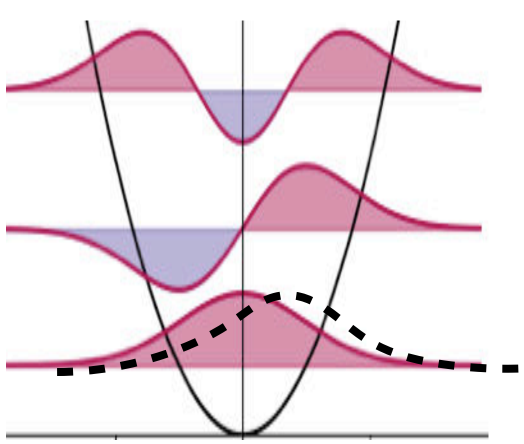
Toy model



Charge translation $e^{iq\phi/\hbar} = e^{i\Lambda(a+a^\dagger)}$

$$\Lambda = \frac{q\phi_{\mathrm{ZPF}}}{\hbar} = \sqrt{\pi \frac{Z_c}{R_K}}$$

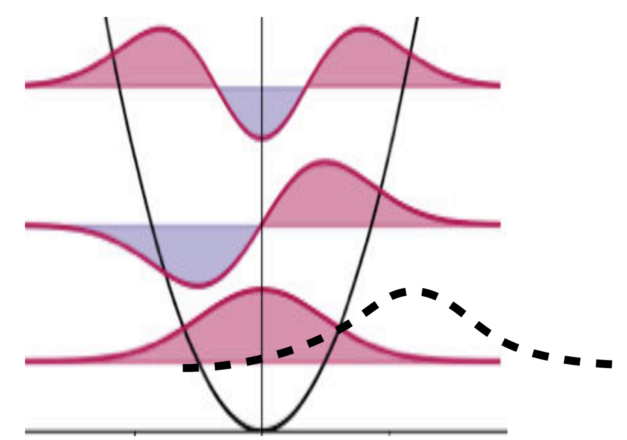
$\Lambda \ll 1$



$$|\langle 0 | e^{i\Lambda(a+a^\dagger)} | 0 \rangle|^2 \approx 1$$

Elastic tunneling prevails

$\Lambda \sim 1$

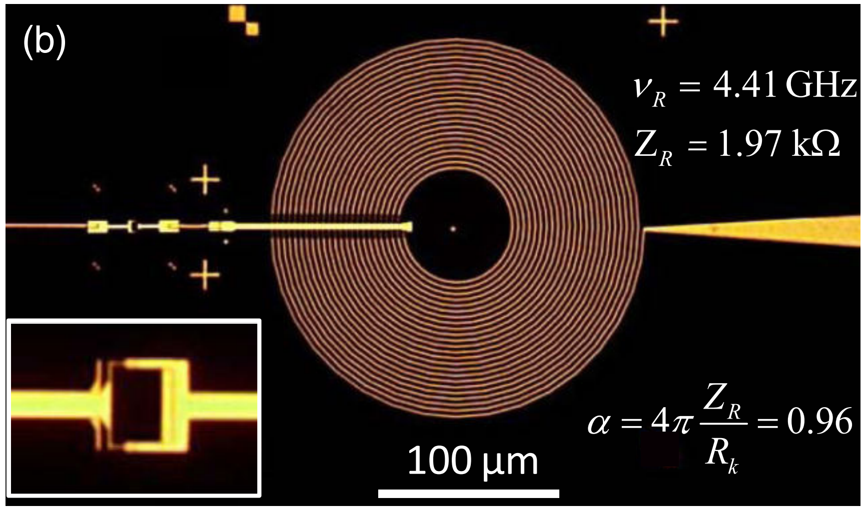


$$|\langle 0 | e^{i\Lambda(a+a^\dagger)} | 0 \rangle|^2 = e^{-\Lambda^2}$$

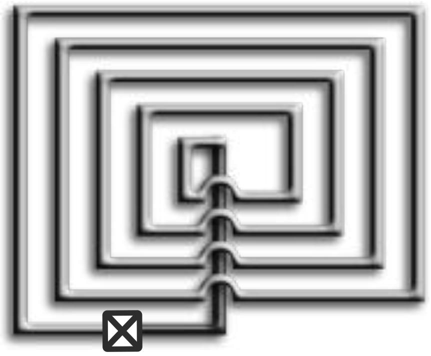
Inelastic tunneling prevails

High Impedance Resonator

- Geometric trick : spiral resonator



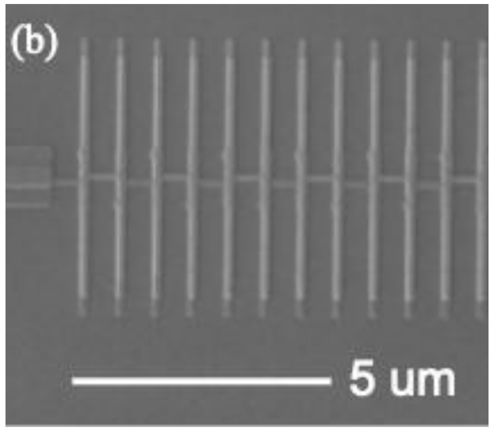
Rolland et al. Phys. Rev. Lett. 122, 186804 (2018)



Peruzzo et al. PRX Quantum **2**, 040341 (2021)

- Metamaterial with high inductance

JJ Chains

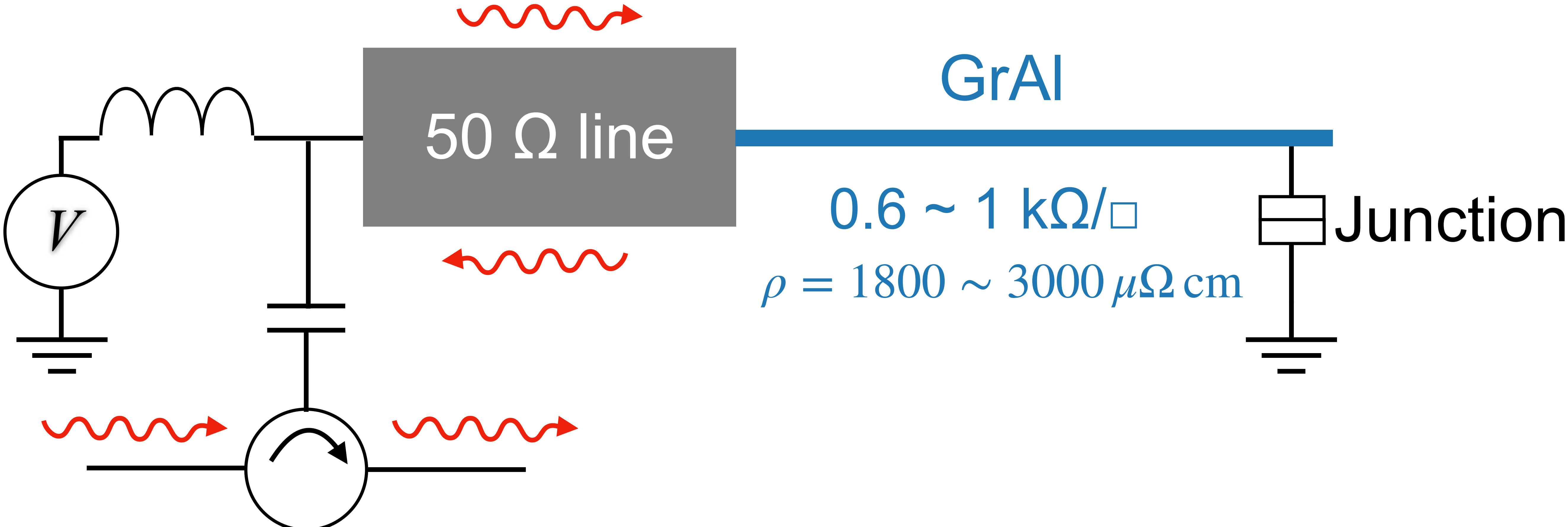


Masluk et al. Phys. Rev. Lett. **109**, 137002 (2012)

Disordered Superconductor

…

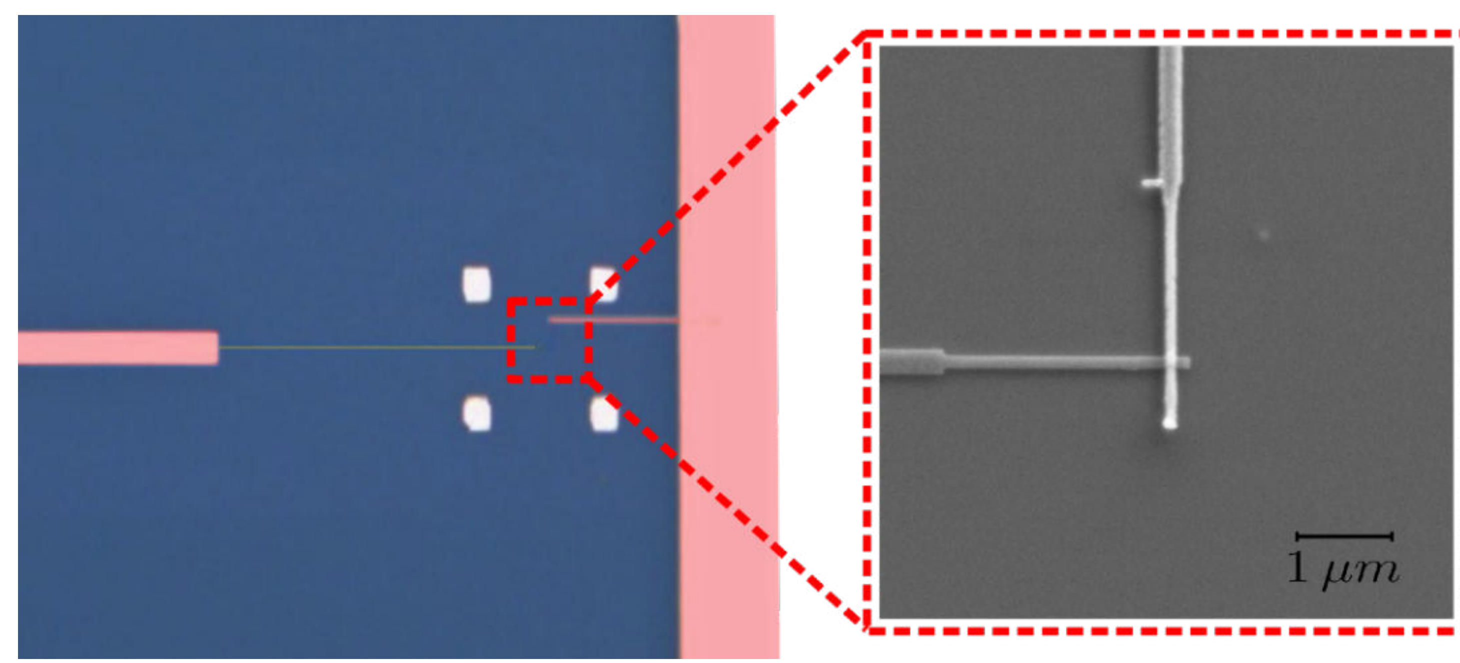
Experimental Setup



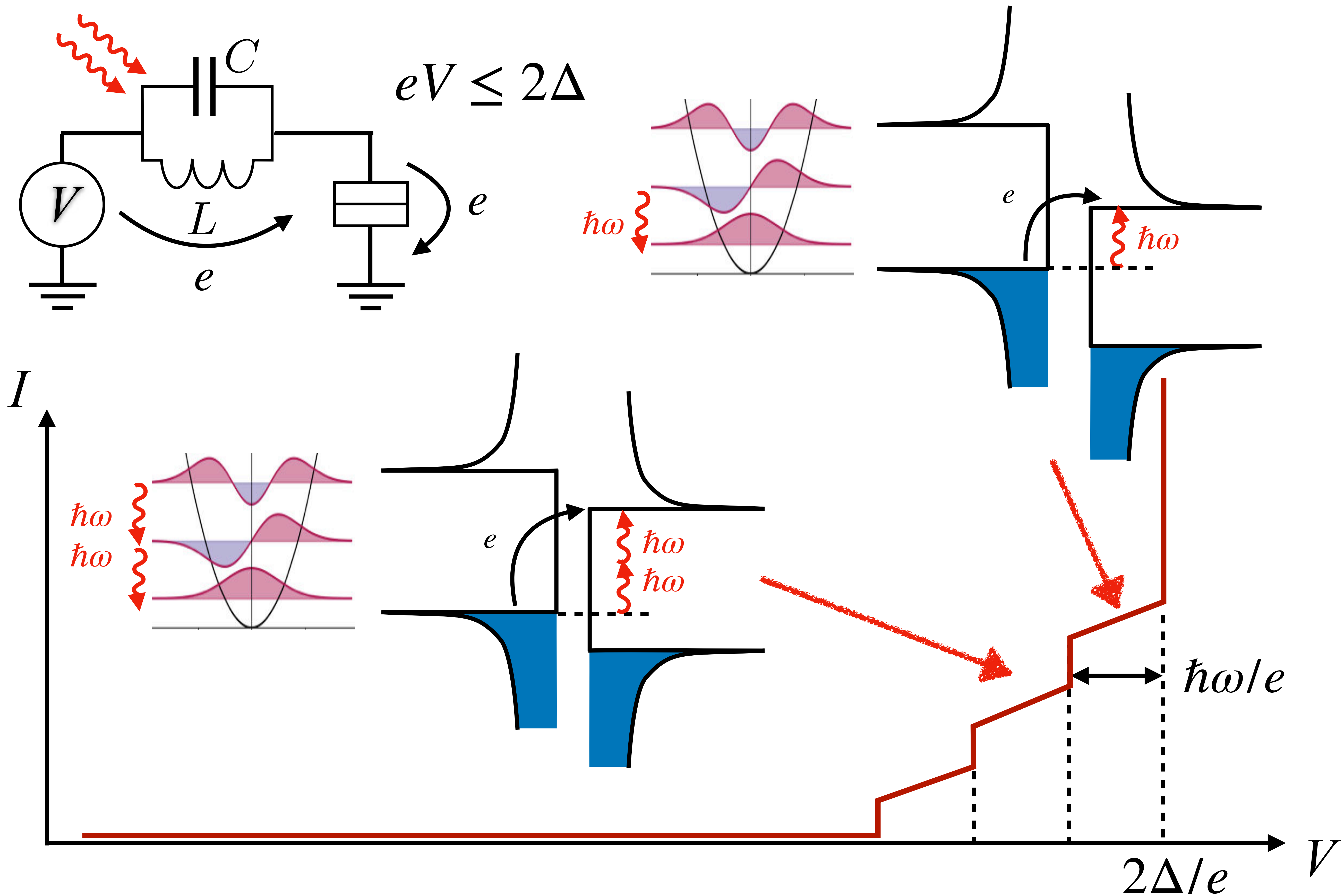
GrAl wire 300 nm x 30 nm

$Z_c \approx 5$ kΩ

$\lambda/4 = 200$ μm @ 6 GHz



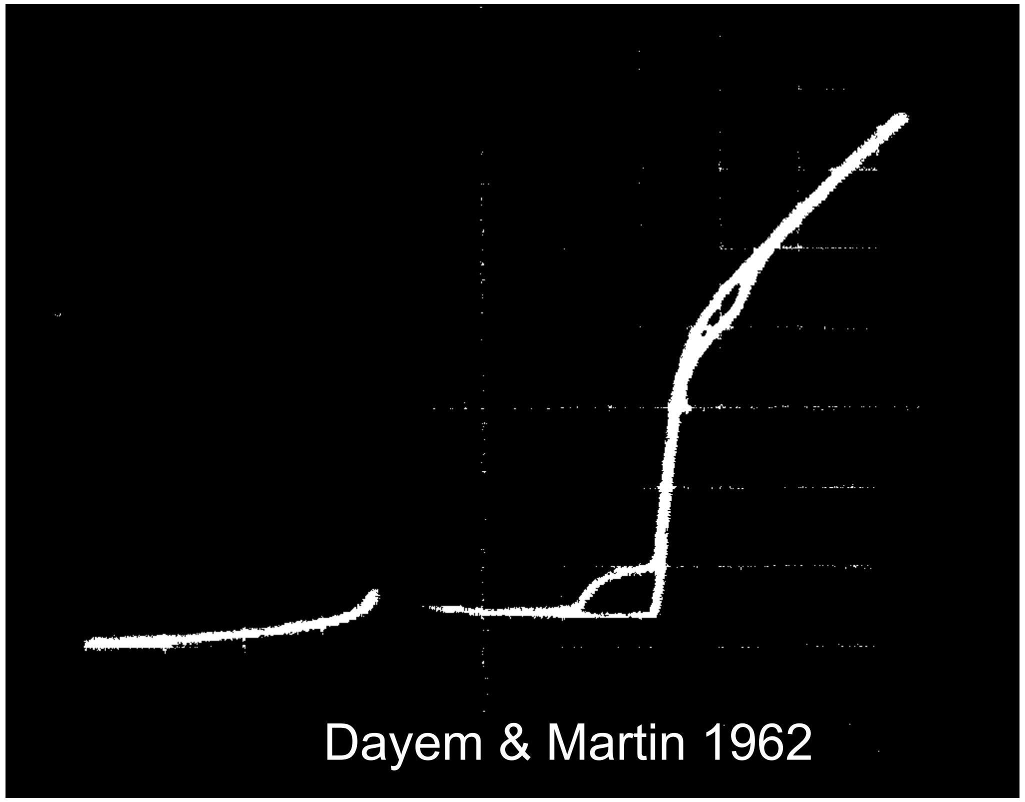
Photoassisted QP Tunneling

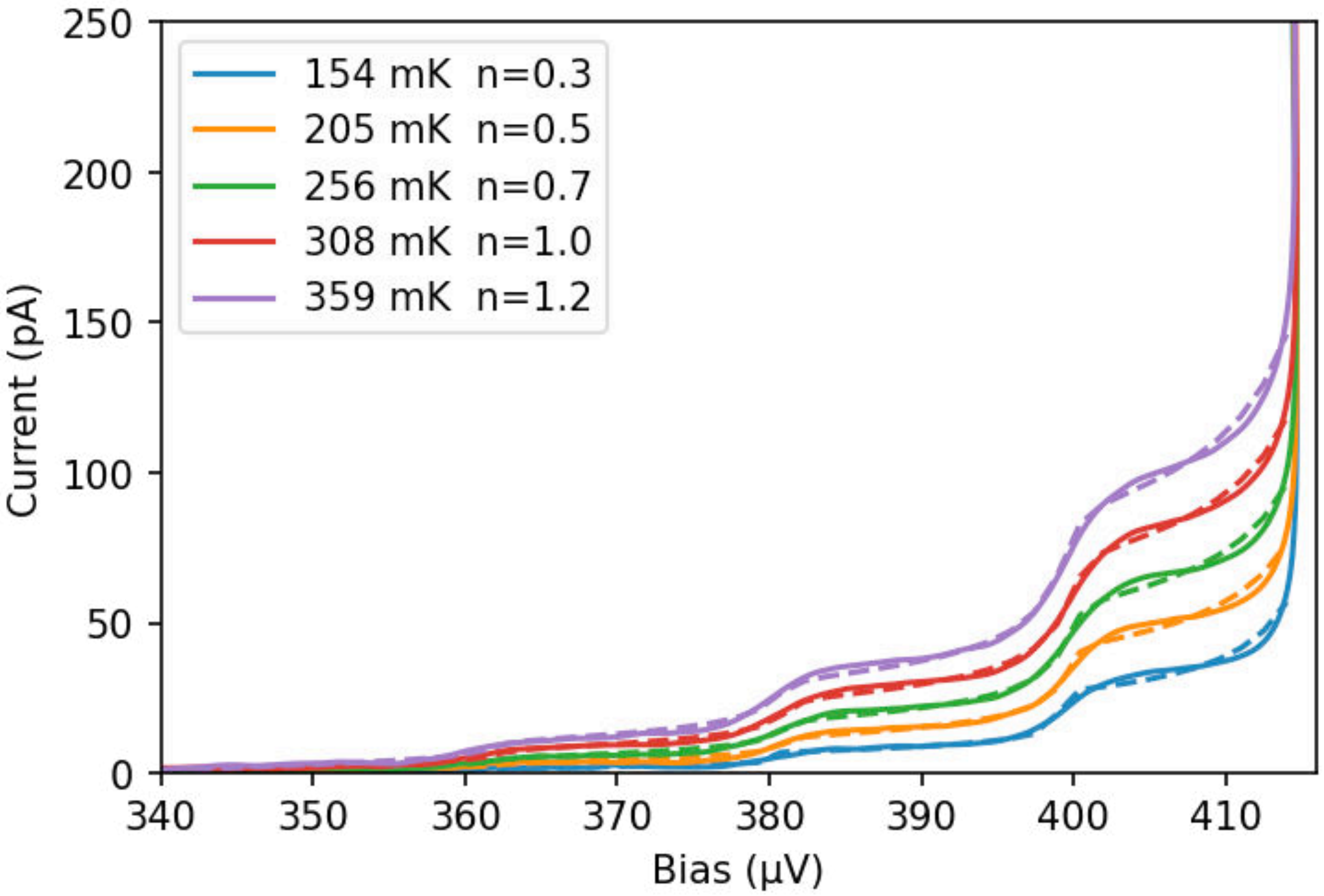


Photoassisted QP Tunneling

Classical Regime

Quantum Regime



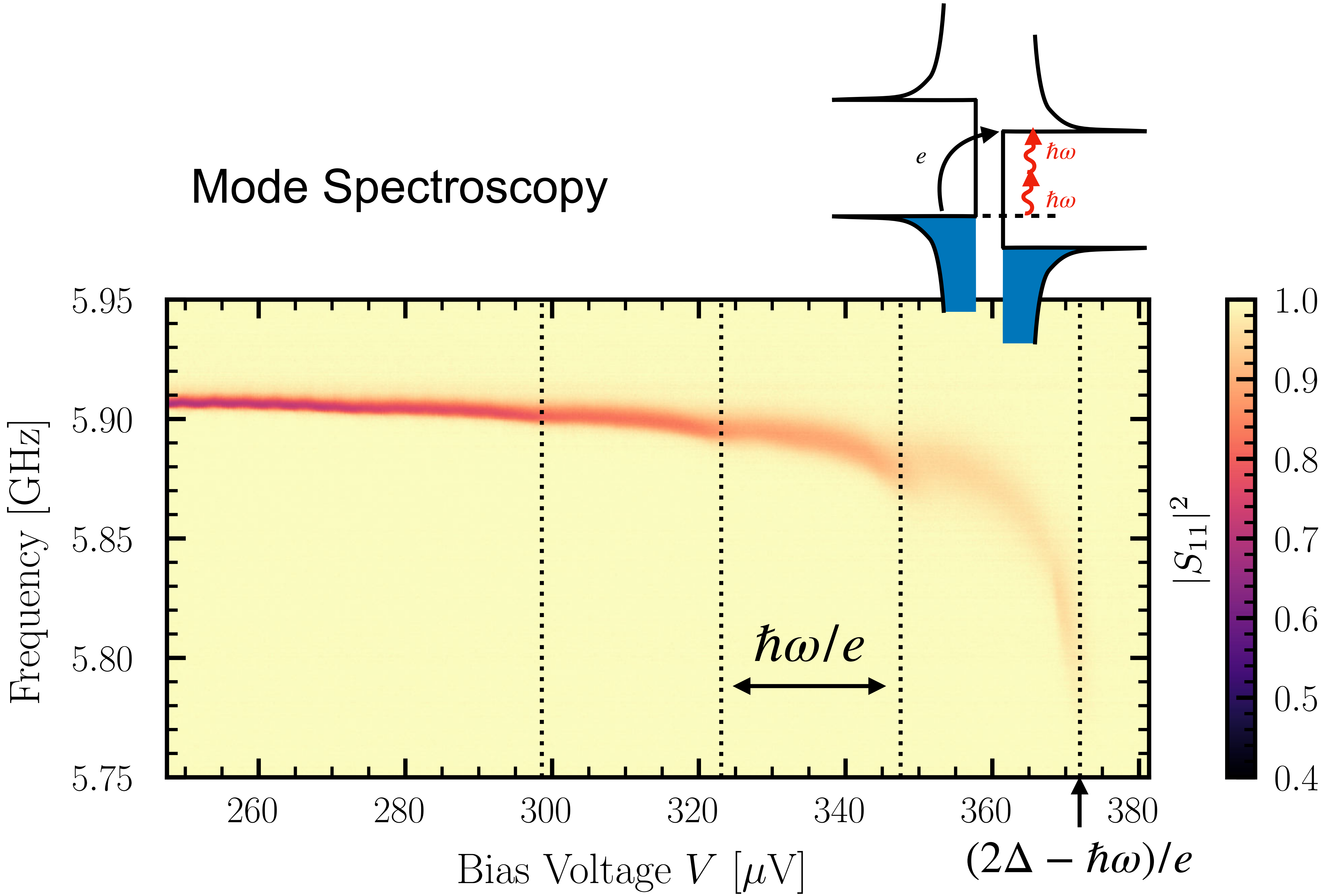


Low Impedance

High Impedance

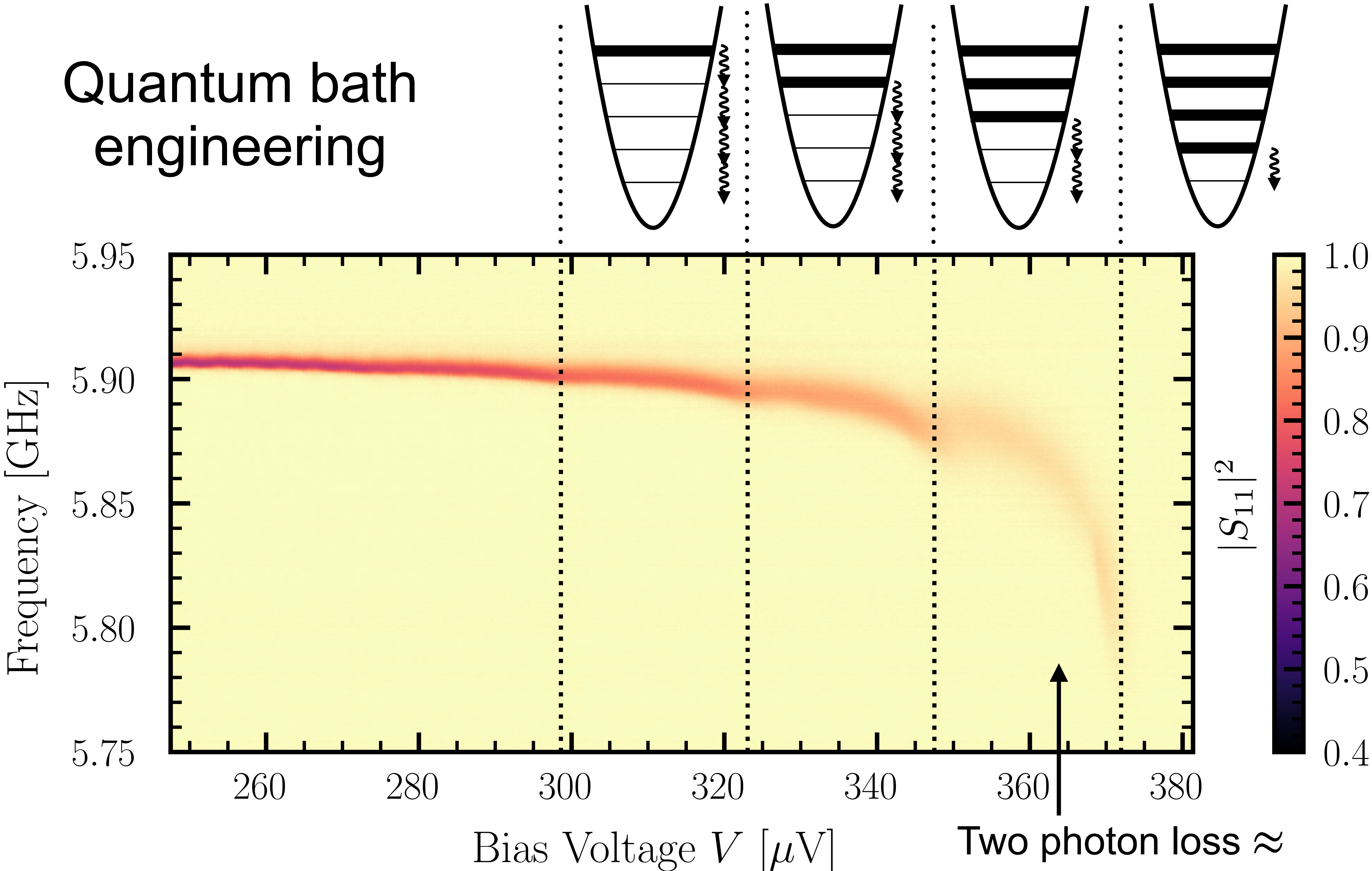
Photoassisted QP tunneling

Mode Spectroscopy



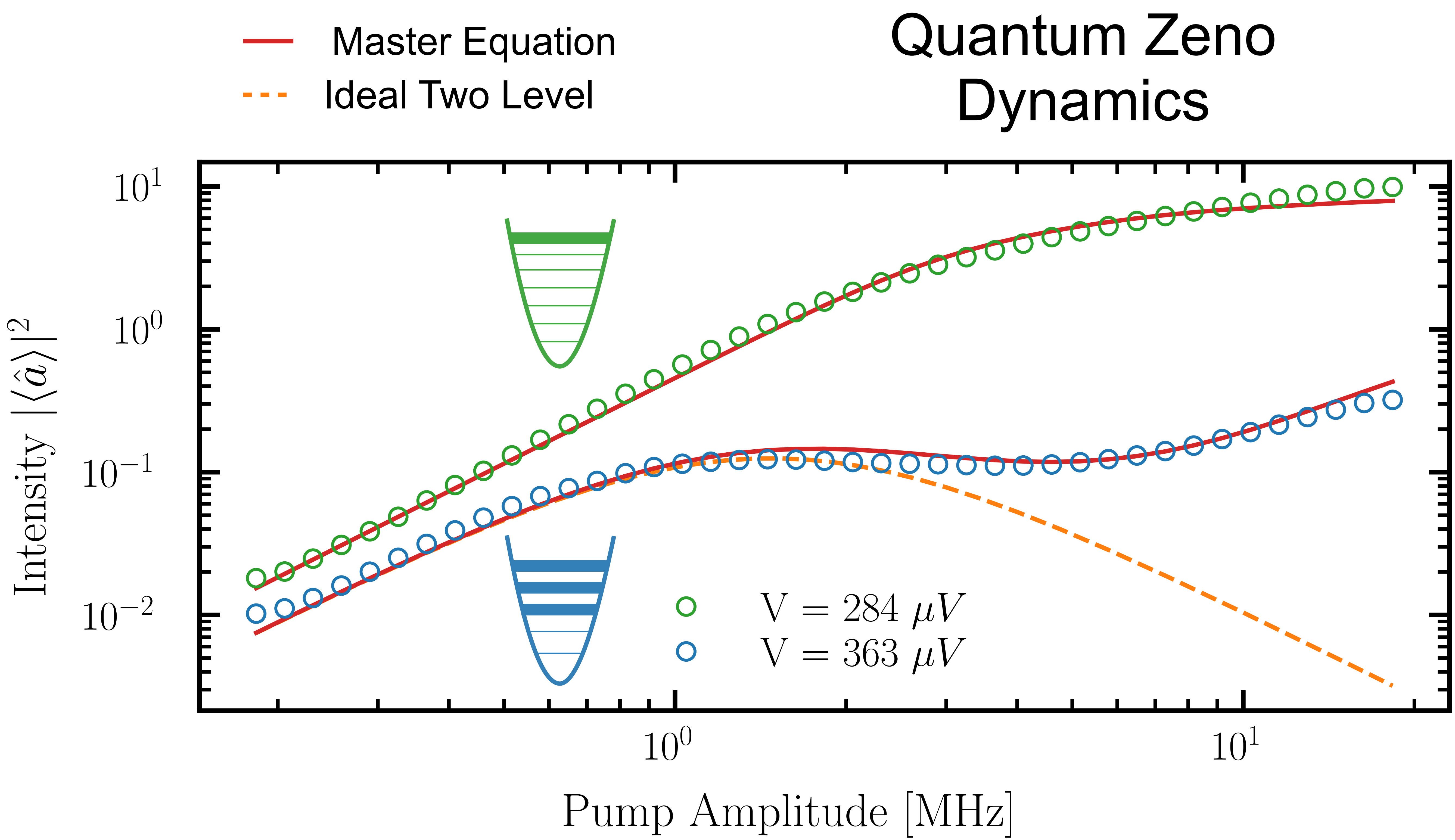
Engineered *n*-photon Losses

Quantum bath engineering

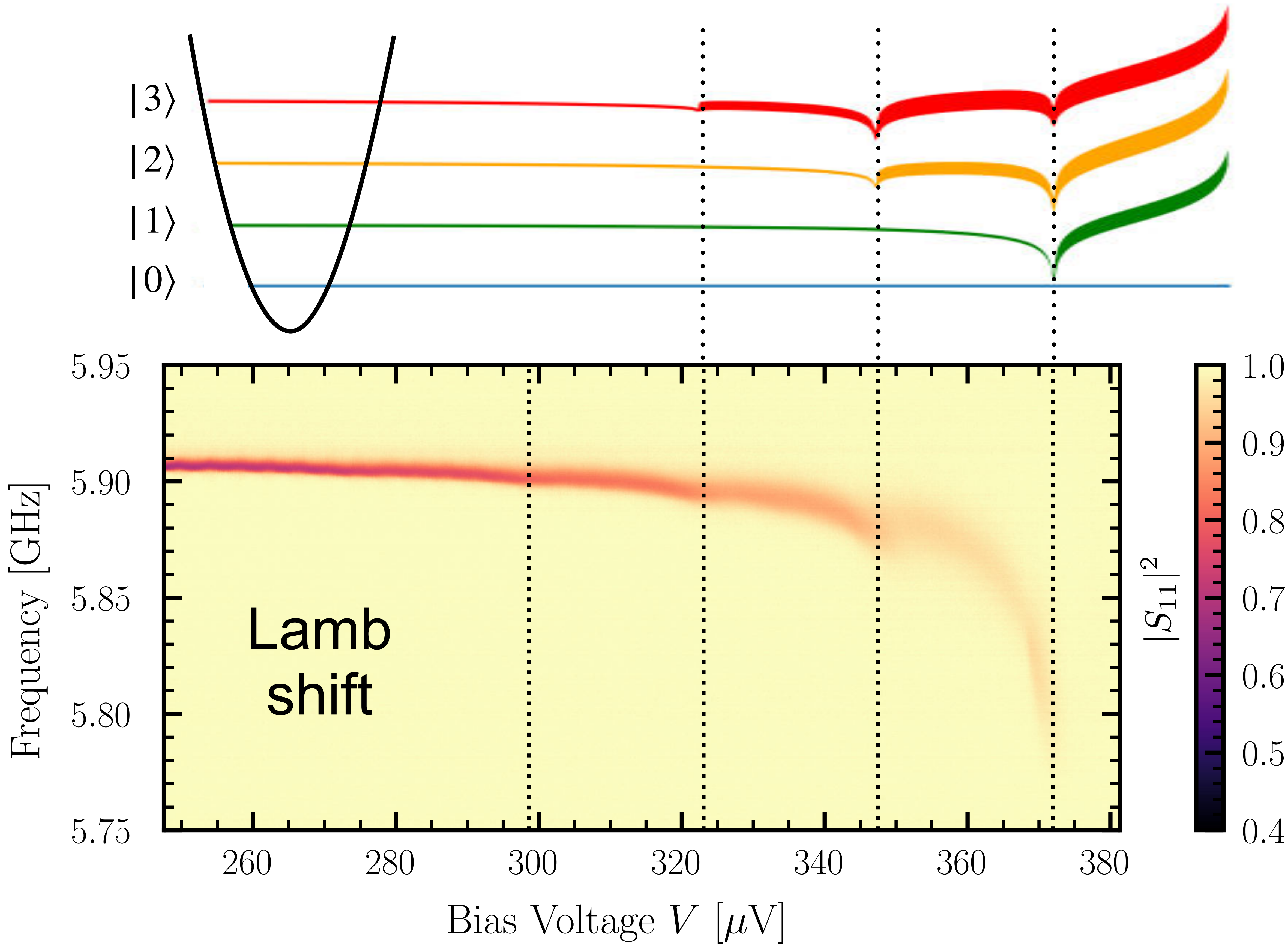


Two photon loss ≈ 25 x one photon loss

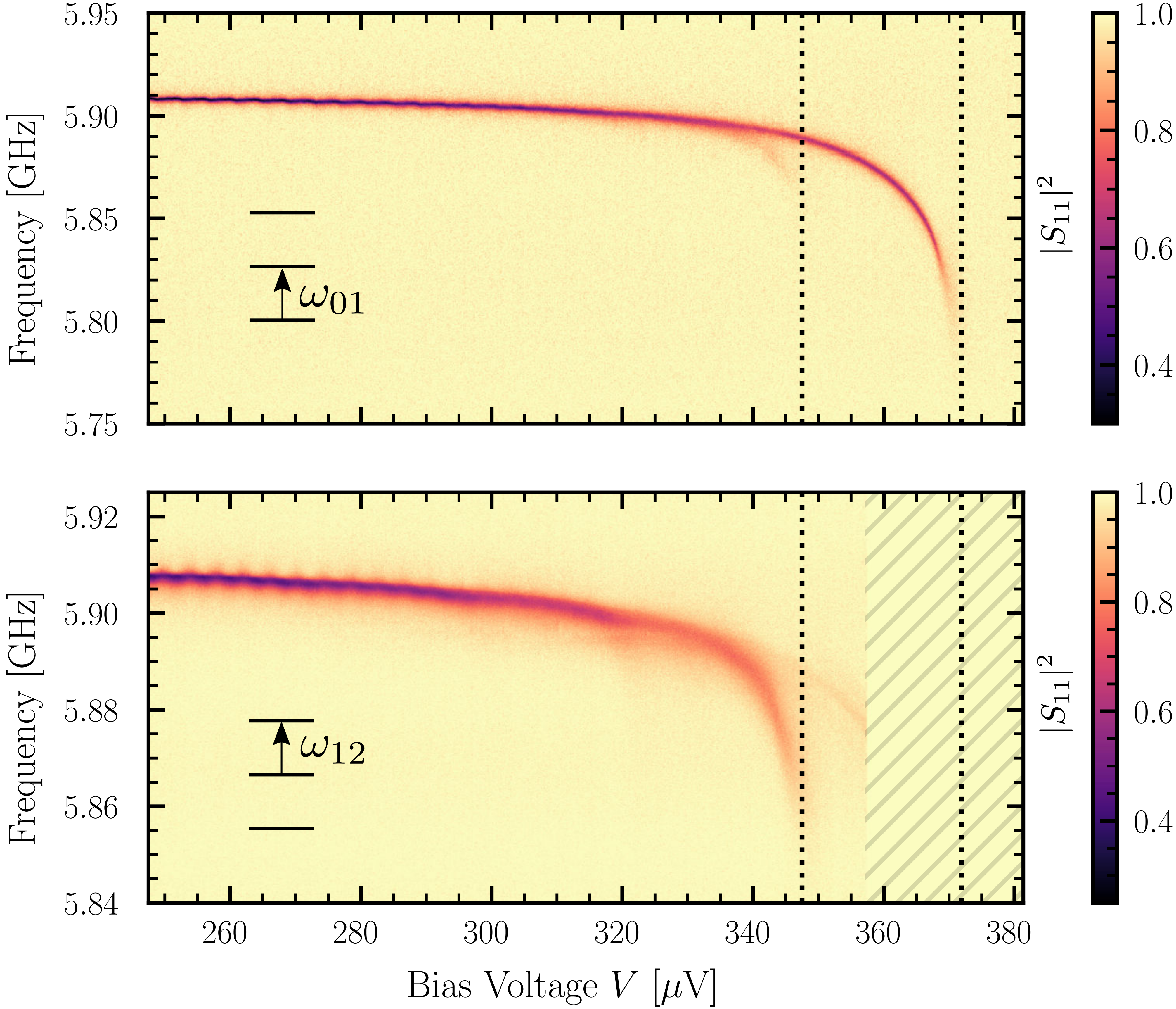
Effective Two-Level System



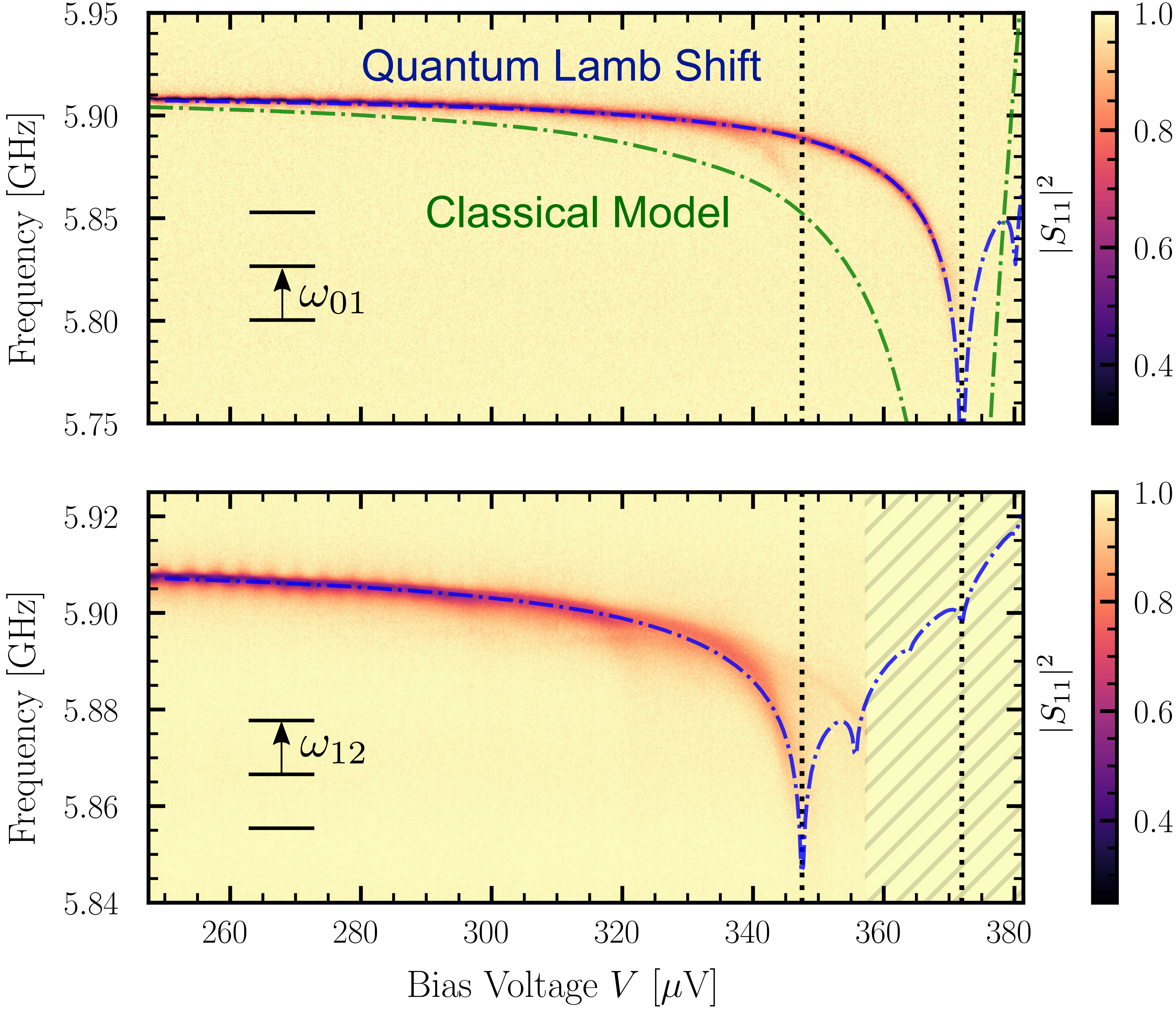
Kramers-Kronig for a single quantum state



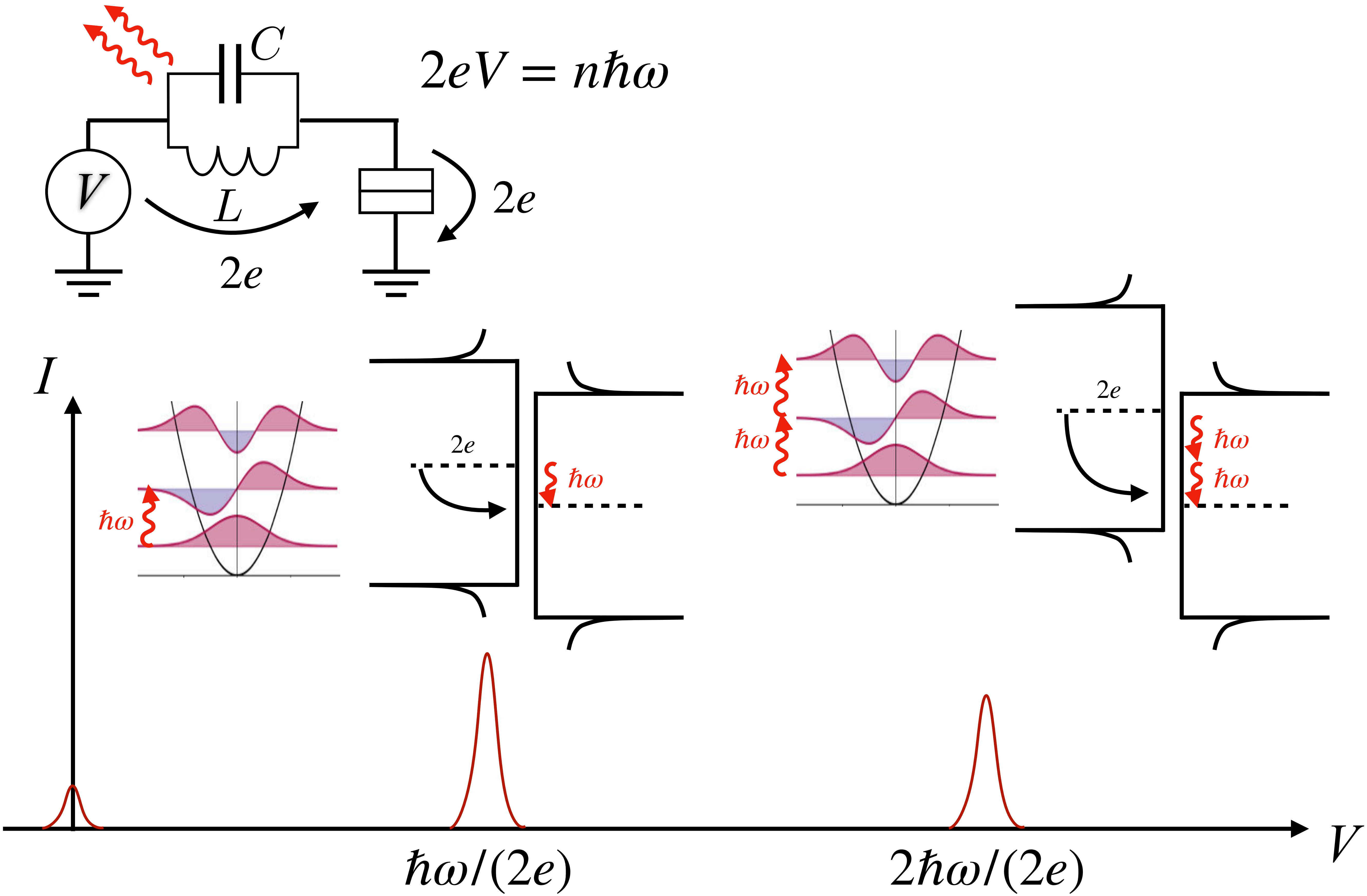
Lamb Shift Spectroscopy



Lamb Shift Spectroscopy



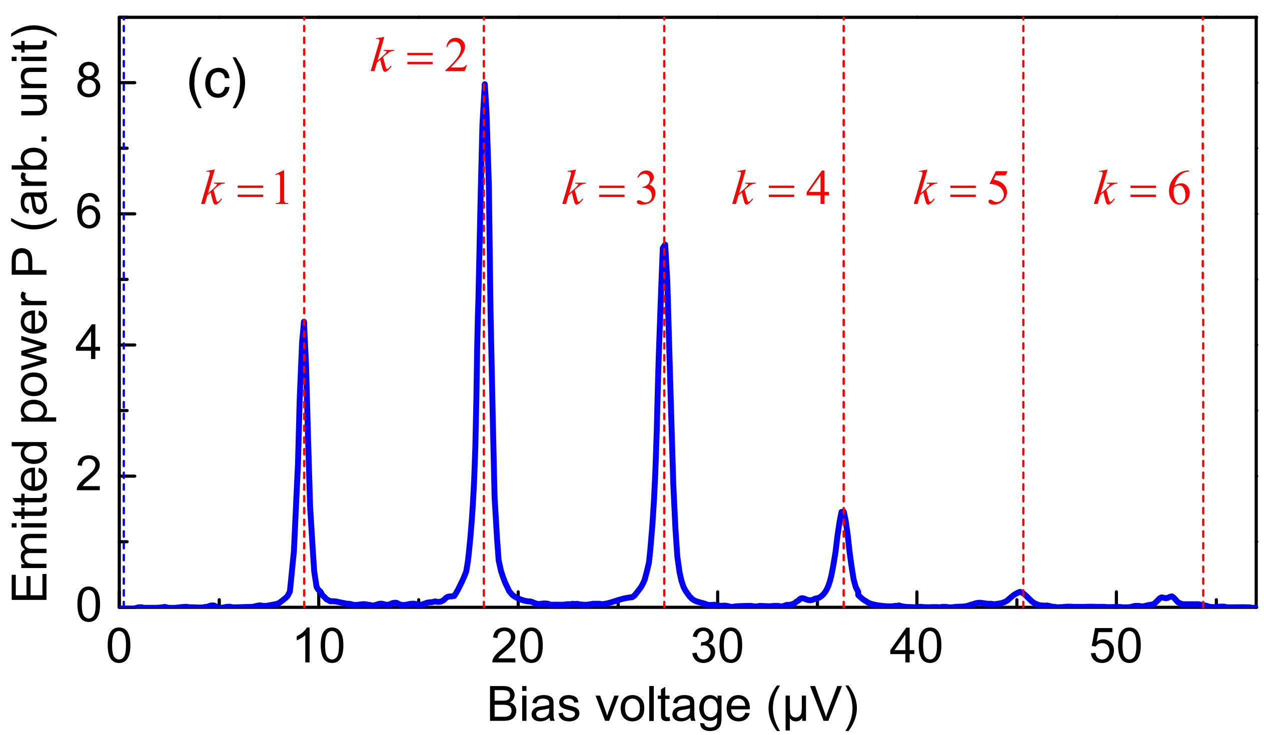
Inelastic CP Tunneling

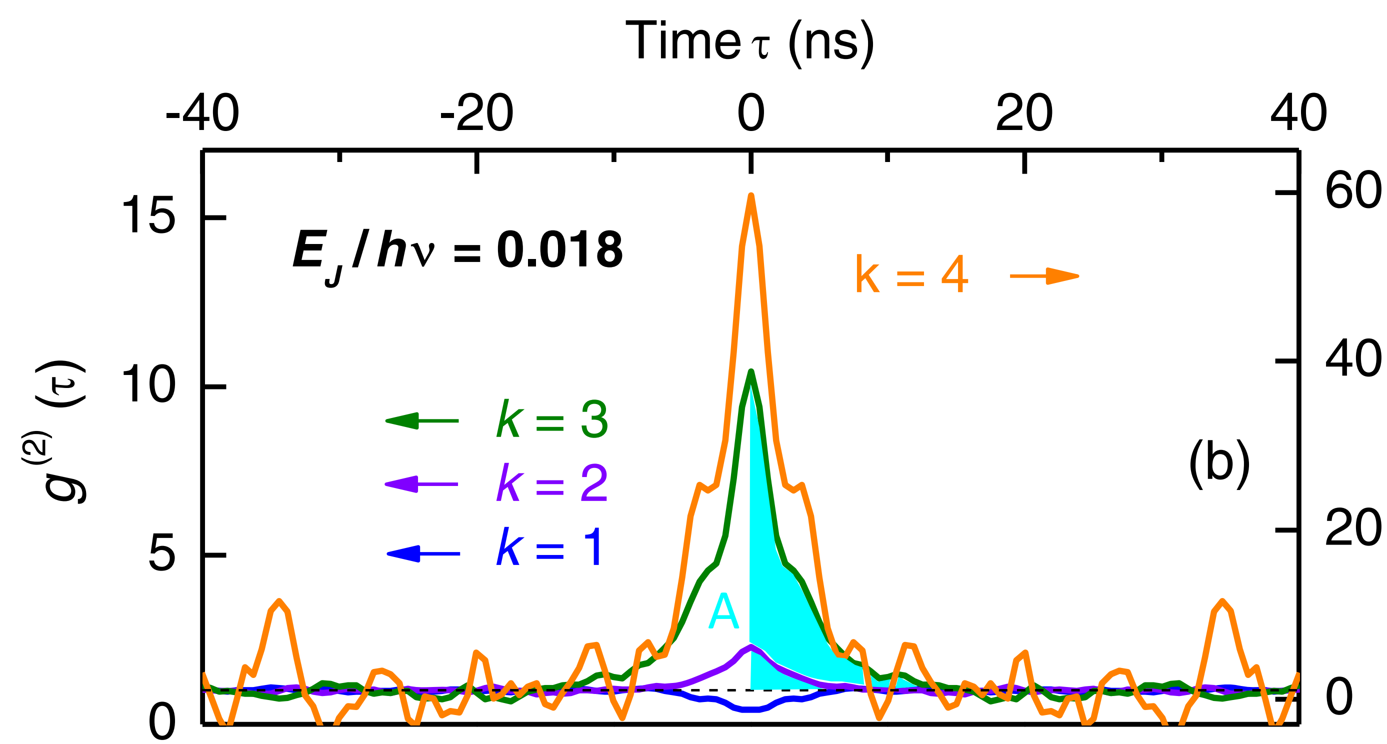


Inelastic CP Tunneling

- Resonator goes back to $|0\rangle$ between two tunneling events
- Single mode physics

$\Rightarrow$ One Cooper pair gives k photons when $2eV = k\,\hbar\omega$



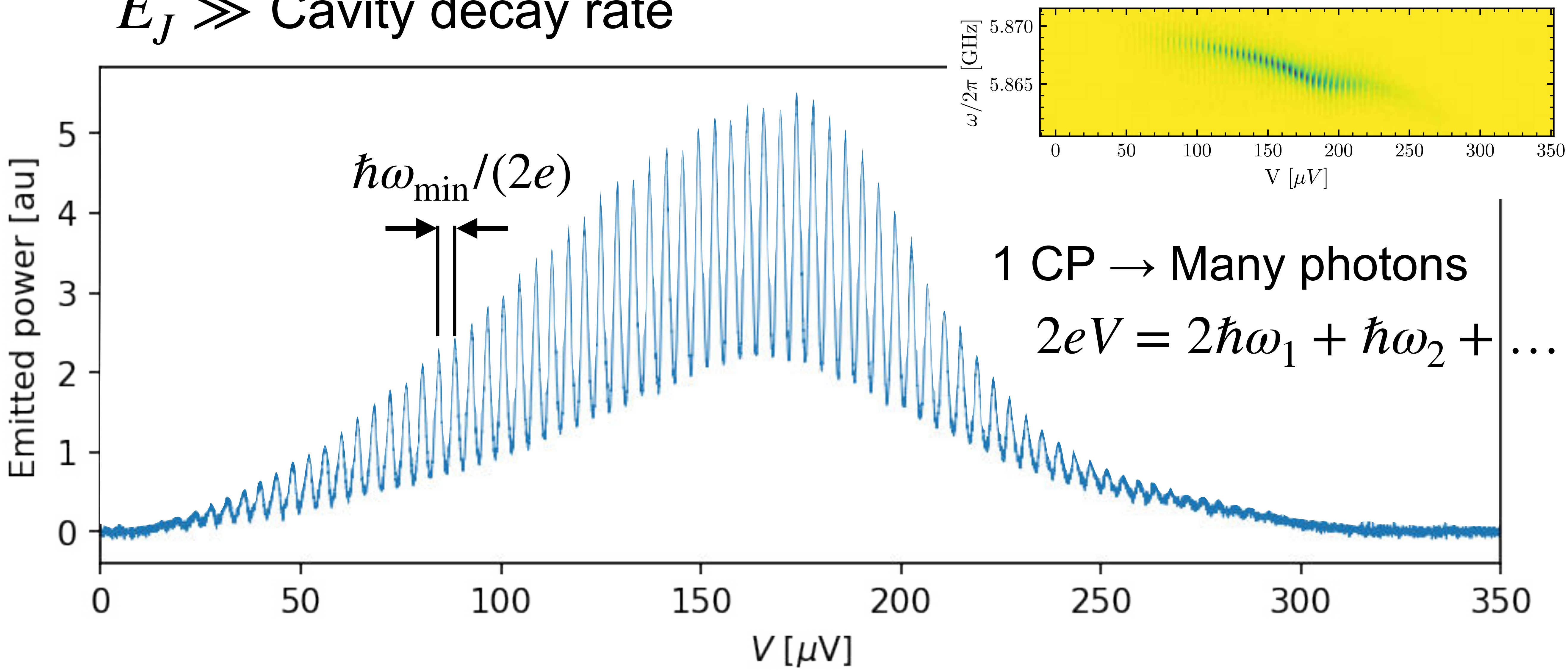


Hofheinz et al. PRL **106**, 217005 (2011)
Rolland et al. PRL **122**, 186804 (2019)
Ménard et al. PRX **12**, 021006 (2022)

F. Portier (Saclay)

Strong coupling regime

$E_J \gg$ Cavity decay rate

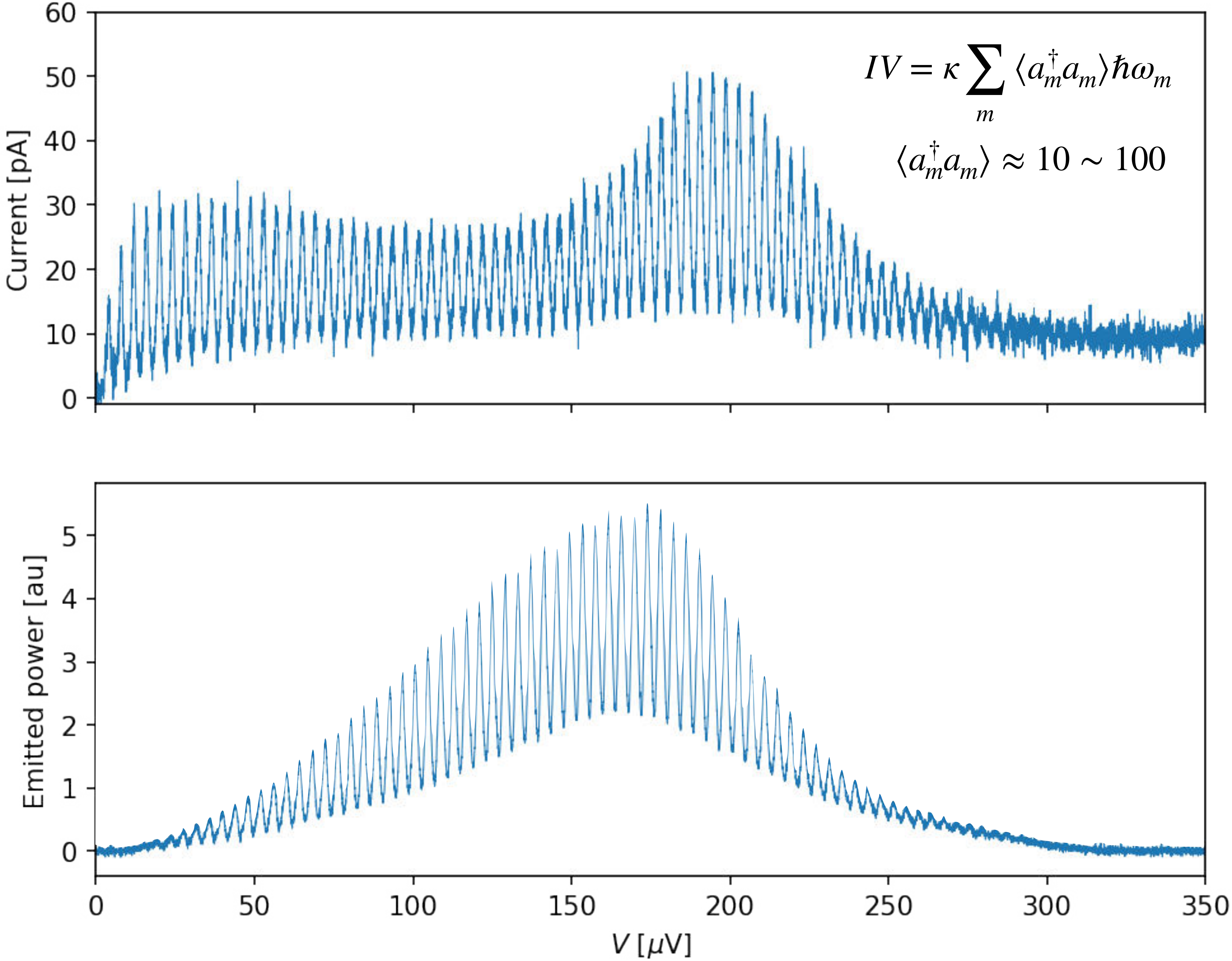


1 CP → Many photons

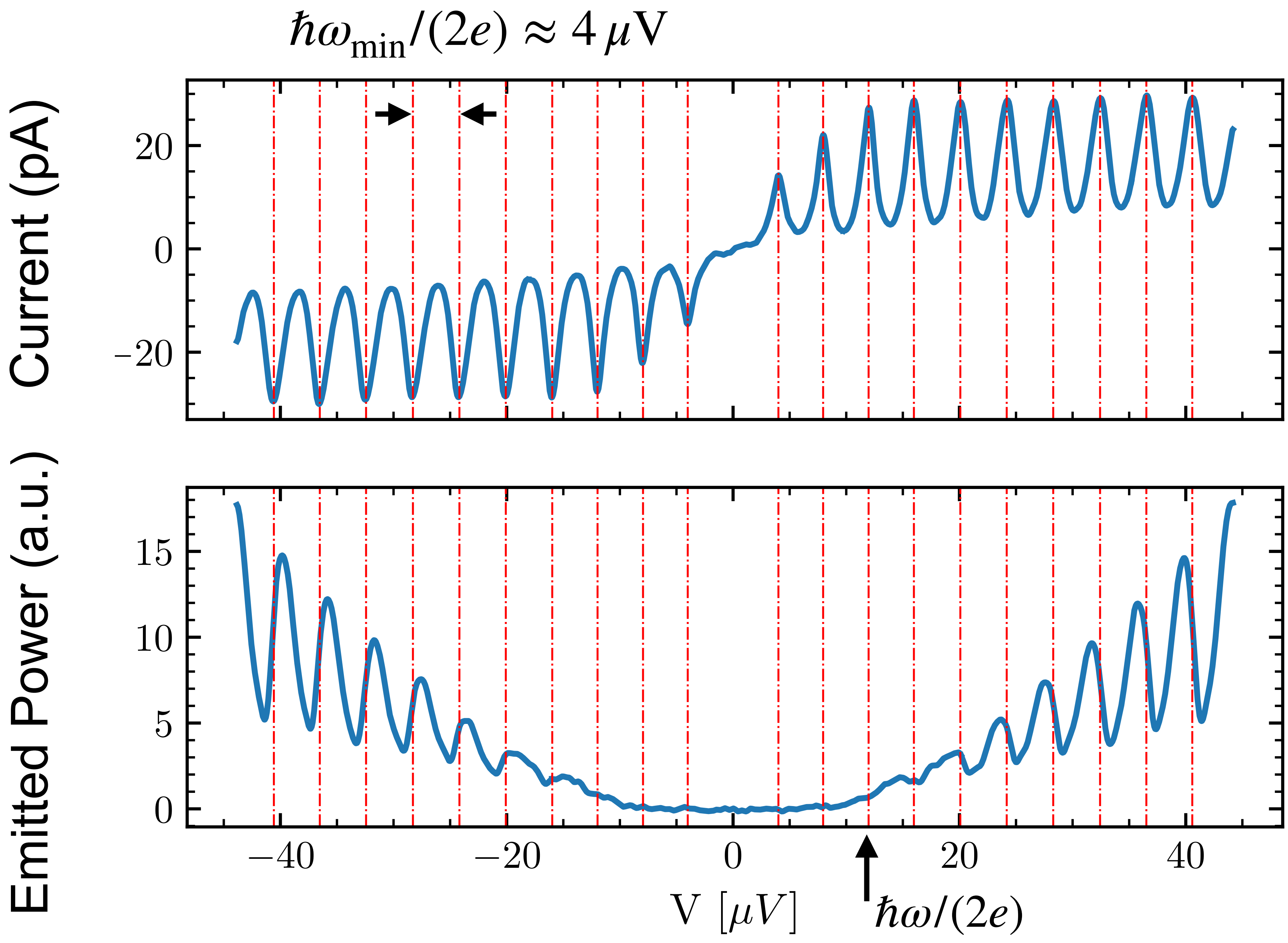
$$2eV = 2\hbar\omega_1 + \hbar\omega_2 + \ldots$$

- Periodic pattern with period $2e\,\delta V = \hbar\omega_{\text{min}}$
- Maximum of emission at ω when $2e\,V \gg \hbar\omega$

Strong coupling regime

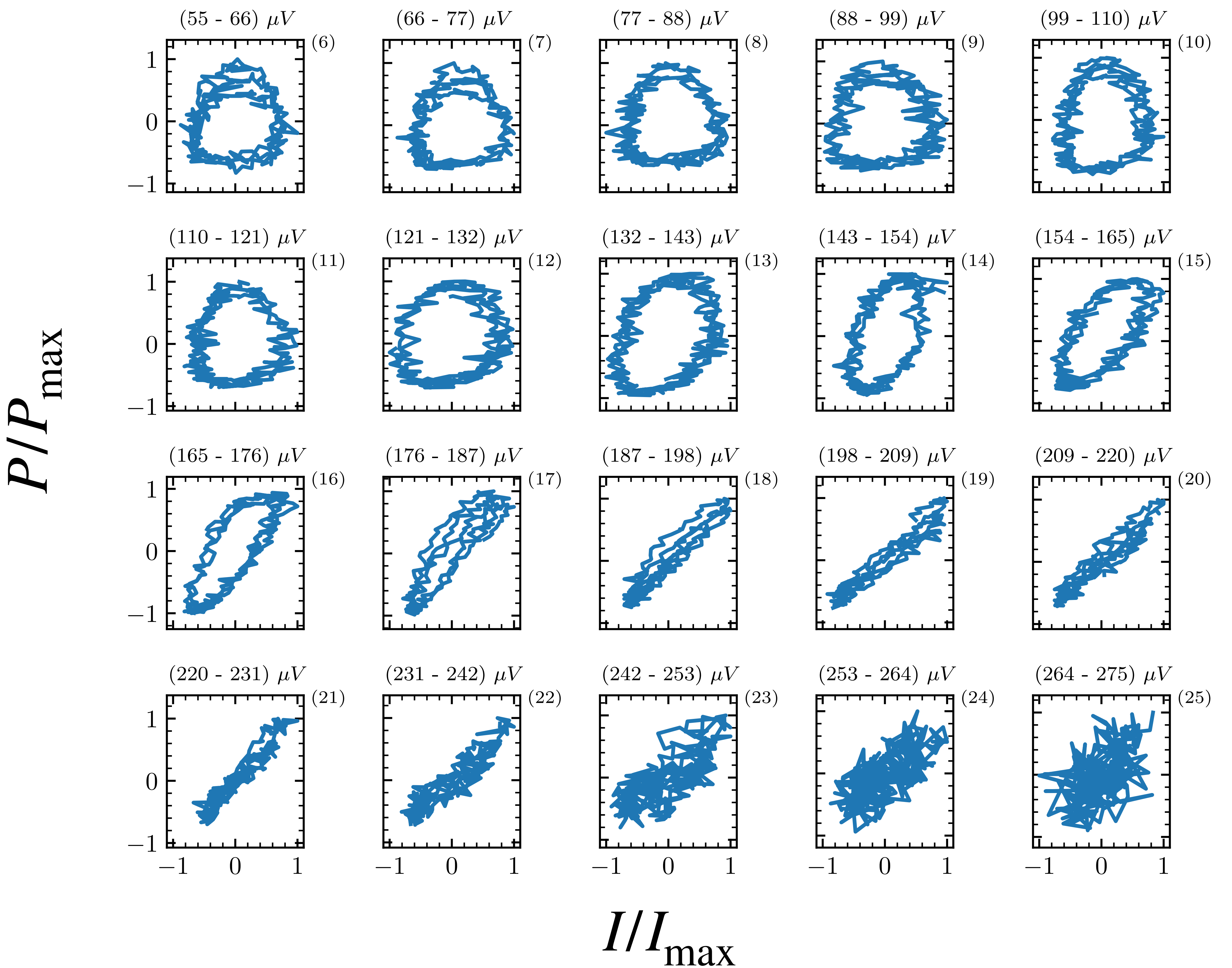


Inelastic Cooper Pair Tunneling

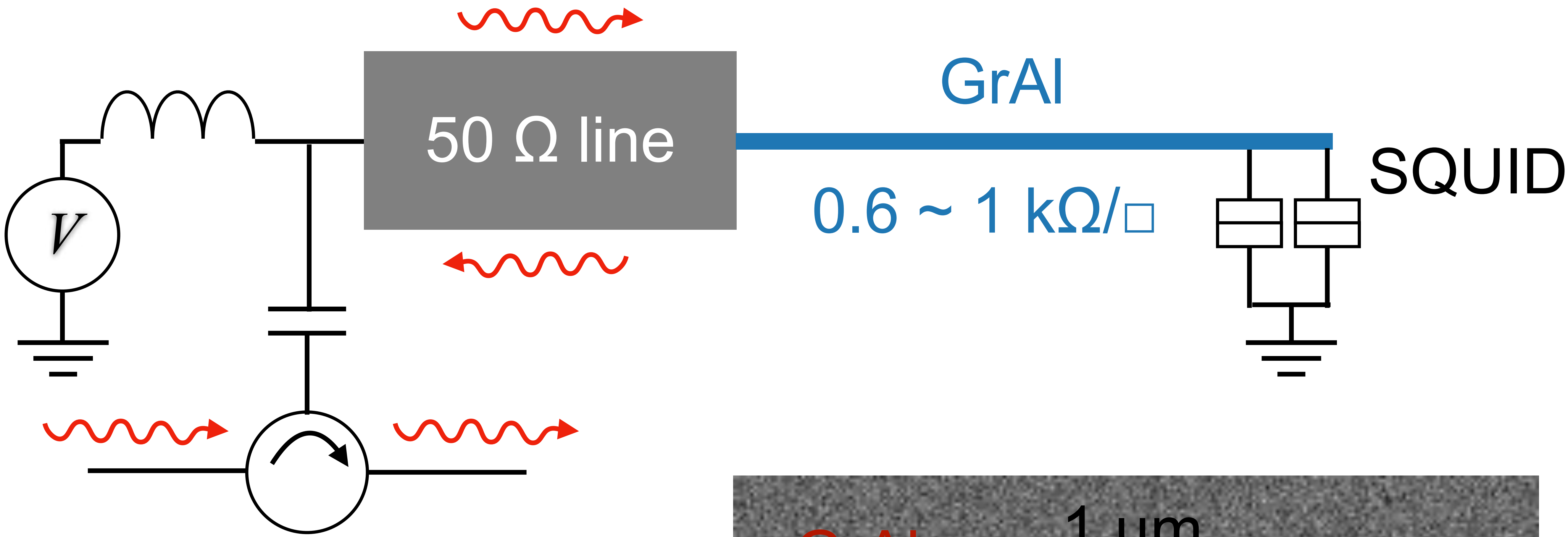


Maxima of MW emission do not always coincide with maxima of current.

Current-Emission Dephasing



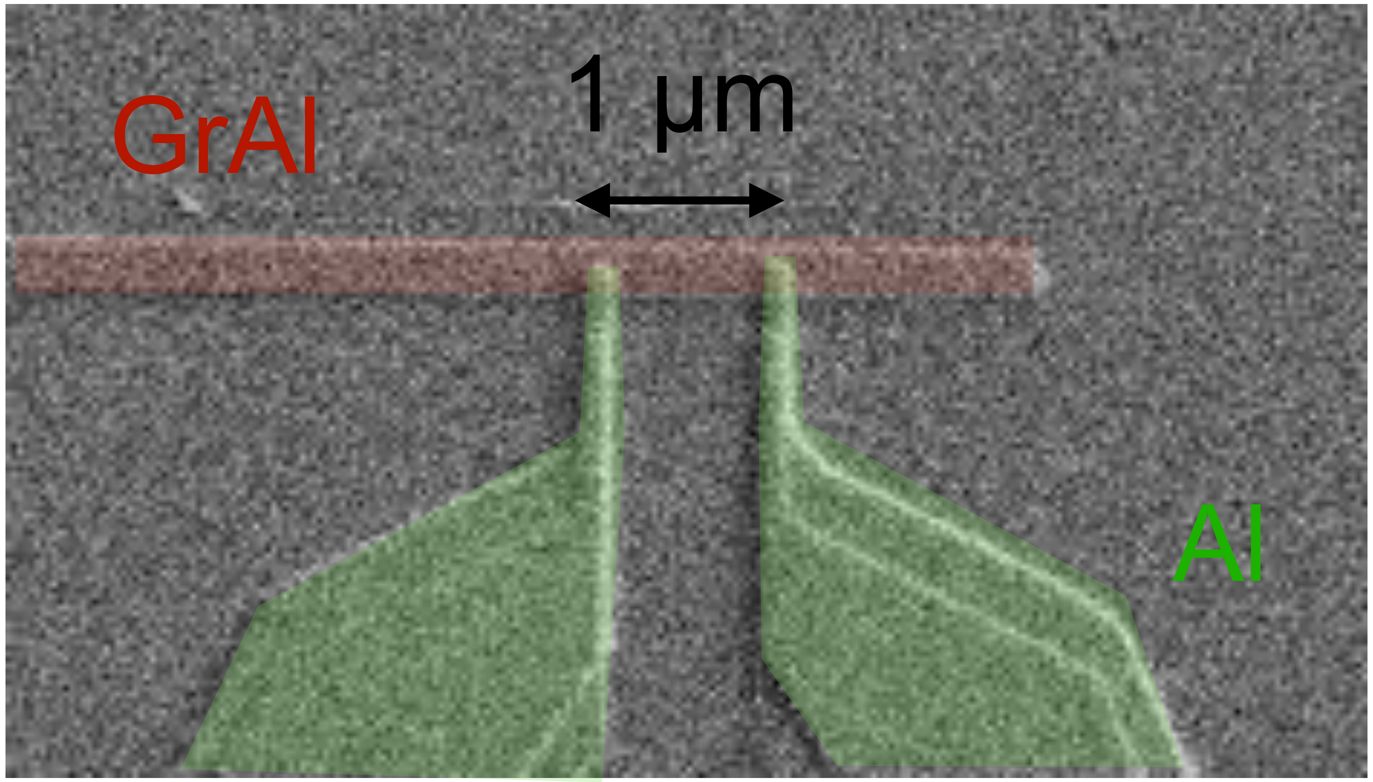
Experimental Setup



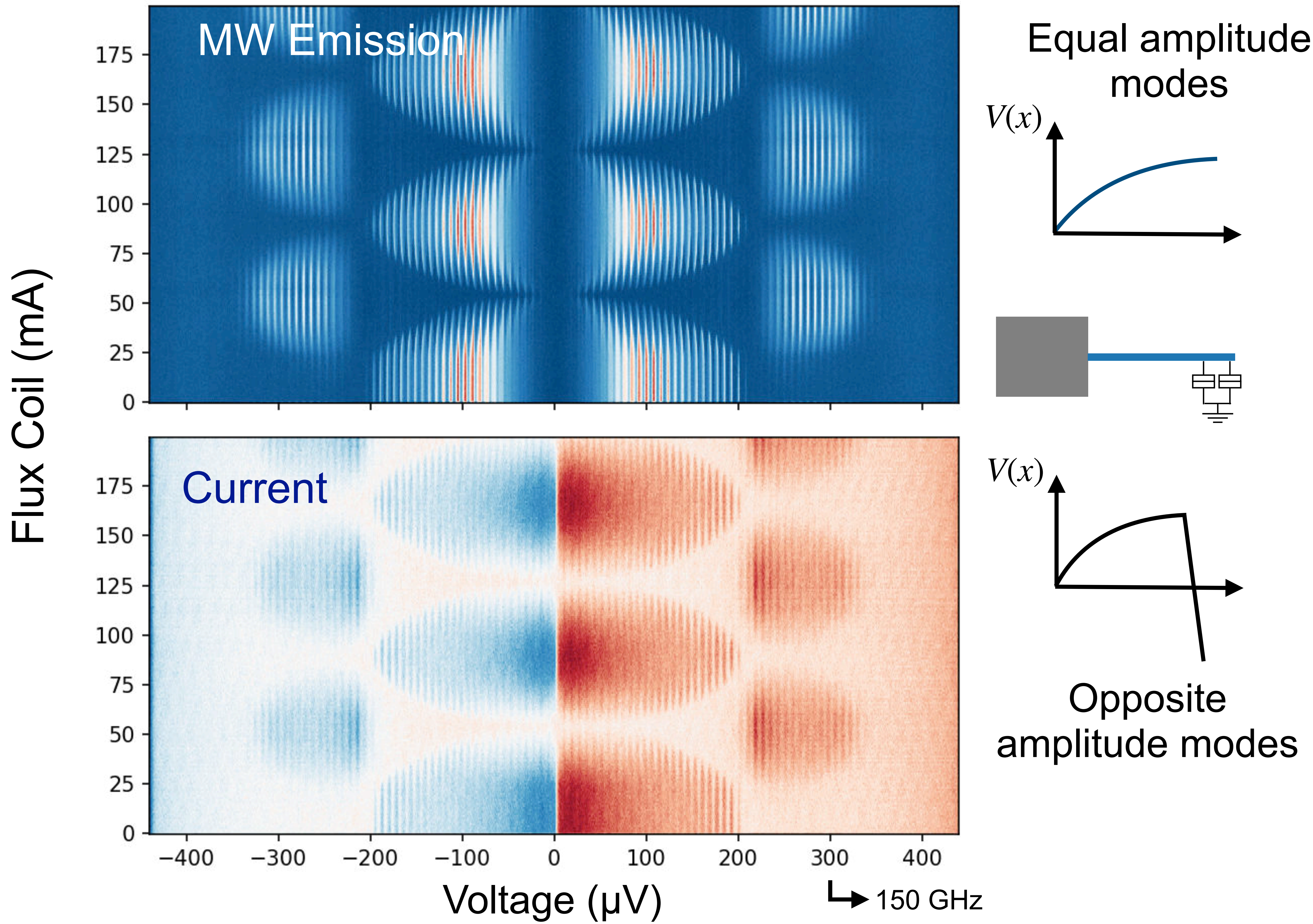
GrAl wire 300 nm x 30 nm

$Z_c \approx 5$ kΩ

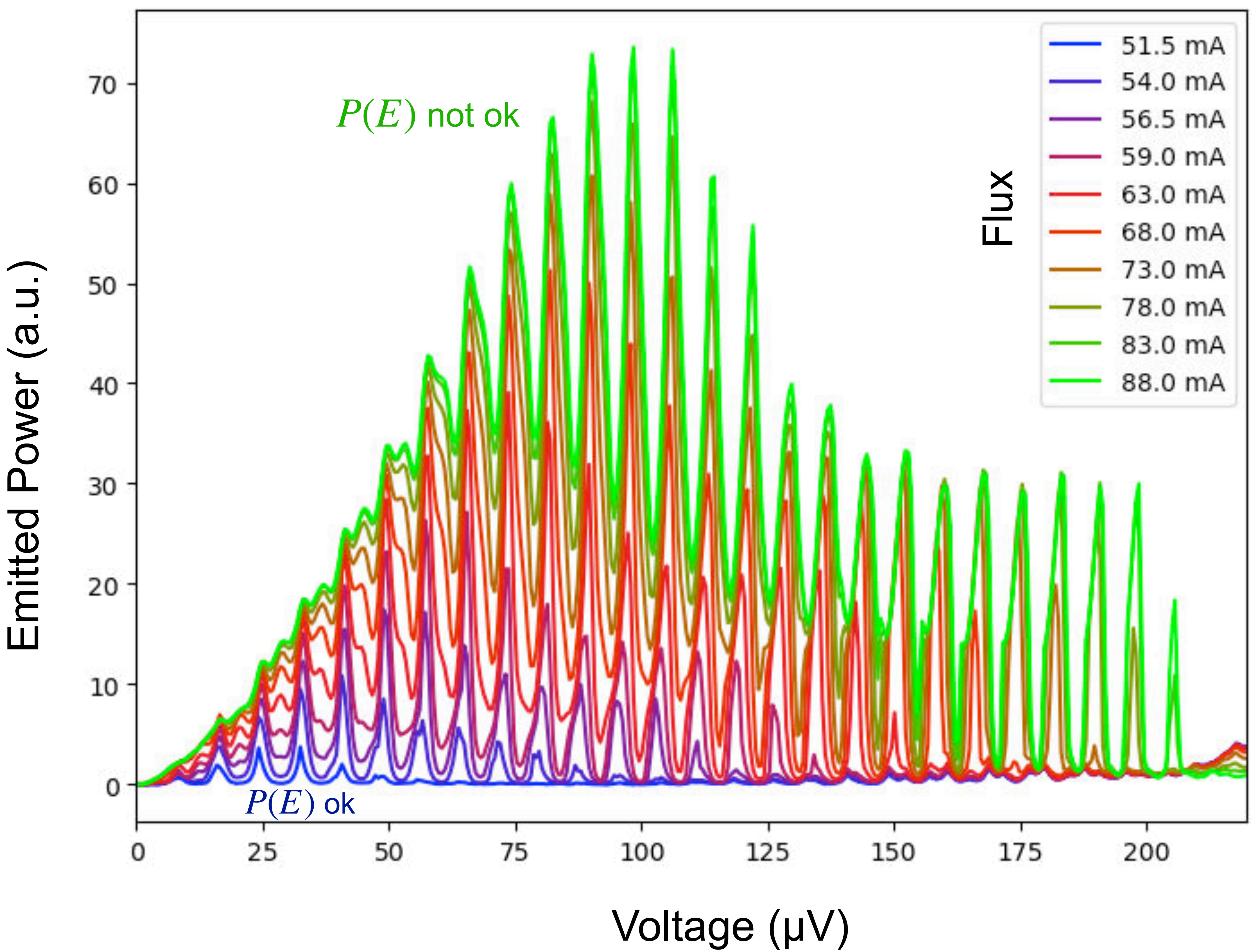
$\lambda/4 = 200$ μm @ 6 GHz



SQUID Configuration



Evolution of MW emission



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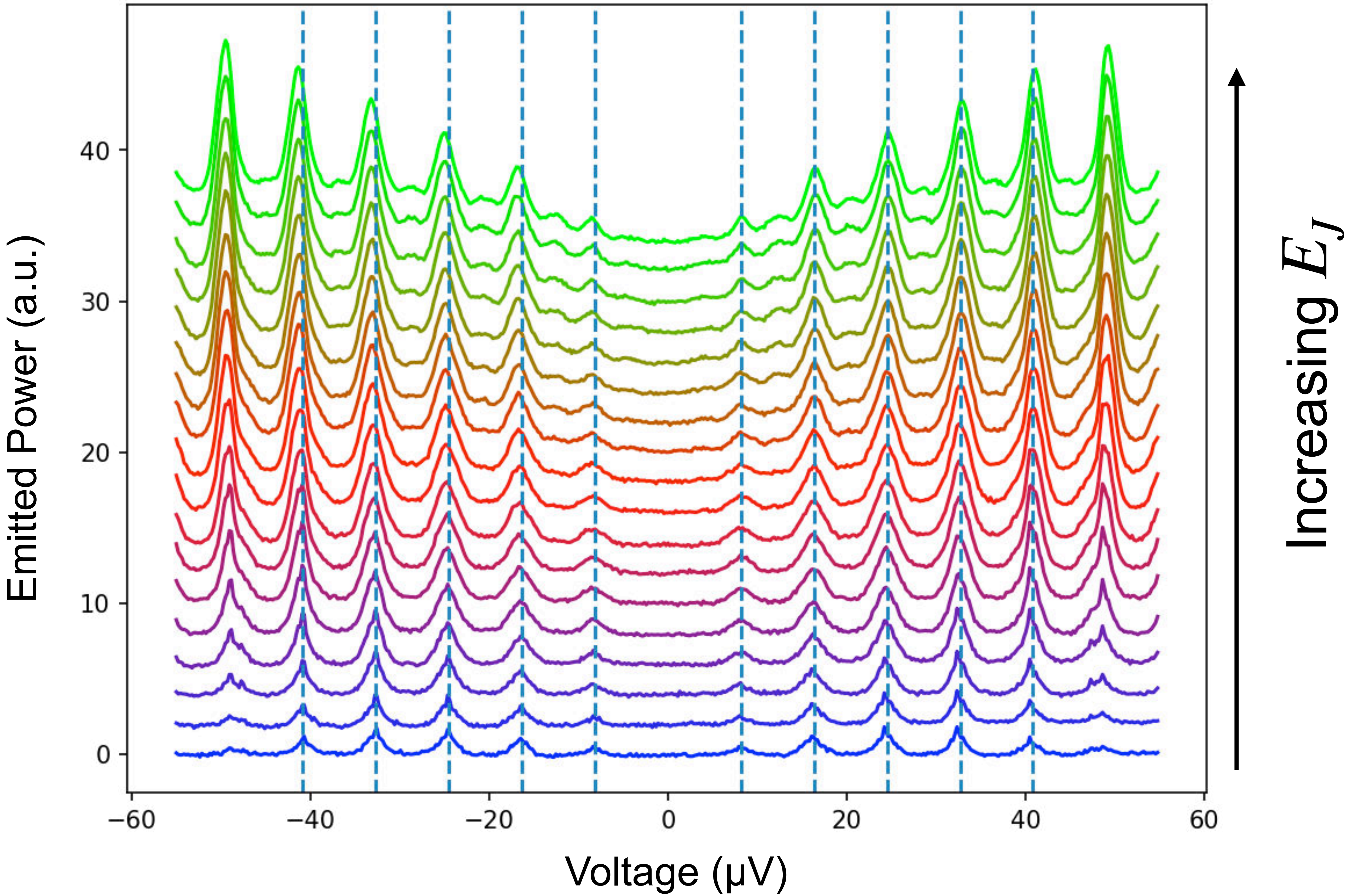
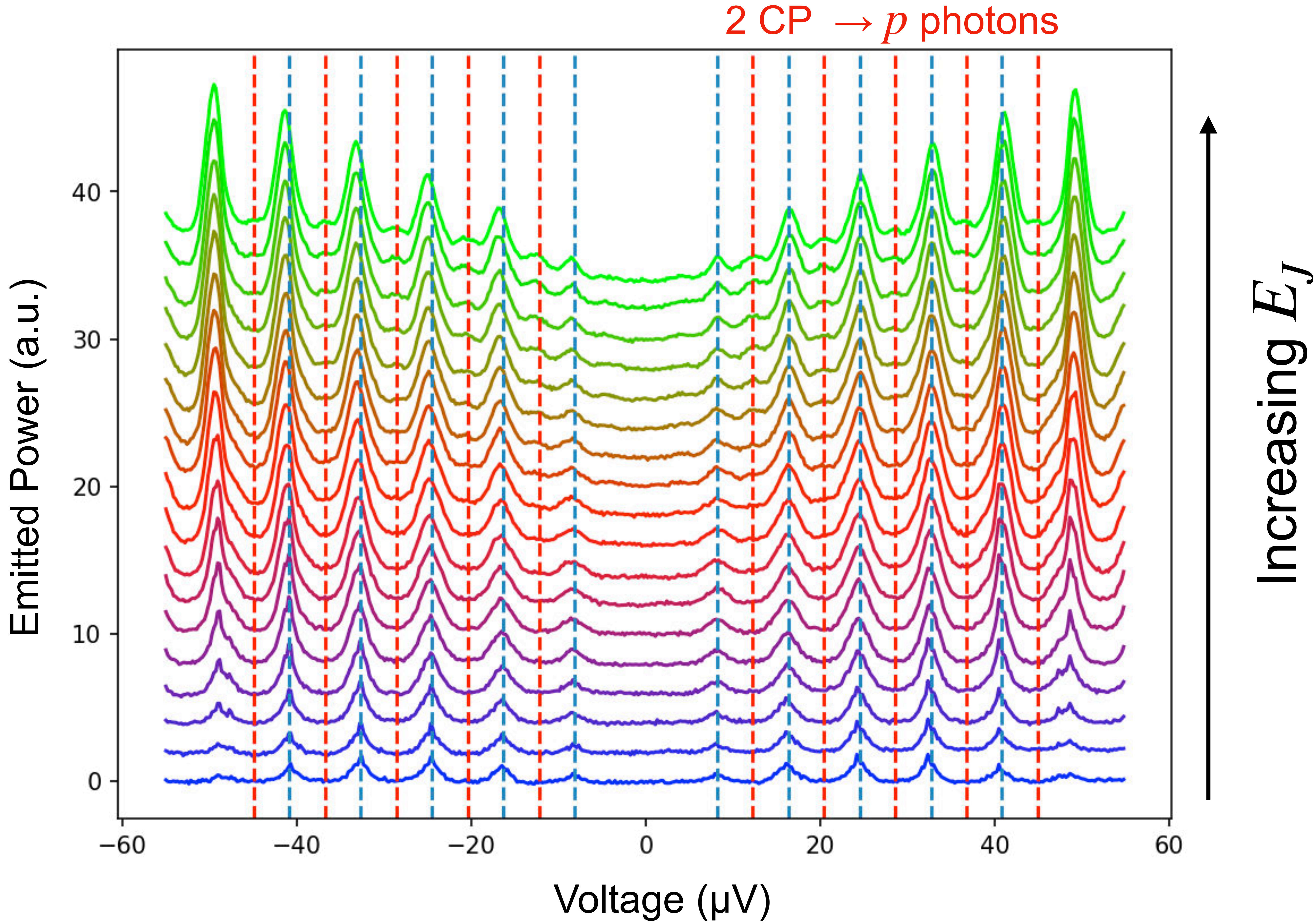
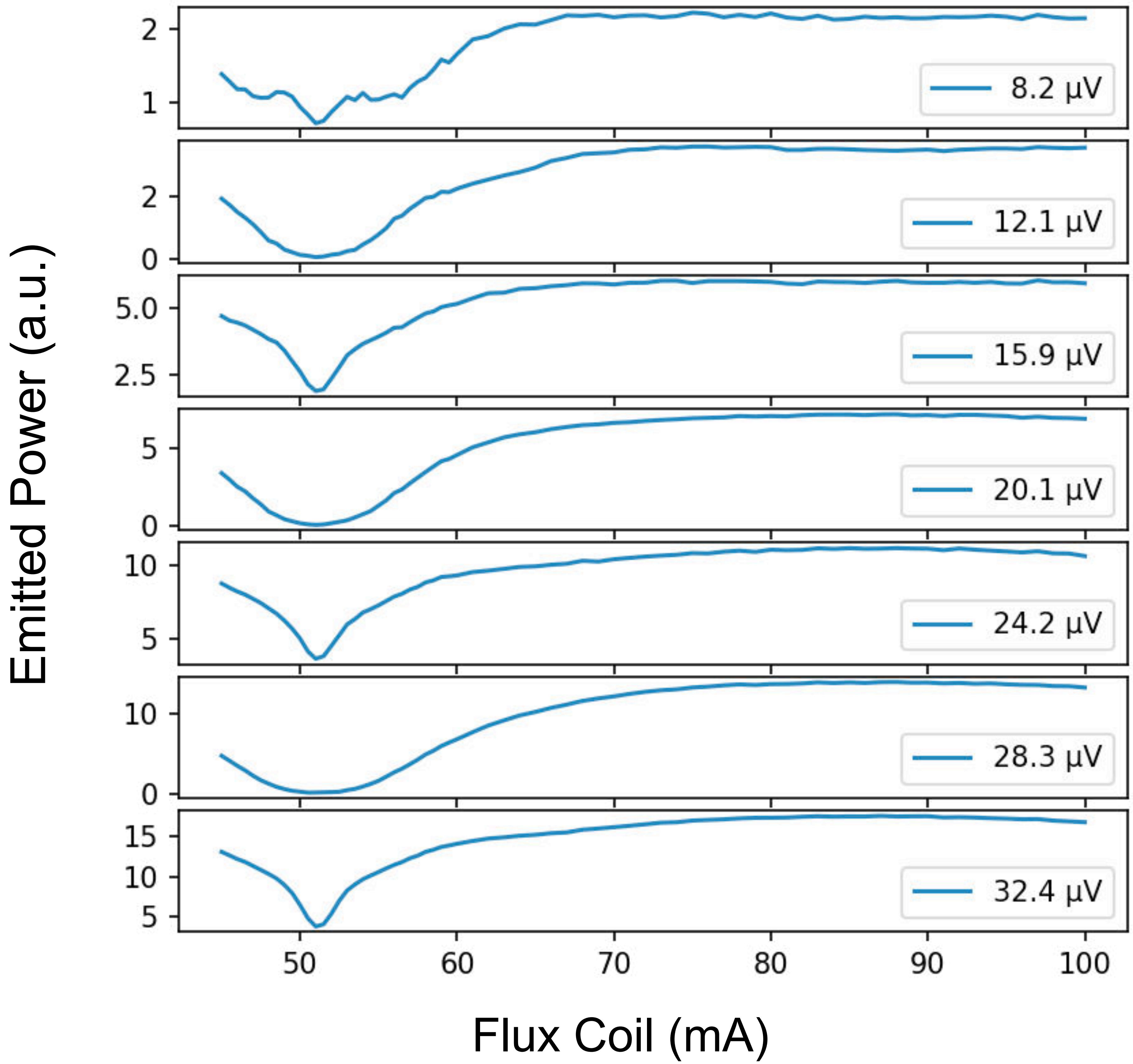


Evolution of MW emission



Flux Dependence



$\omega_J = \omega_0$

$\omega_J = 3\omega_0/2$

$\omega_J = 2\omega_0$

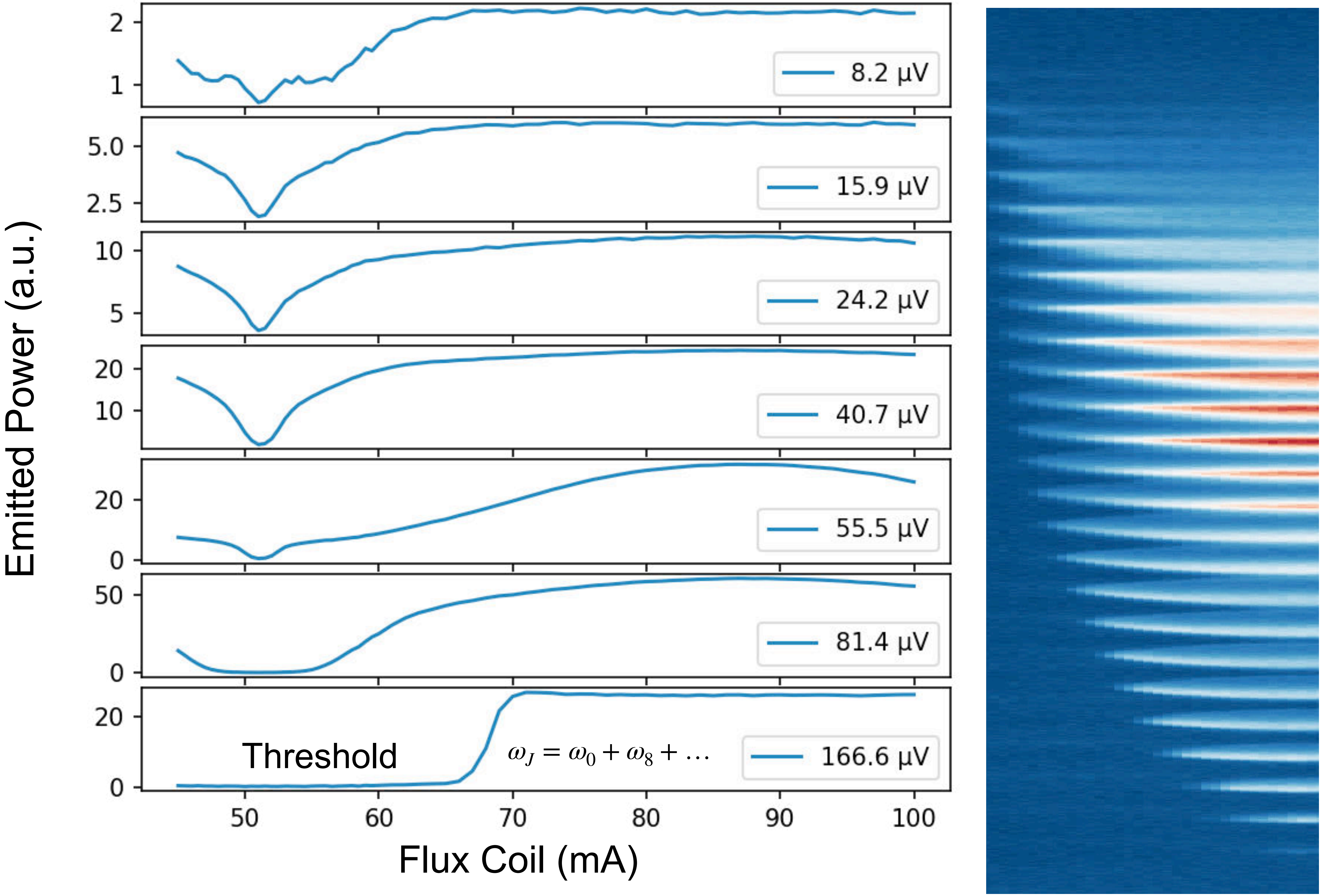
$\omega_J = 5\omega_0/2$

$\omega_J = 3\omega_0$

$\omega_J = 7\omega_0/2$

$\omega_J = 4\omega_0$

Flux Dependence

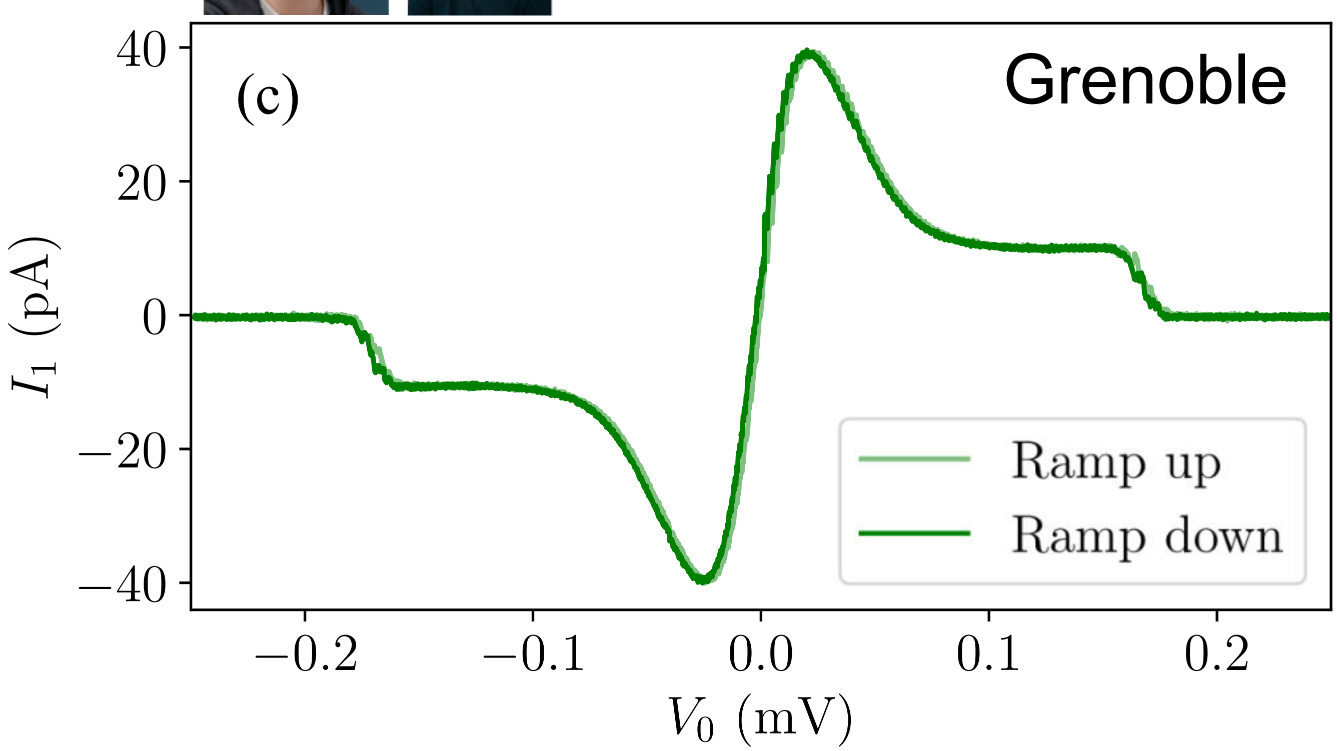


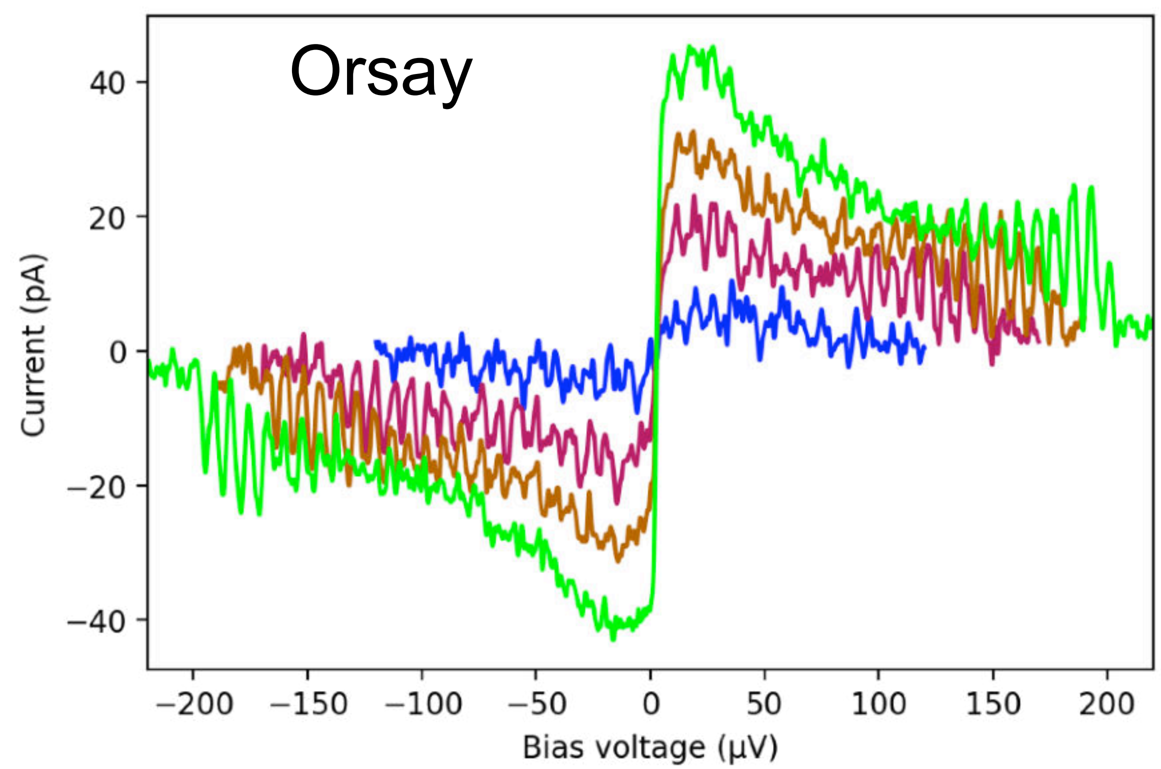
Long Wire Limit

$$H = \sum_m \hbar\omega_m a_m^\dagger a_m - E_J \cos\left[\omega_J t + \sum_m \lambda_m (a_m + a_m^\dagger)\right] \qquad \lambda_m^2 \propto 1/(2m+1)$$

$\omega_{m+1} - \omega_m \propto 1/\text{length}$ Elastic contribution vanishes $\Pi_m \, e^{-\Lambda_m^2} \to 0$

What is the limiting $I(V)$?





Josephson Junction Chain

GrAl wire

Long Wire Limit

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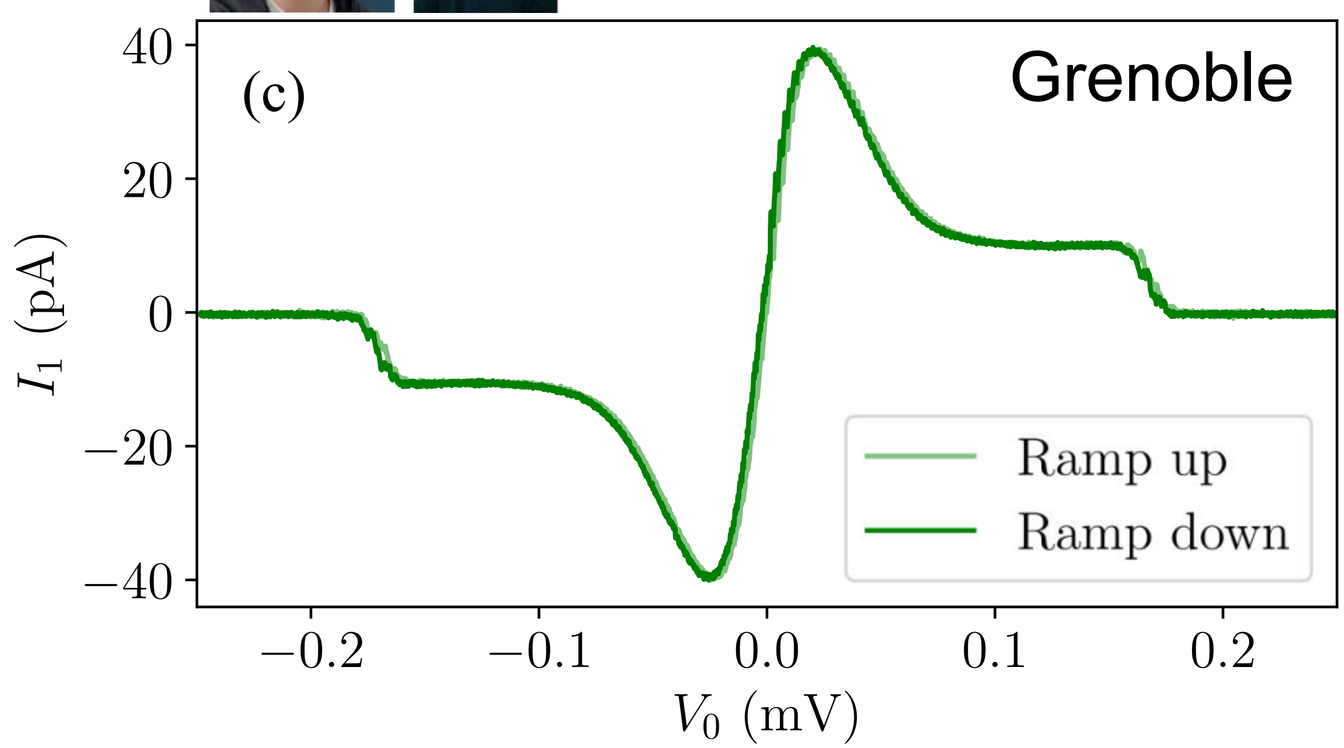
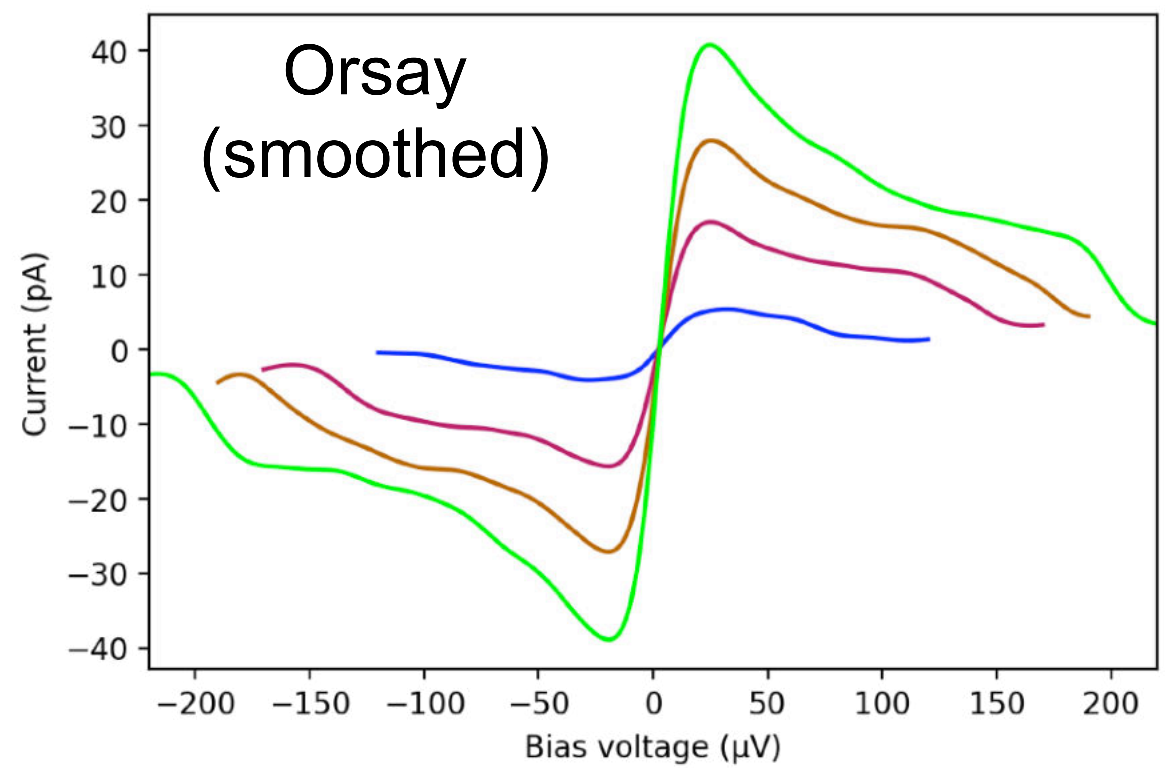




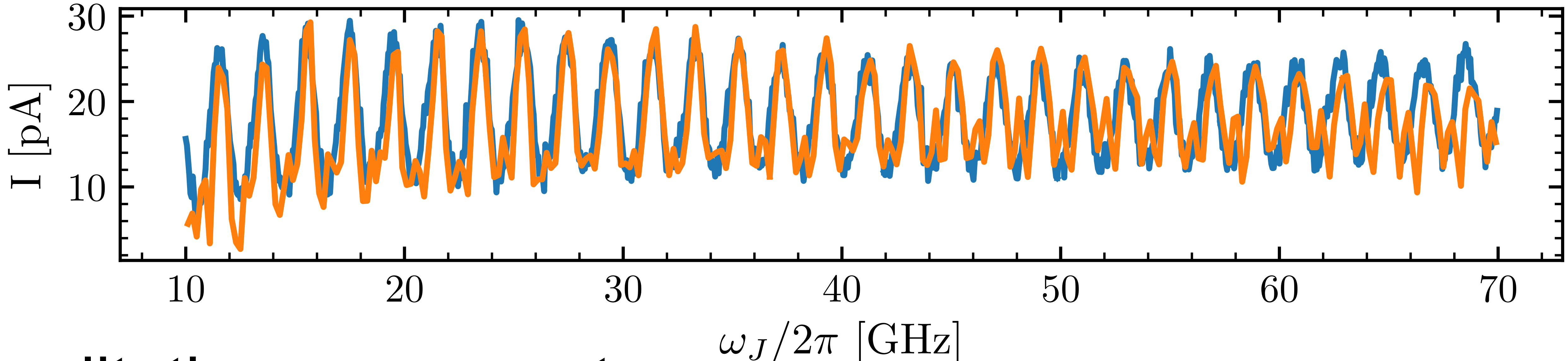
Josephson Junction Chain

GrAl wire

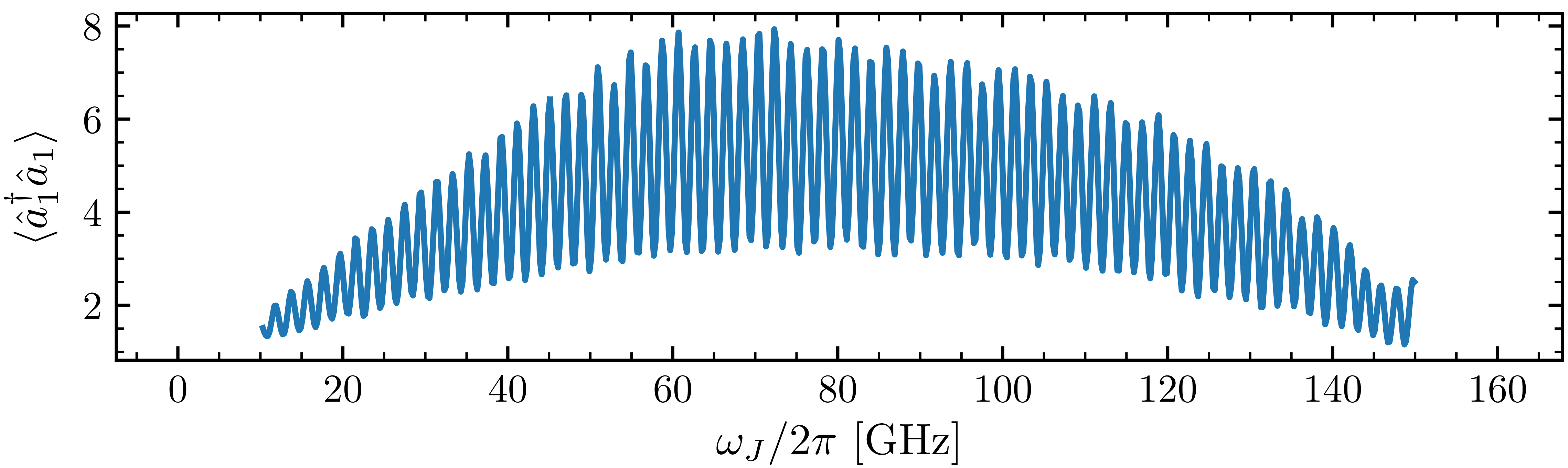
Numerical Simulation

Truncated Wigner Approximation : Stochastic classical traj.

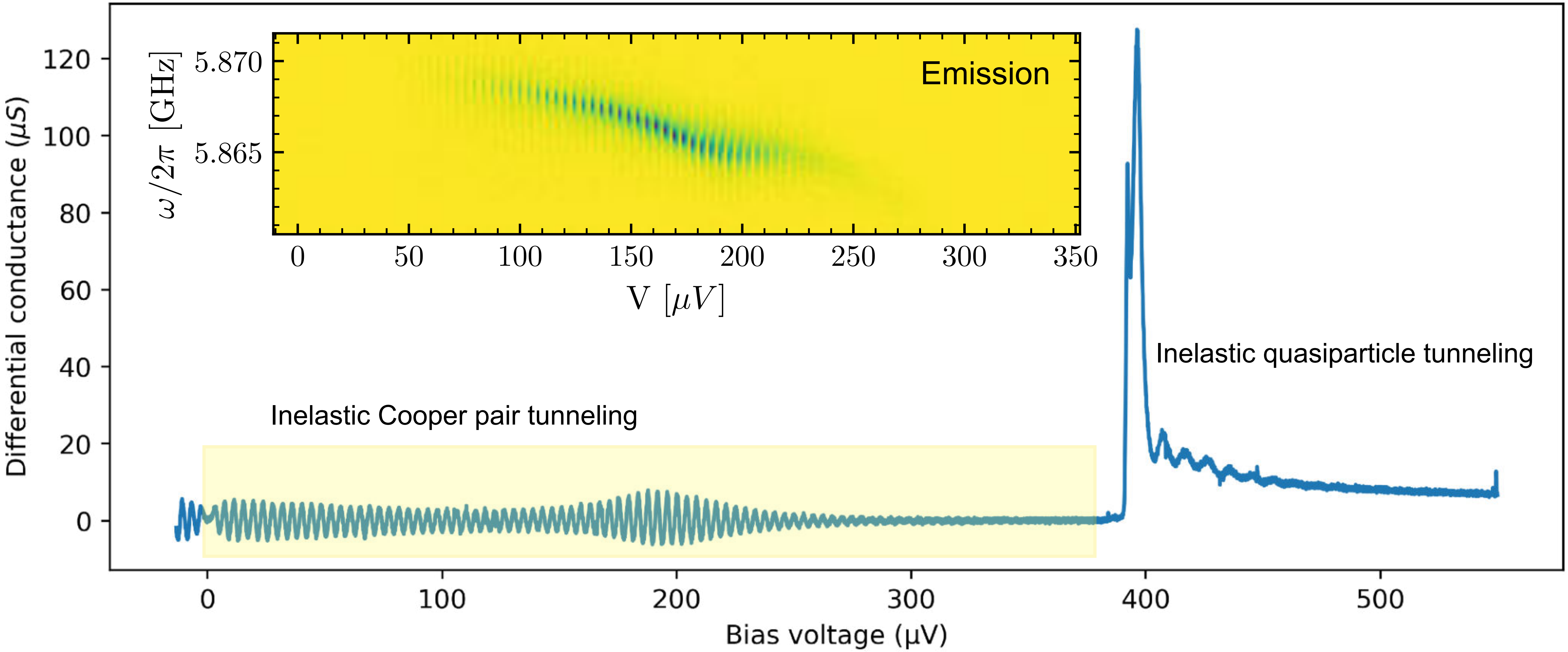
$$W(x+\lambda) \approx W(x) + \lambda \frac{\partial W}{\partial x}$$

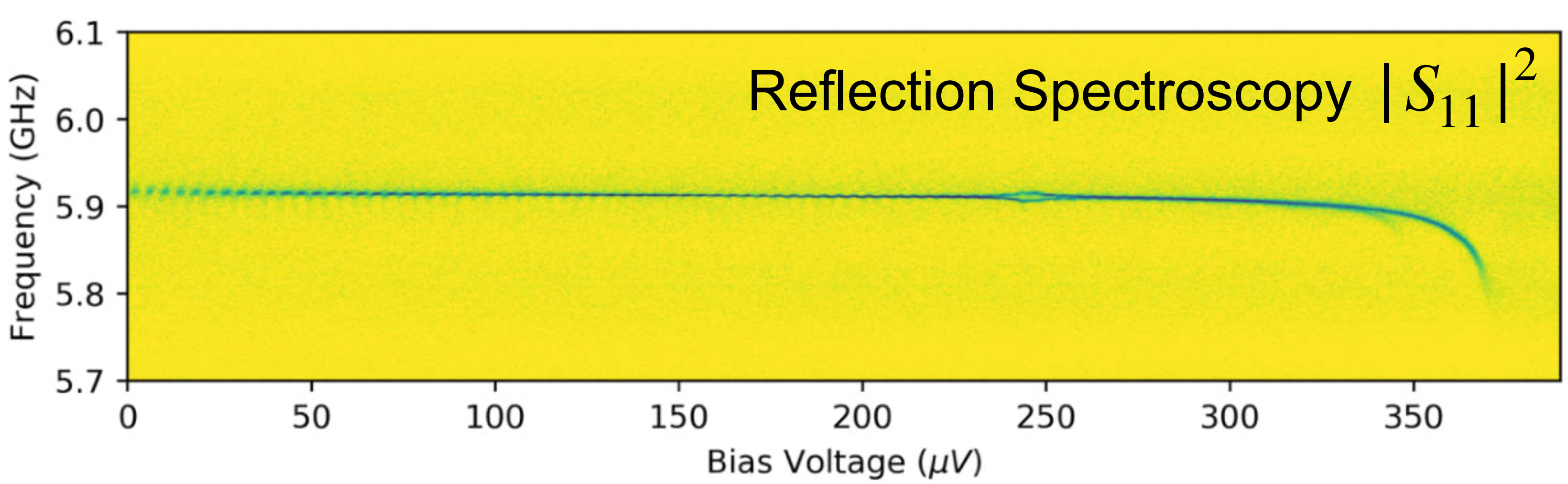


Qualitative agreement

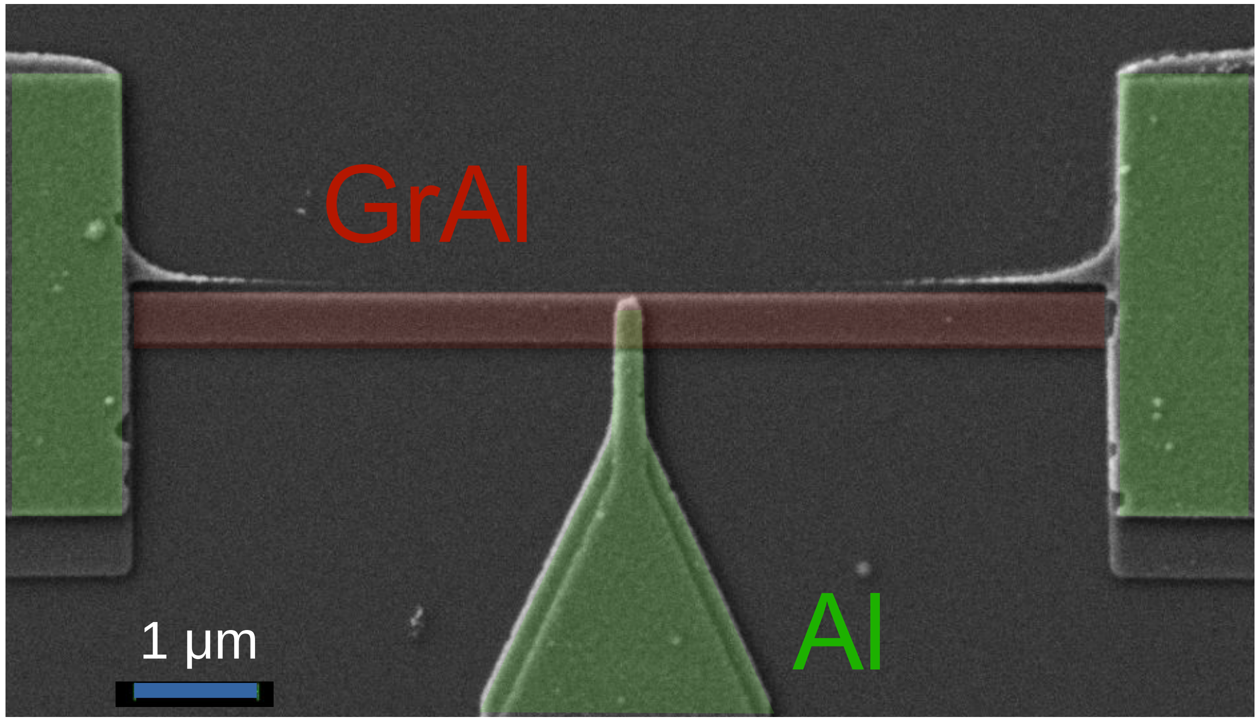


Summary


Emission
$\omega/2\pi$ [GHz]
5.870
5.865
0 50 100 150 200 250 300 350
V [μV]
Differential conductance (μS)
120 100 80 60 40 20 0
Inelastic Cooper pair tunneling
Inelastic quasiparticle tunneling
0 100 200 300 400 500
Bias voltage (μV)

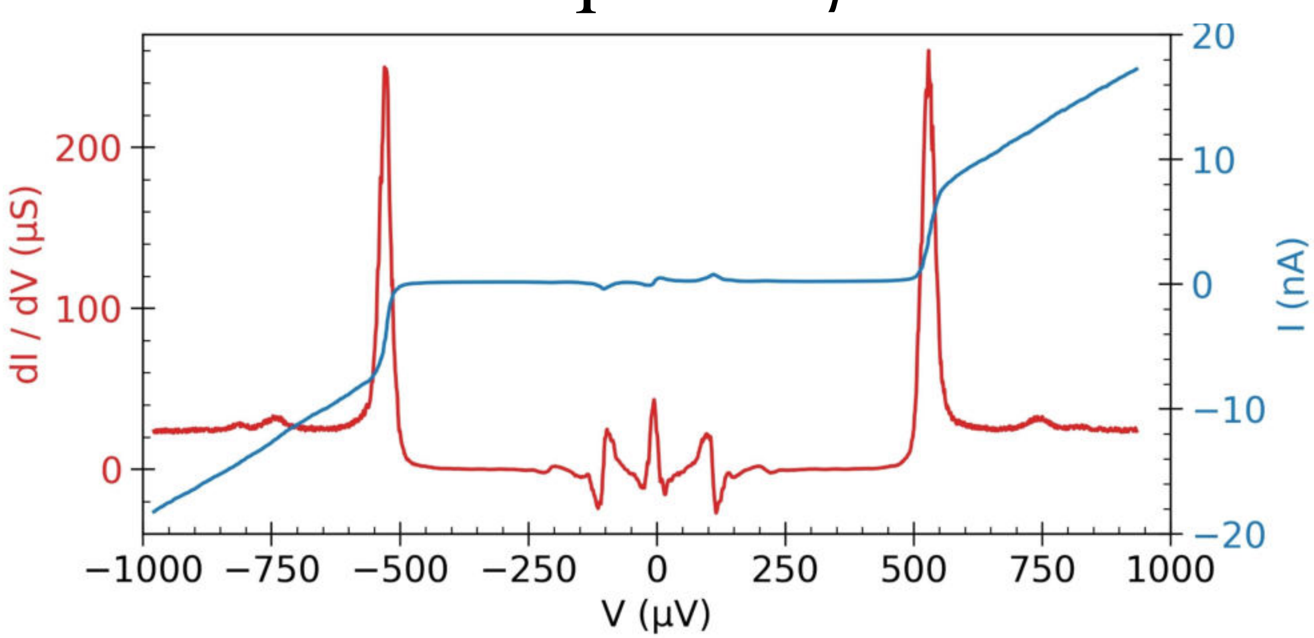

Reflection Spectroscopy $|S_{11}|^2$
Frequency (GHz)
6.1 6.0 5.9 5.8 5.7
0 50 100 150 200 250 300 350
Bias Voltage (μV)

Short Wire Tunneling Spectroscopy

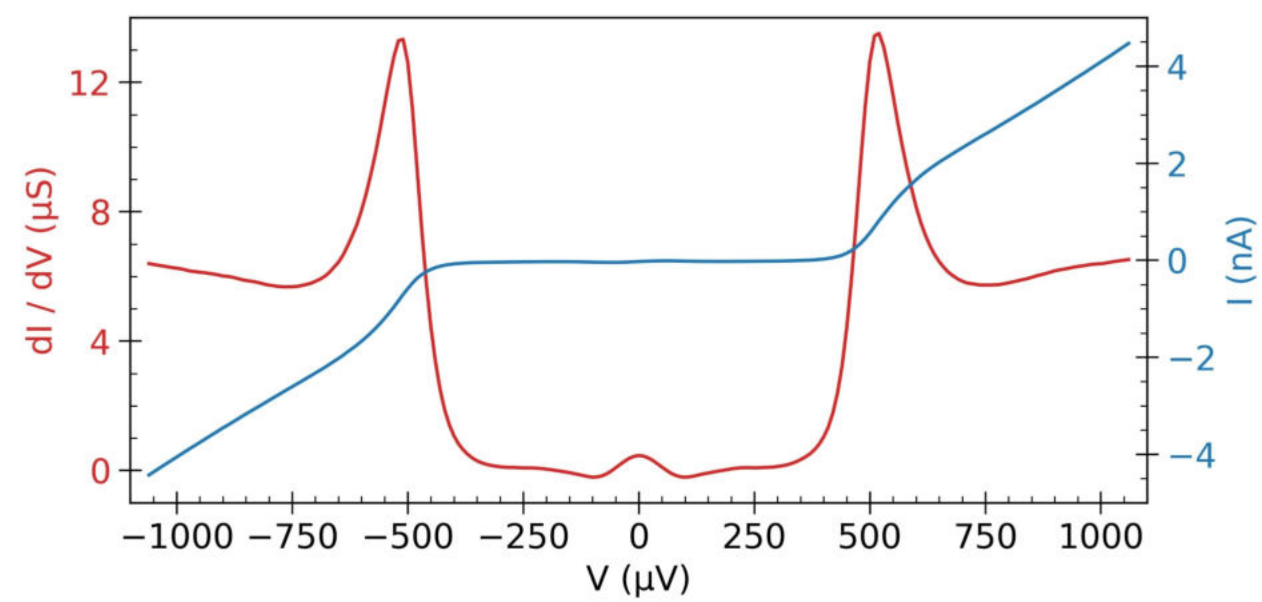


GrAl wire
8 x 0.35 x 0.03 µm^3

$650\,\Omega/\mathrm{sq}\quad 2000\,\mu\Omega\,\mathrm{cm}$



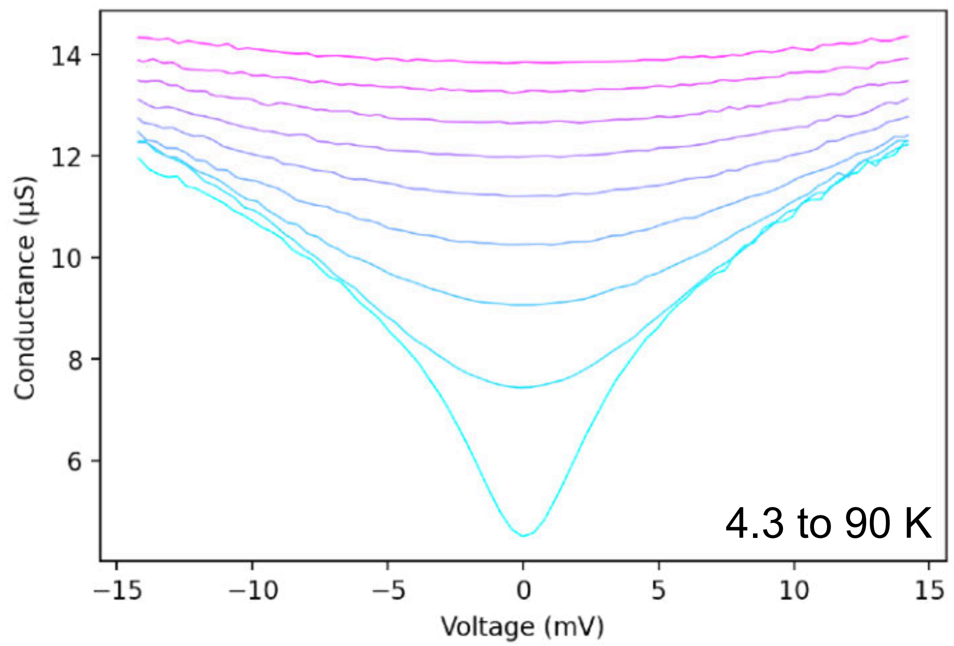
$4\,\mathrm{k}\Omega/\mathrm{sq}\quad 12000\,\mu\Omega\,\mathrm{cm}$



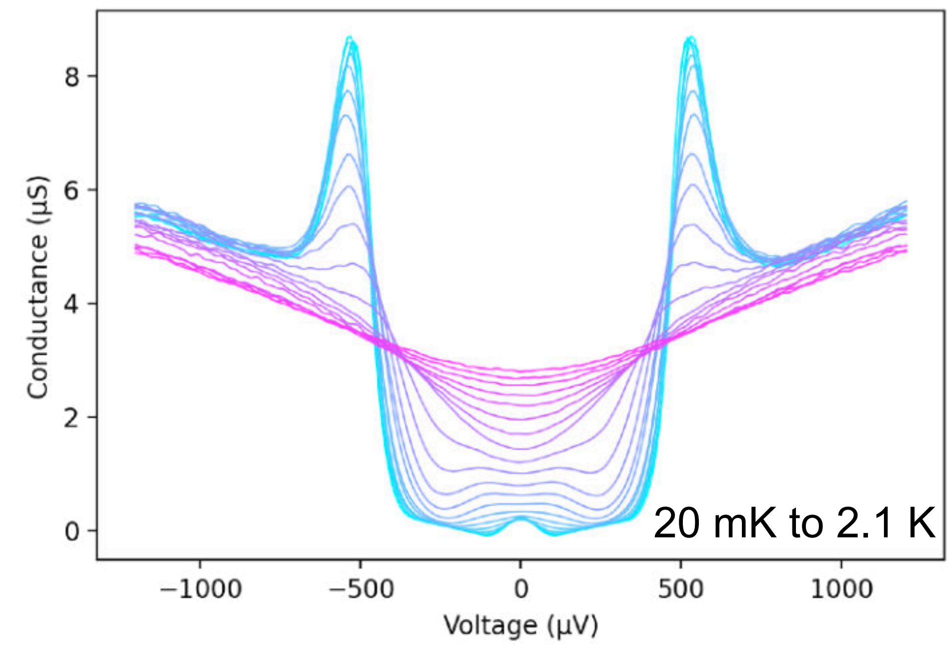
Electrodynamics ≡ MW mode

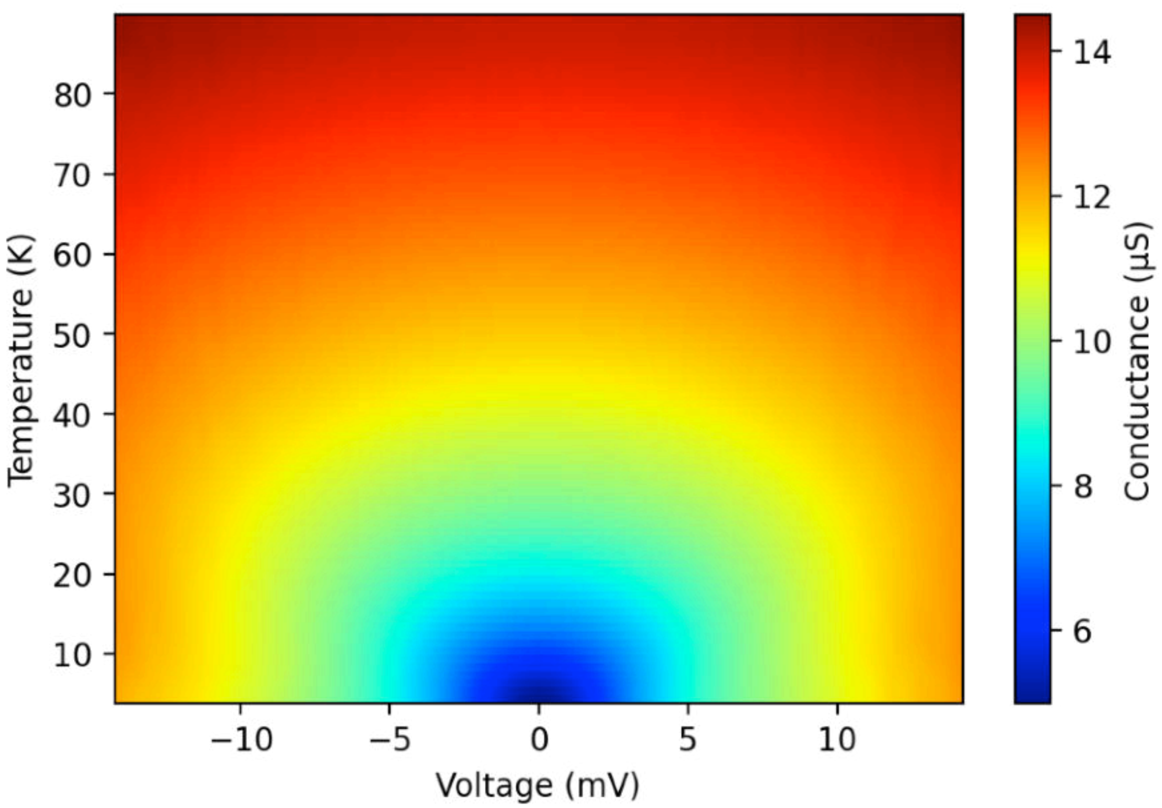
Electrodynamics ≡ *RC* environment

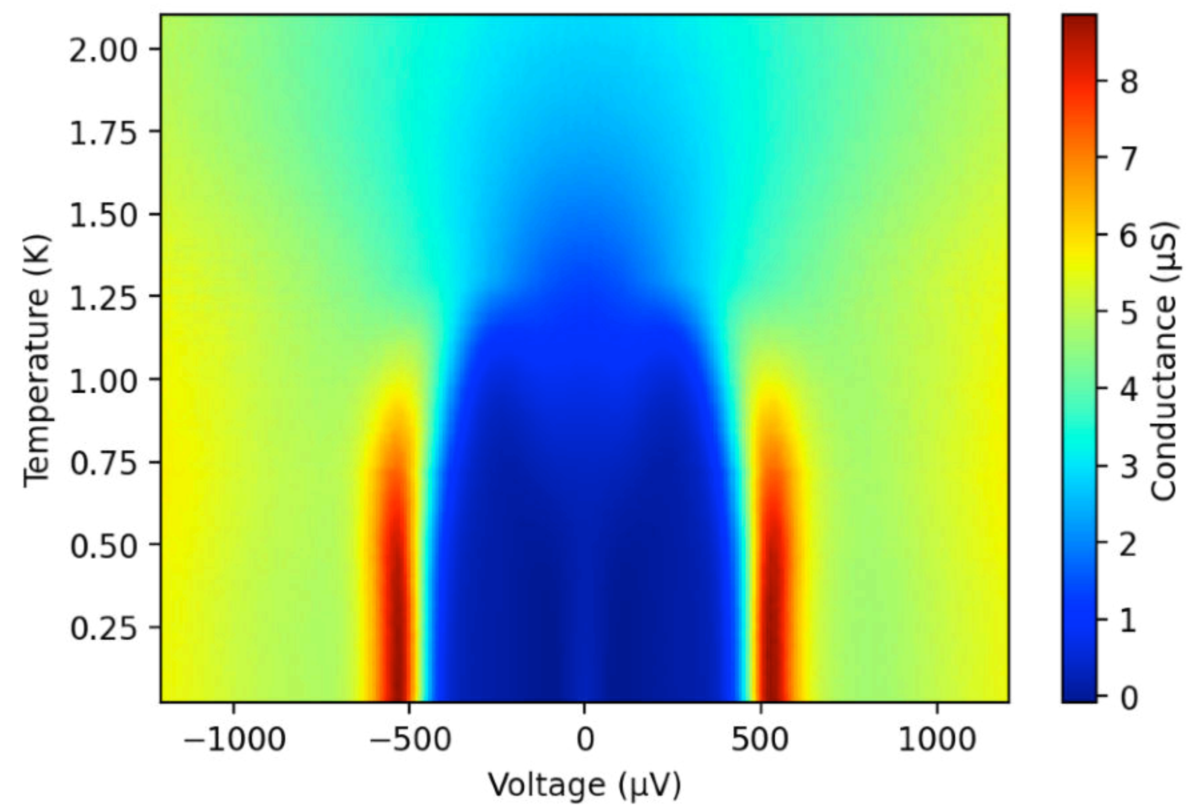
Short Wire Tunneling Spectroscopy



4 kΩ/sq
12 000 μΩ.cm



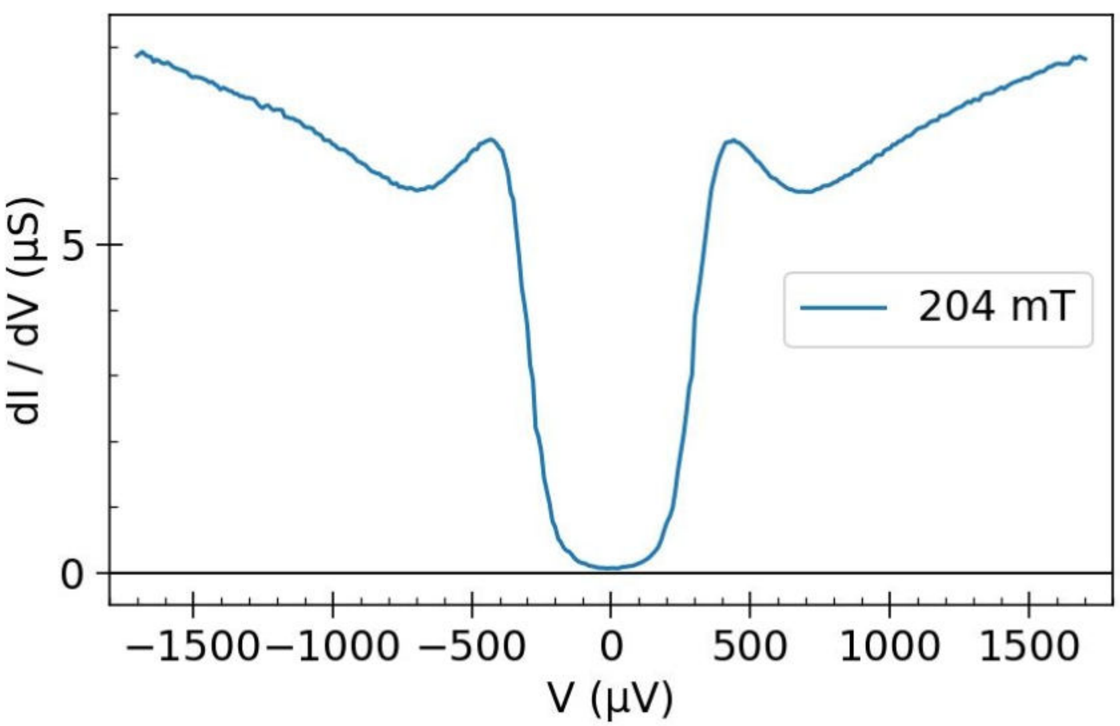


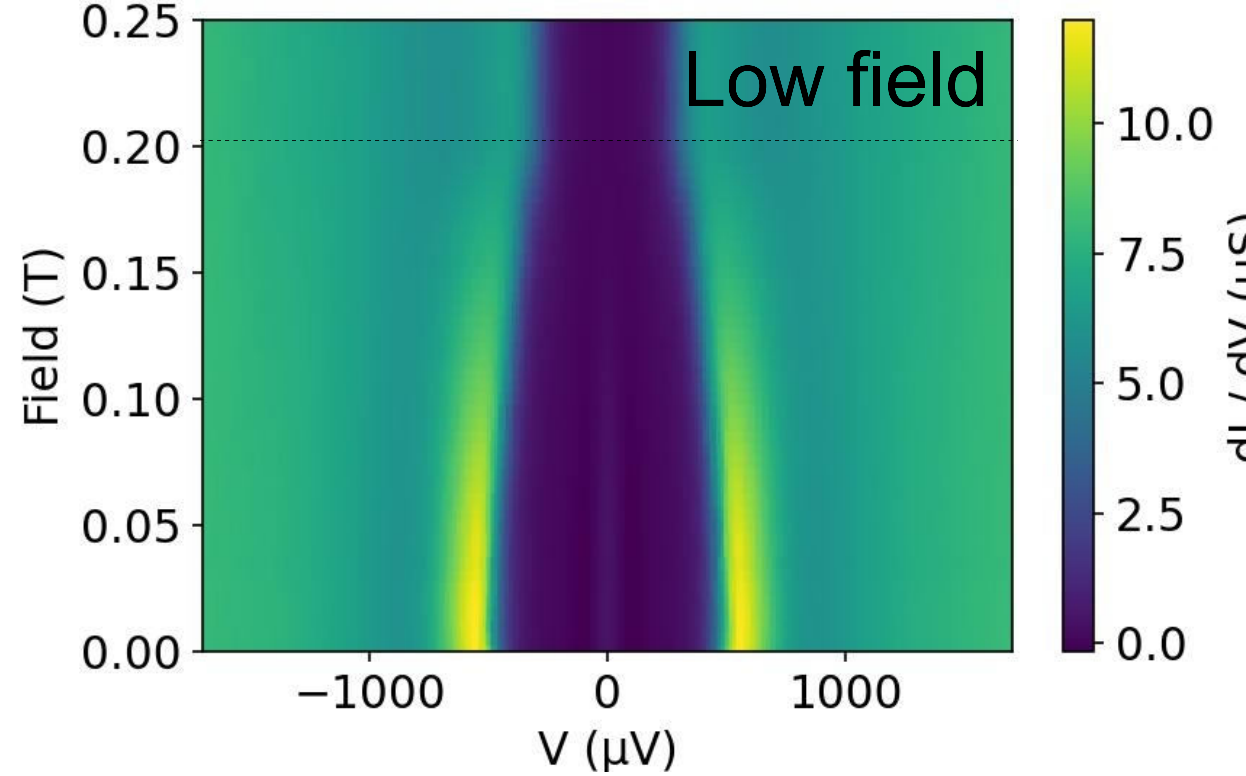


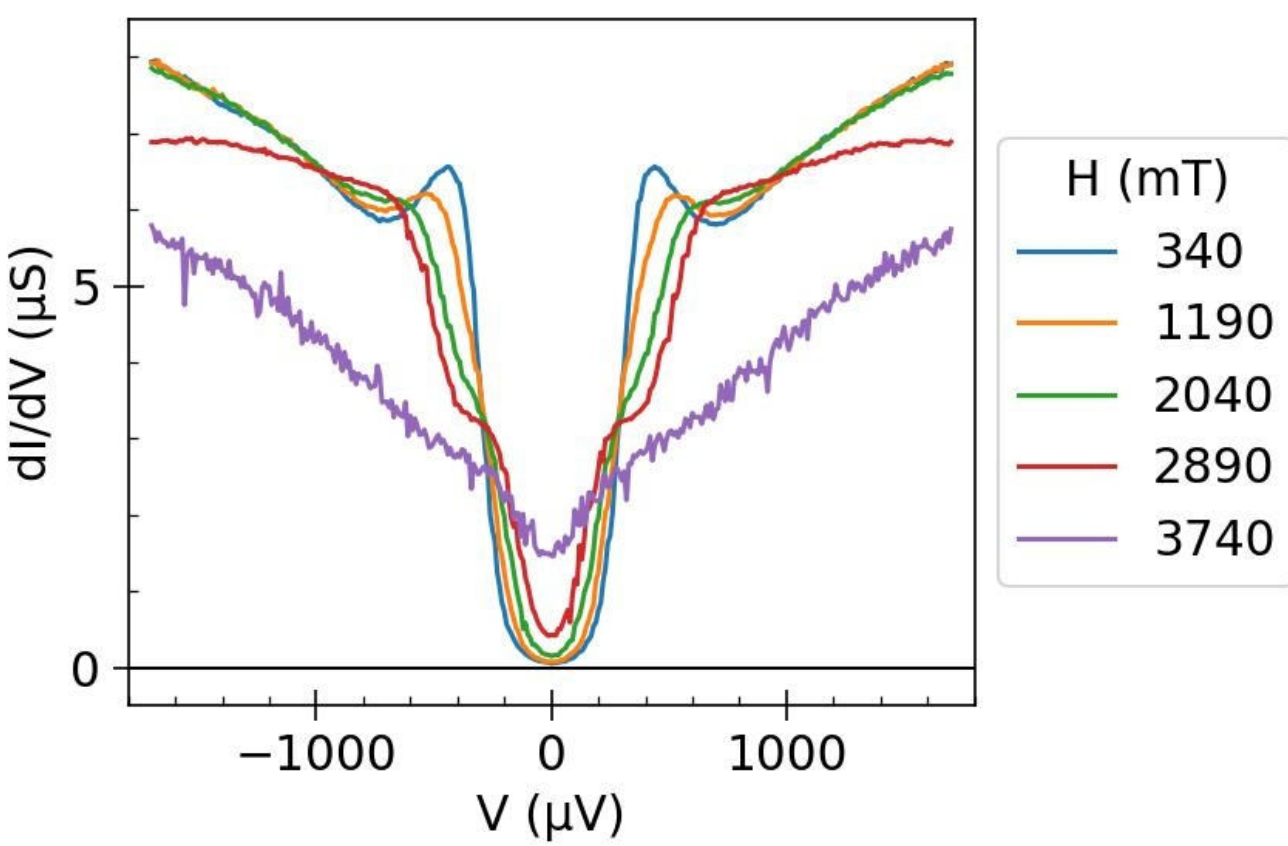
Large zero bias Altshuler-Aronov anomaly → bad for qubit ?

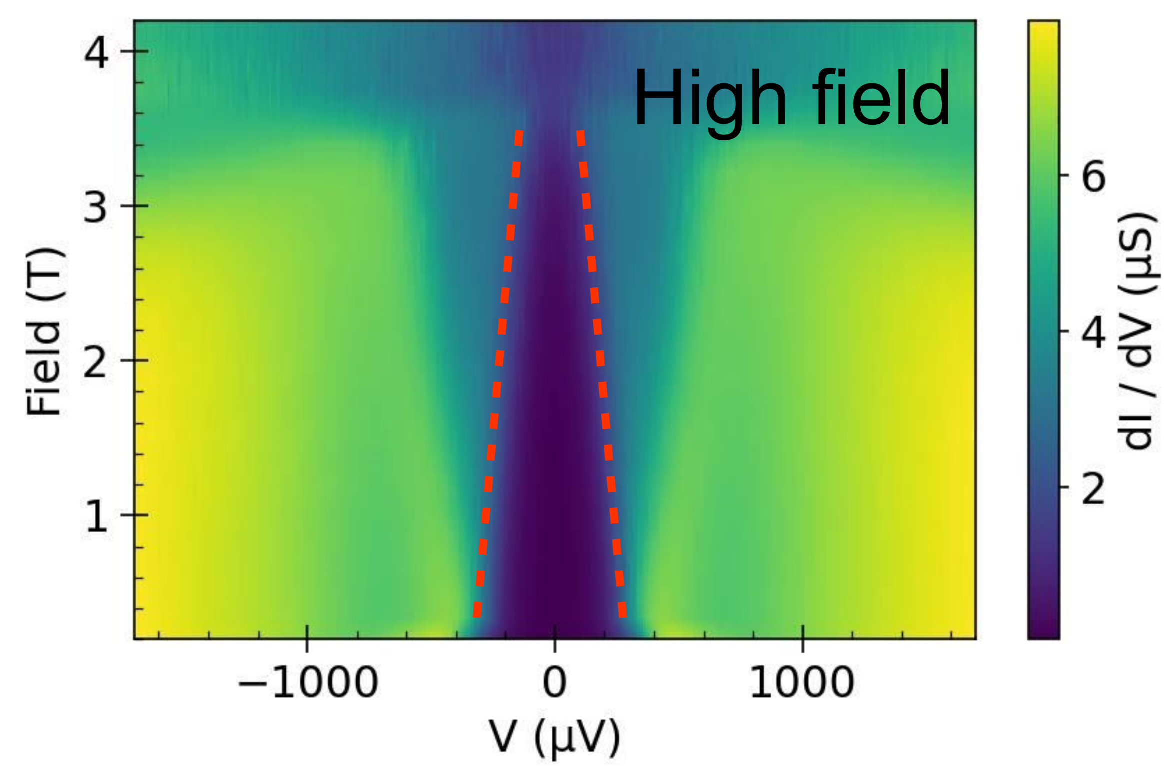
Magnetic Field Dependence

GrAl density of states









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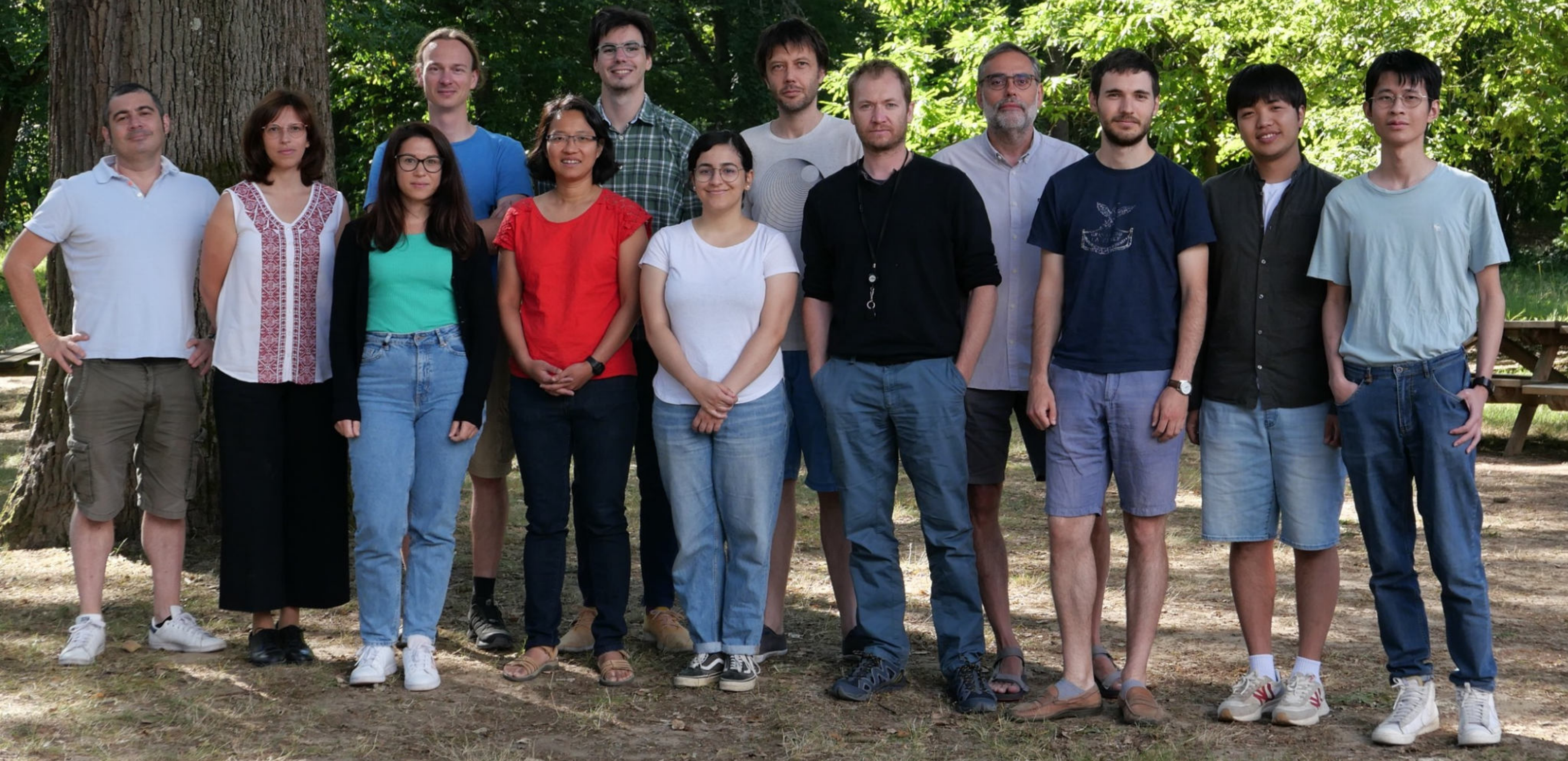


Perspectives

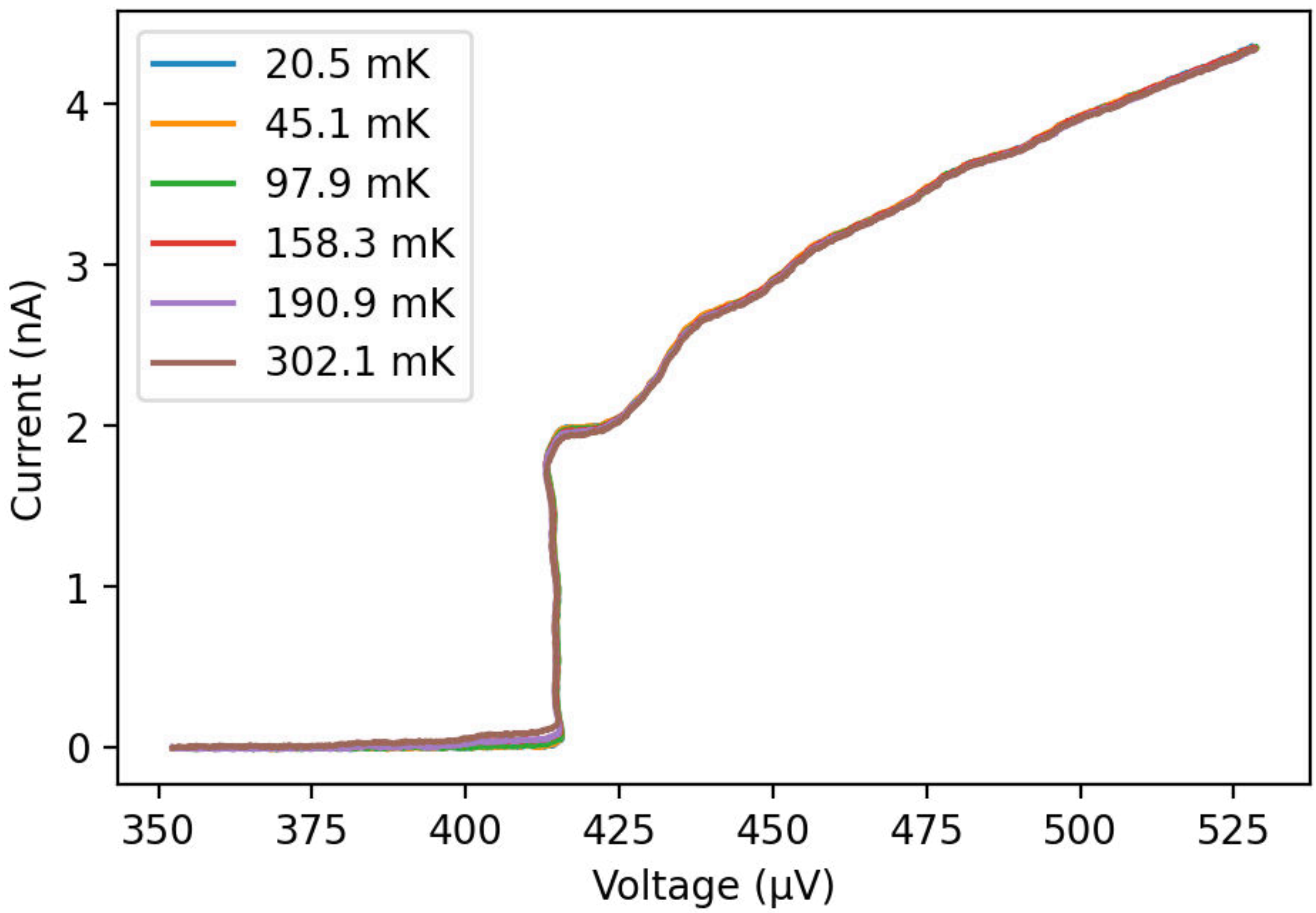
- Quantum bath engineering: Tunable loss channel / Buffer Mode
- Microwave Photon Detector
- Paramp with DC pump
- The very bright side of Coulomb blockade: Many body open quantum system. Bridge ICPT & Bloch oscillations.

ArXiv:1807.02364
HAL tel-03165358
ArXiv:2204.08701

Many thanks to:

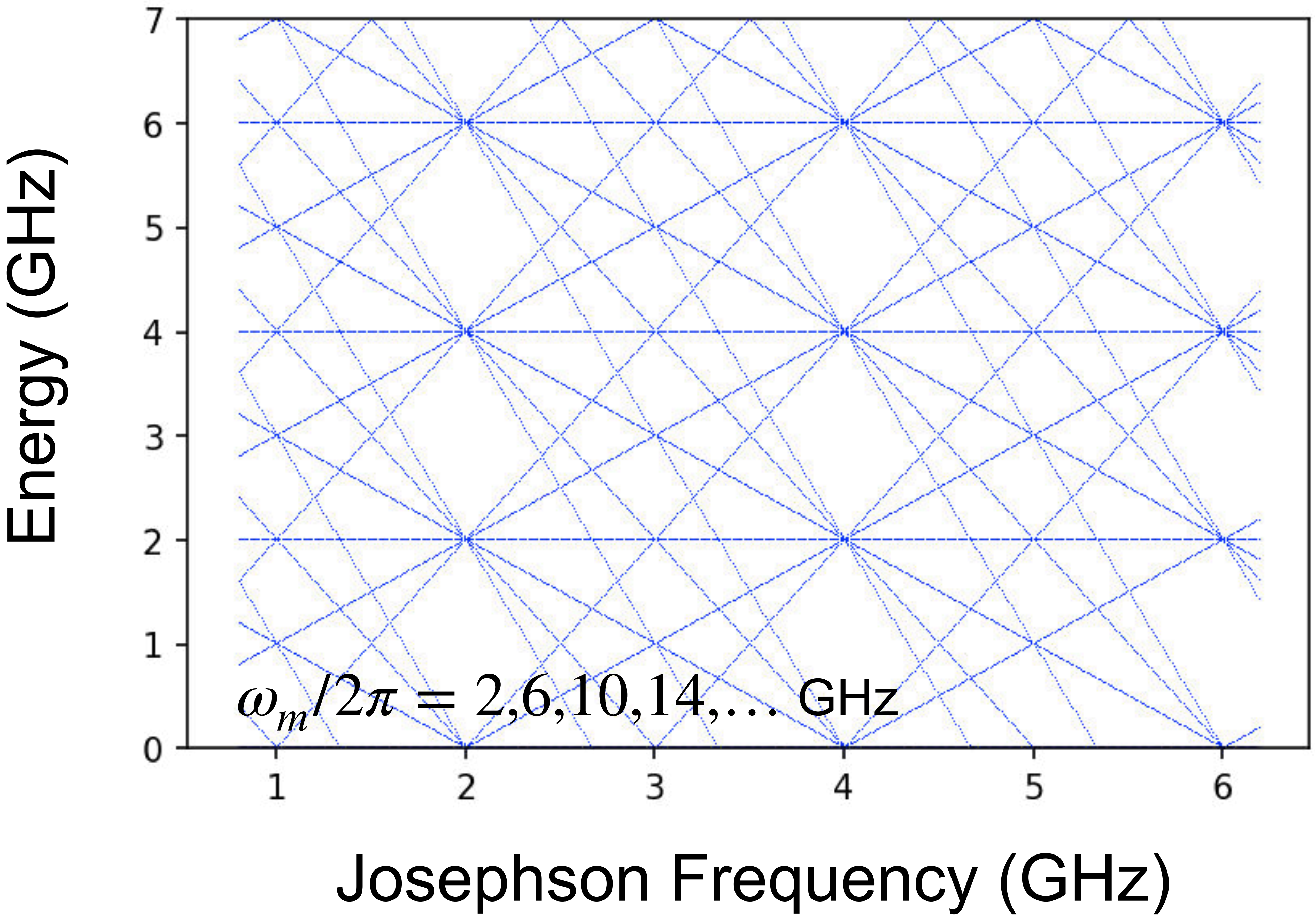
- Fabien Portier, Carles Altimiras, Yuri Mukharski, Ambroise Peugeot
- Claire Marrache-Kikuchi, Hélène Lesueur, Louis Dumoulin
- Nicolas Roch

Inelastic QP tunneling



Dressed State Picture

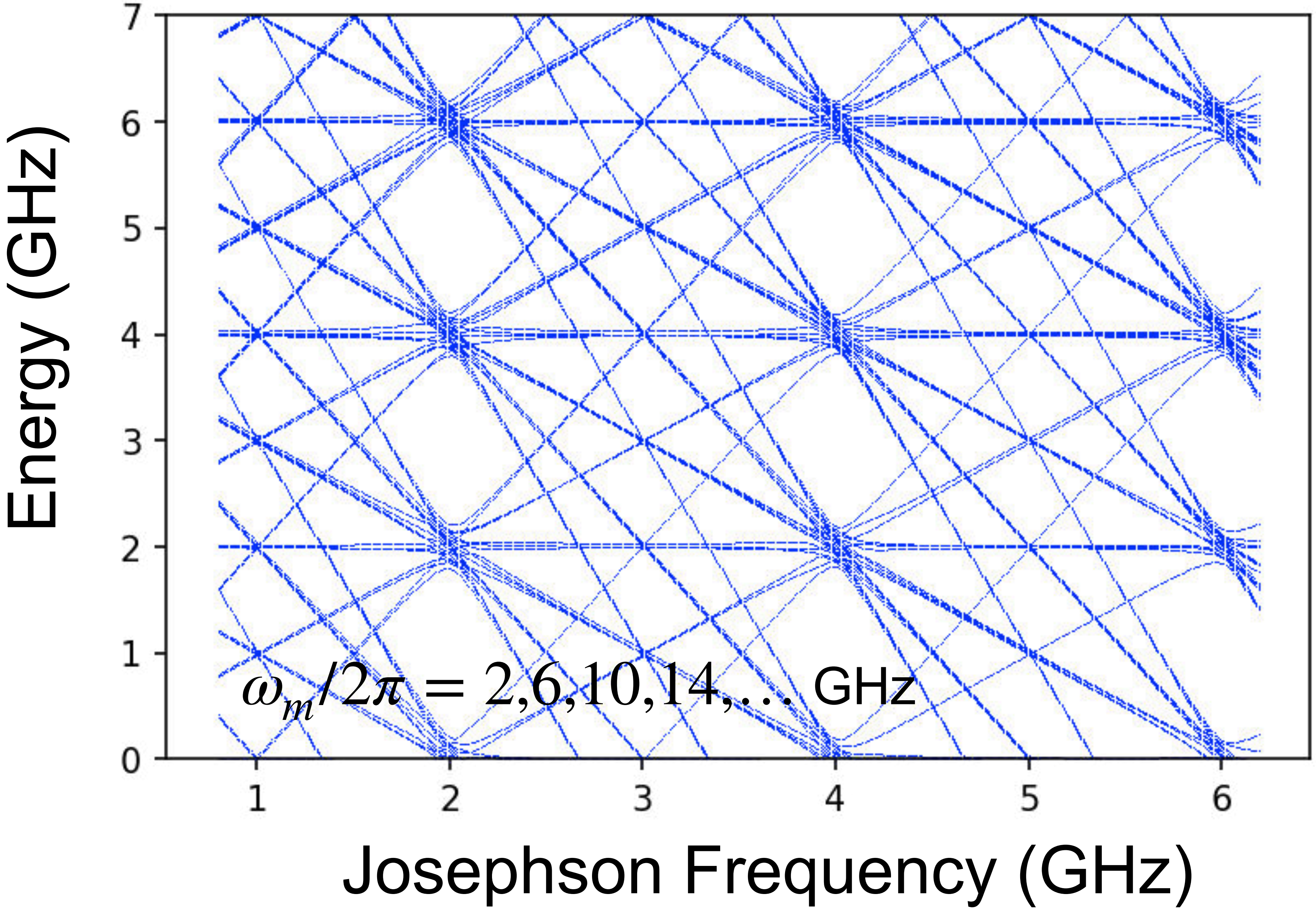
$$|\, n_0, n_1, \ldots; N\rangle \quad \varepsilon = n_0 \hbar\omega_0 + n_1 \hbar\omega_1 + \ldots + N\hbar\omega_J$$



$$\begin{bmatrix} \ddots & & & \\ & H_0 - \omega_J & & \\ & & H_0 & \\ & & & H_0 + \omega_J \\ & & & & \ddots \end{bmatrix}$$

Dressed State Picture

$$H = \sum_m \hbar\omega_m a_m^\dagger a_m - E_J \cos\left[\omega_J t + \sum_m \lambda_m (a_m + a_m^\dagger)\right] = \sum_m \hbar\omega_m a_m^\dagger a_m - (E_J/2) D_\lambda e^{i\omega_J t} - \text{hc}$$



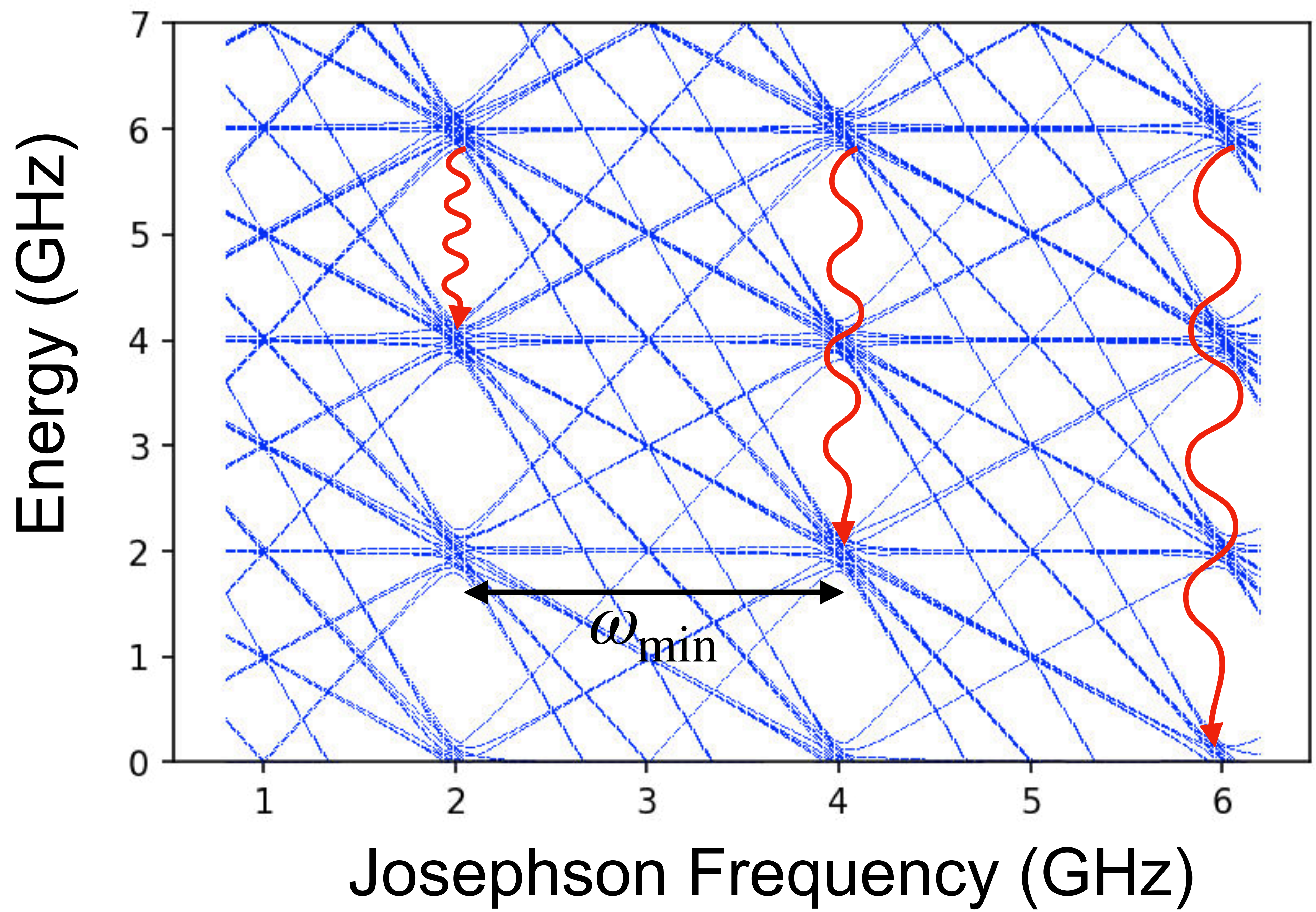
$$D_\lambda = \Pi e^{i\lambda_m (a_m + a_m^\dagger)}$$

$$\begin{bmatrix} \ddots & & & \\ & H_0 - \omega_J & D_\lambda & \\ & D_\lambda^\dagger & H_0 & D_\lambda \\ & & D_\lambda^\dagger & H_0 + \omega_J \\ & & & \ddots \end{bmatrix}$$

Floquet spectrum

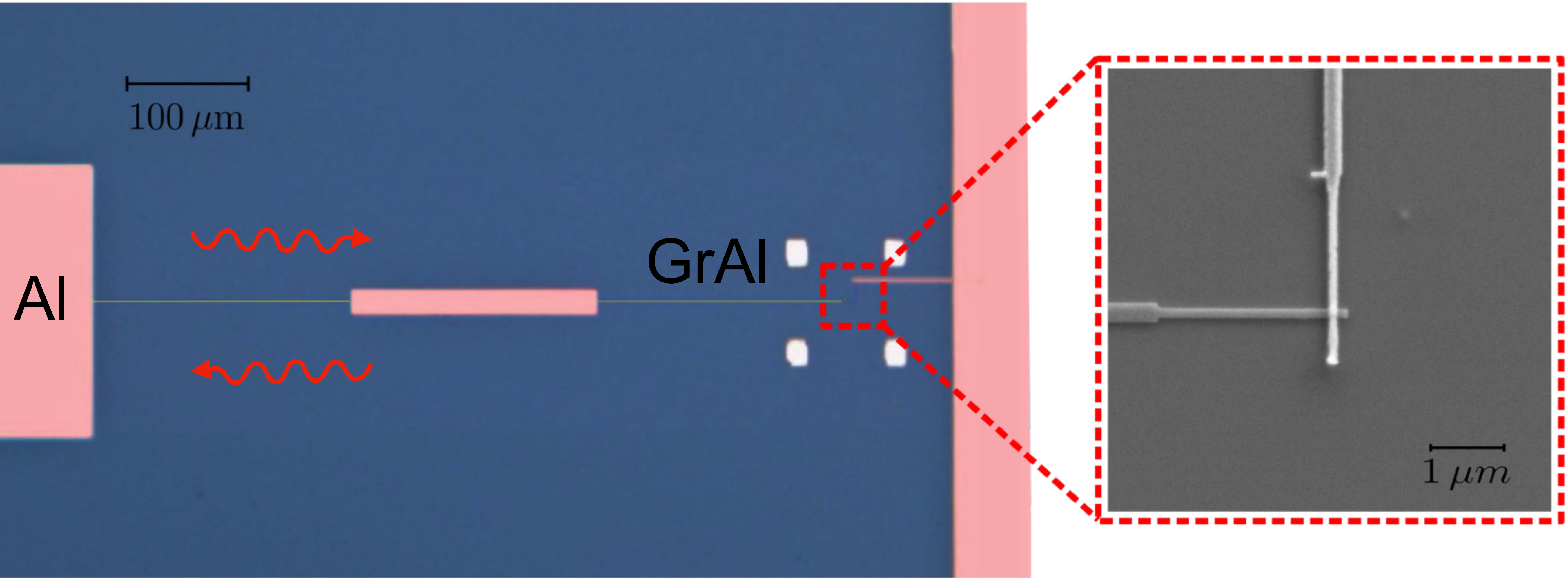
Dressed State Picture

Inelastic Cooper Pair Tunneling ≡ Fluorescence cascade



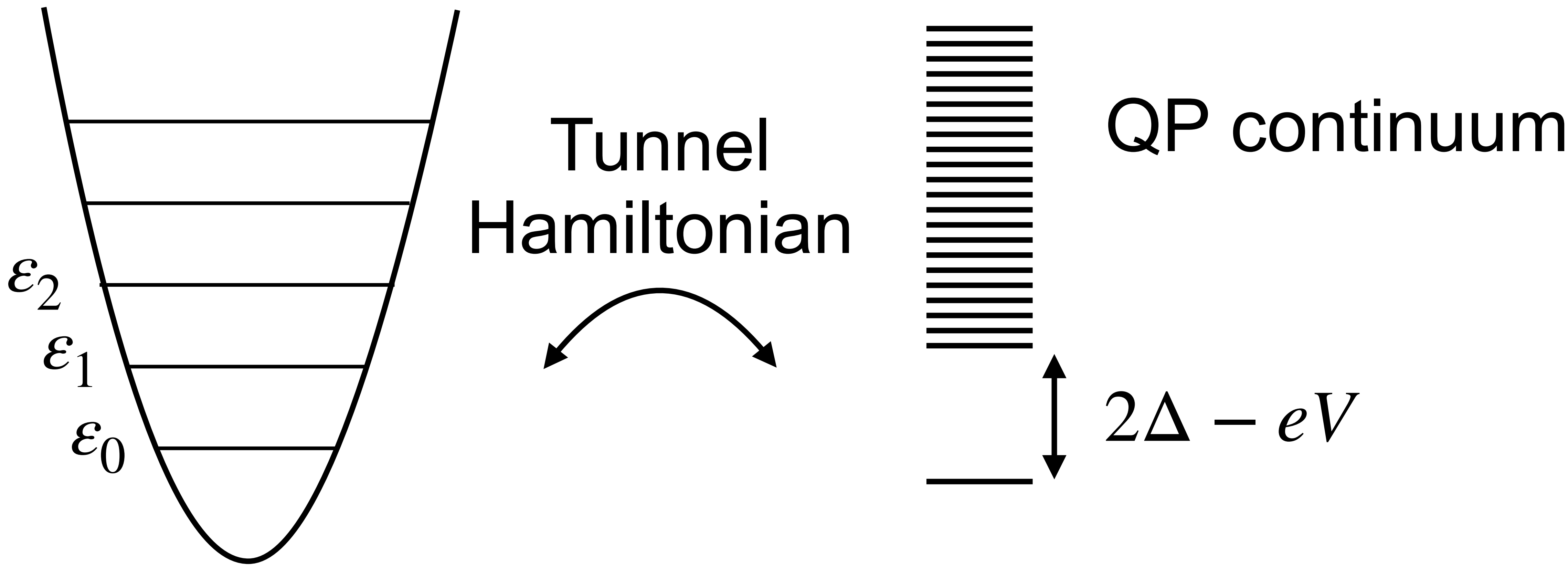
Solve master equation between dressed states

Granular Aluminium Resonator



6 GHz mode $\Lambda = 0.74$

Lamb Shift Series Expansion

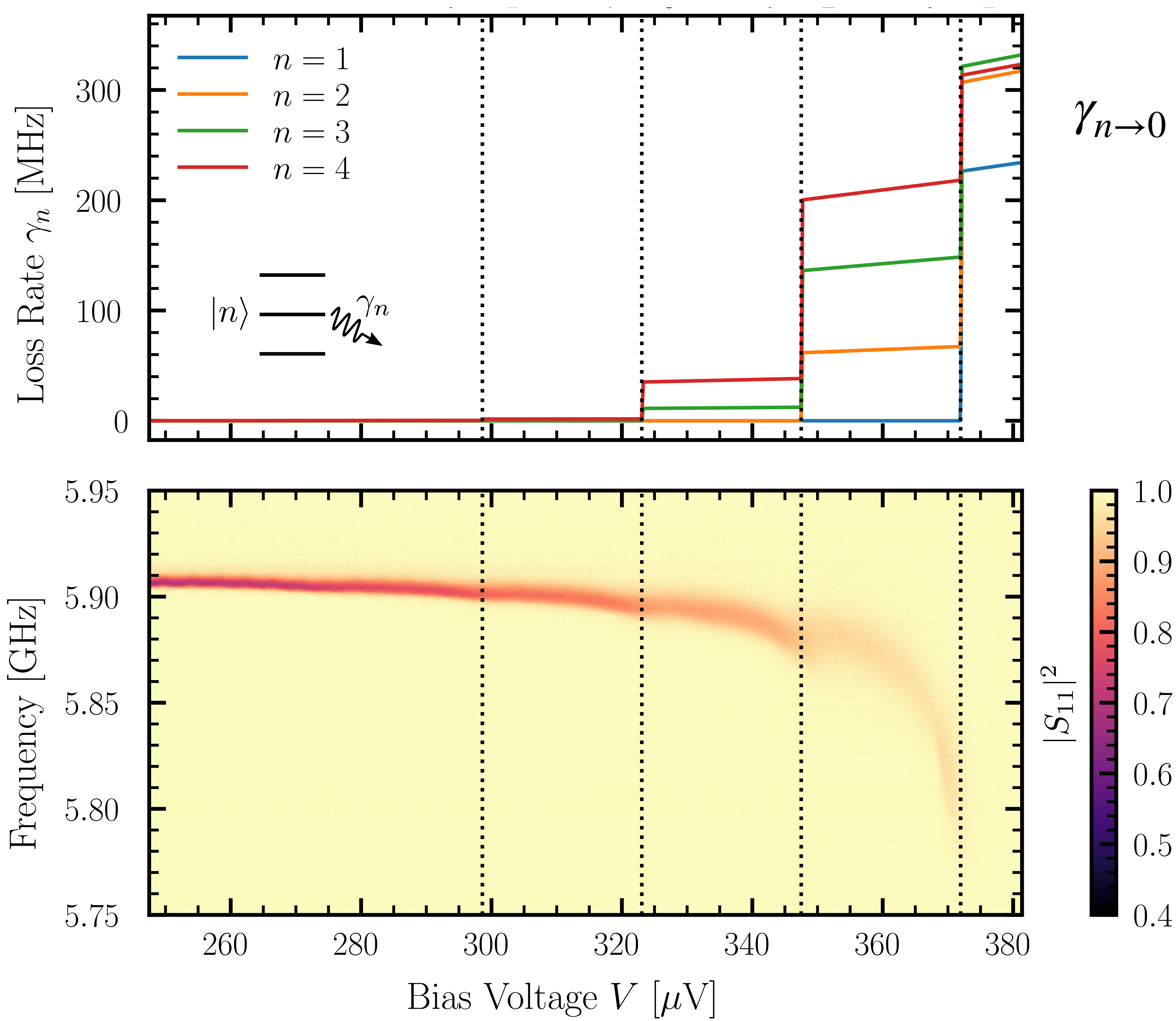


- Perturbation series diverges for ε_n but converges for $\varepsilon_n - \varepsilon_m$
- Similar to the original Lamb shift effect

$$\delta\omega_{01} = -\frac{\Lambda^2 e^{-\Lambda^2}}{2e} \sum_{l=0}^{\infty} \frac{\Lambda^{2l}}{l!} \left(I^{KK}_{-l-1} + I^{KK}_{-l+1} - 2I^{KK}_{-l}\right) \qquad I^{KK}_l = I^{KK}(V + l\hbar\omega/e)$$

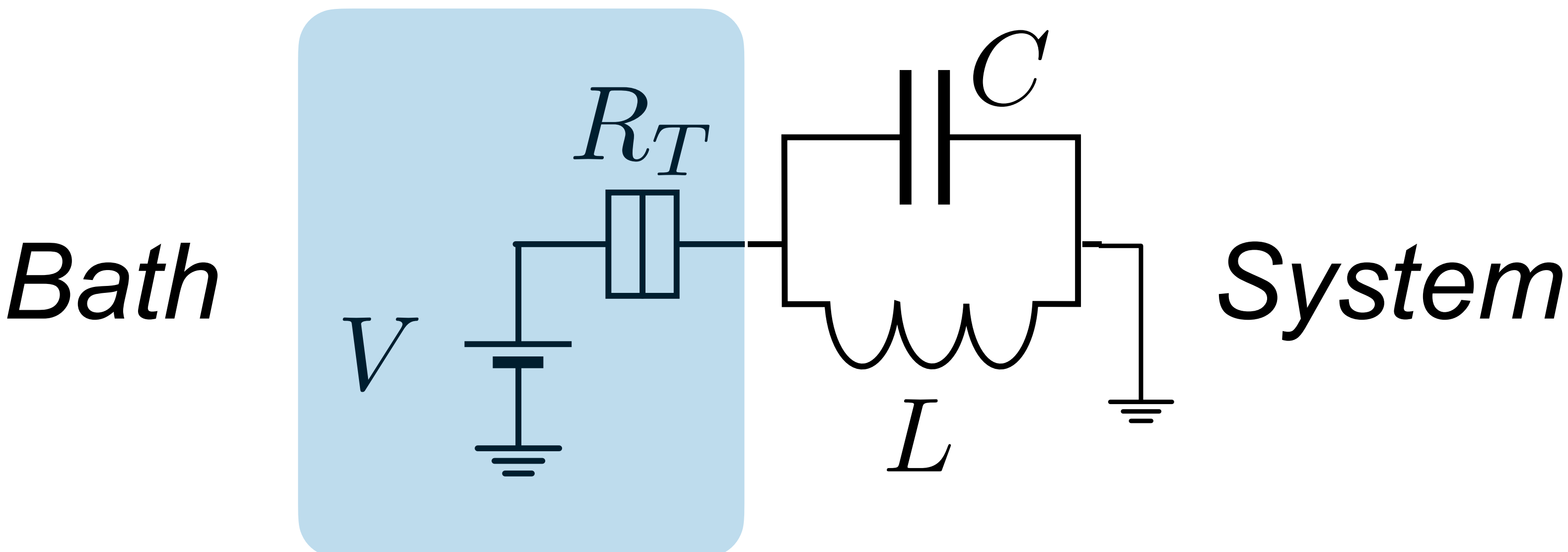
QED expansion $\alpha, \alpha^2, \ldots$

n-Photon Loss Rate



$$\gamma_{n\to 0} = |\langle 0|e^{i\Lambda(a+a^\dagger)}|n\rangle|^2 \times \frac{I(V + n\hbar\omega/e)}{e}$$

Master Equation



$$\Lambda^2 = \pi \frac{Z_c}{R_K}$$

Tunnel Hamiltonian $t \sum_{k,q} c_k^\dagger d_q \, e^{i\Lambda(a+a^\dagger)} + \mathrm{hc}$

$$e^{i\Lambda(a+a^\dagger)} = \sum_{l=-\infty}^{l=+\infty} A_l$$

- creates l photons if $l > 0$
- annihilates l photons if $l < 0$

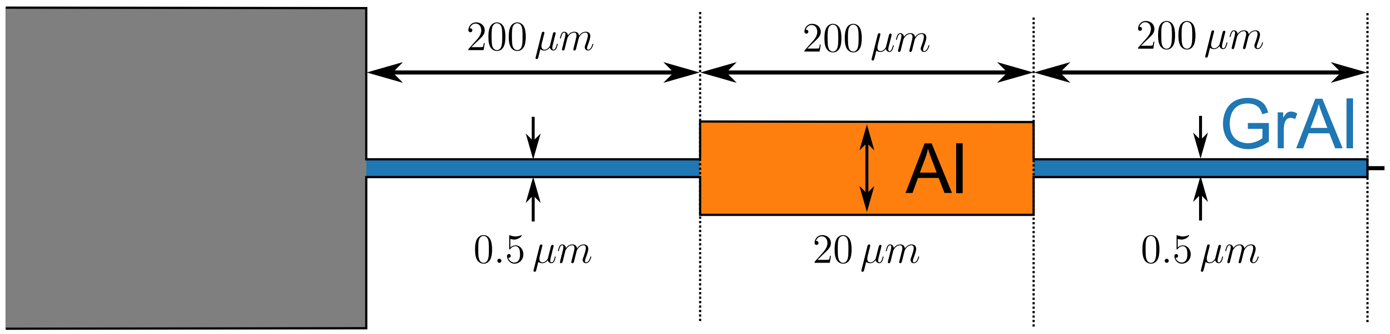
$$\frac{d\rho}{dt} = \sum_l -i\,\epsilon_l \left[A_l A_l^\dagger, \rho\right] + \gamma_l \left(2A_l^\dagger \rho A_l - \{A_l A_l^\dagger, \rho\}\right)$$

Lamb shift

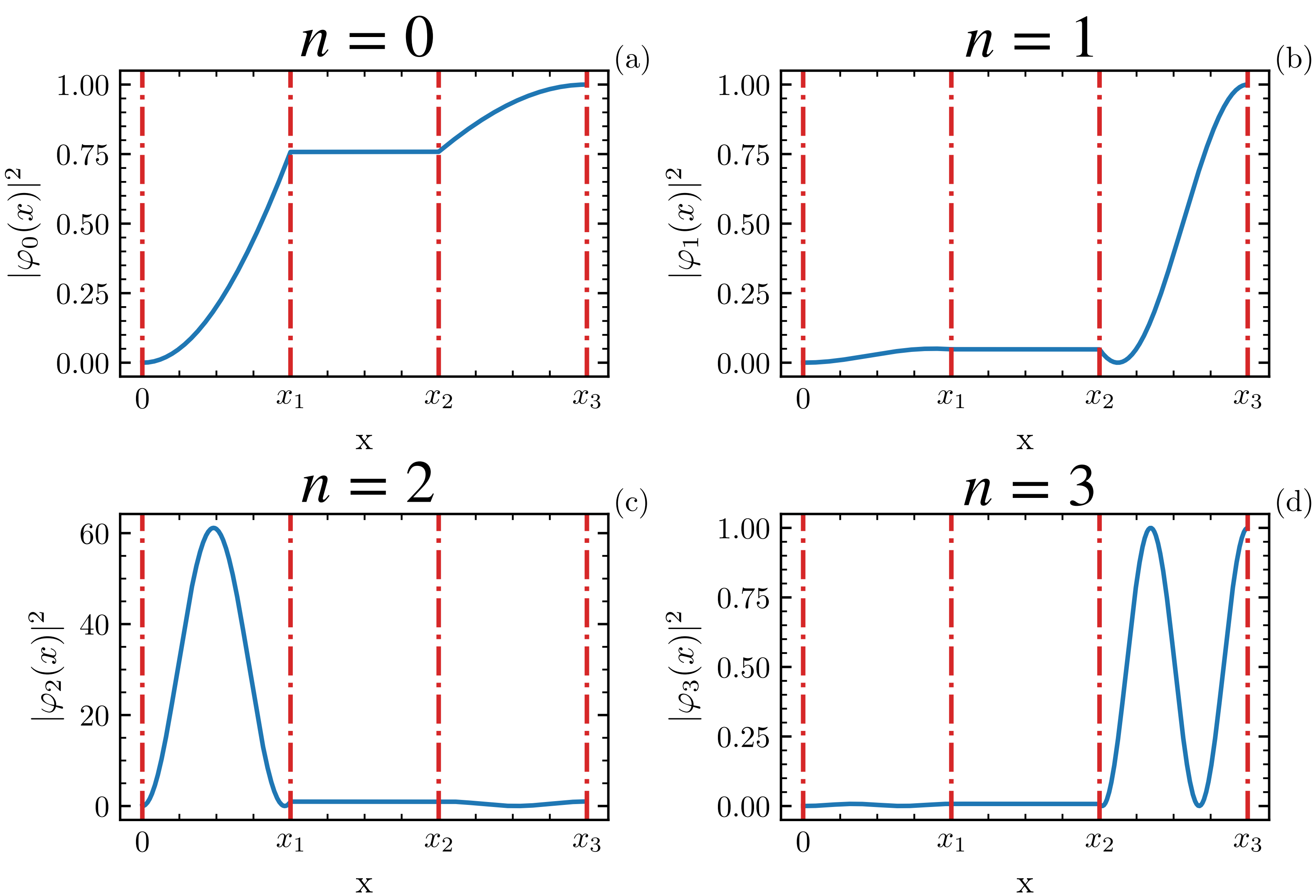
Tunneling events
& Photon exchange

ArXiv:1807.02364

Multimode Structure



$$L_{\square} = 0.65\ \text{nH}$$



	$\omega_n/2\pi$ [GHz]	$\tilde{Z}_{c_n}$ [kΩ]
n = 0	1.8	3.6
n = 1	6.2	5.8
n = 2	11.8	0.017
n = 3	16.9	1.5
n = 4	22.4	2×10^{-3}
n = 5	27.5	0.710
n = 6	33	3×10^{-4}
n = 7	38.2	0.380