First-order superconductor-insulator transition: interplay between superconductivity and Coulomb glass

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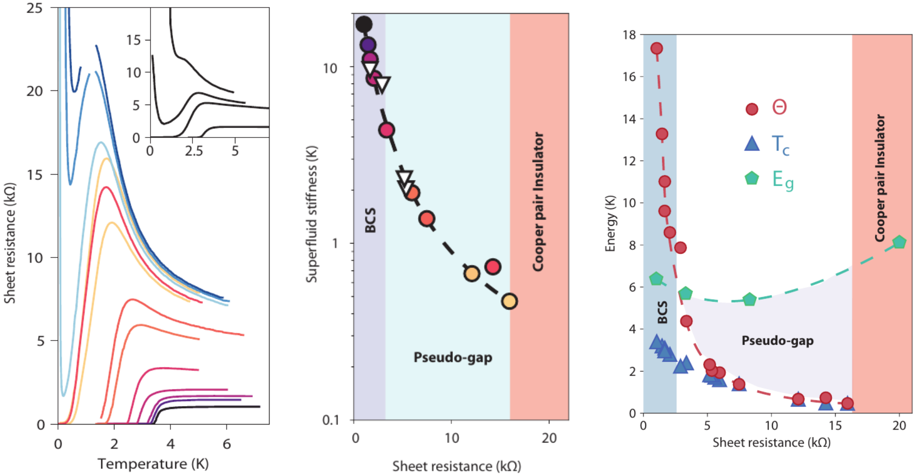
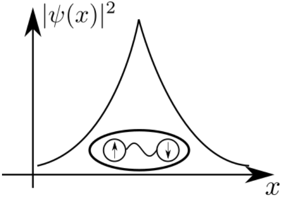
Experimental motivation (InO)



- Jump in superfluid stiffness ($T \approx 20$ mK)
- Large pseudogap

Thibault Charpentier et al. (not yet published)

Anderson pseudospins



- No single-electron excitations $\Rightarrow$ Anderson pseudospin representation:

$$\hat{\sigma}_i^z = 1 - \hat{a}^\dagger_{i,\uparrow}\hat{a}_{i,\uparrow} - \hat{a}^\dagger_{i,\downarrow}\hat{a}_{i,\downarrow}$$

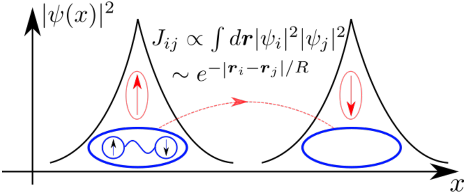
$$\hat{\sigma}_i^+ = 2\hat{a}^\dagger_{i,\uparrow}\hat{a}^\dagger_{i,\downarrow}, \quad \hat{\sigma}_i^- = 2\hat{a}_{i,\downarrow}\hat{a}_{i,\uparrow}$$

- Index i enumerates **localized (multifractal) states** (Anderson localization)

Theoretical model: Hamiltonian

$$\hat{H} = \sum_{\boldsymbol{r}} \varepsilon_{\boldsymbol{r}} \hat{\sigma}^z_{\boldsymbol{r}} - \frac{1}{8} \sum_{\boldsymbol{r} \neq \boldsymbol{r}'} J_{\boldsymbol{r}\boldsymbol{r}'} (\hat{\sigma}^+_{\boldsymbol{r}} \hat{\sigma}^-_{\boldsymbol{r}'} + h.c.) + \frac{1}{2} \sum_{\boldsymbol{r} \neq \boldsymbol{r}'} U_{\boldsymbol{r}\boldsymbol{r}'} \hat{\sigma}^z_{\boldsymbol{r}} \hat{\sigma}^z_{\boldsymbol{r}'}$$

- Random localized band: $\nu_0(\varepsilon_{\boldsymbol{r}}) = \exp\left(-\varepsilon^2_{\boldsymbol{r}}/2W^2\right)/\sqrt{2\pi}W$
- Long-range pair tunelling: $J(\boldsymbol{q}) \simeq J_0(1 - q^2 R^2)$, $R \gg 1$



- Coulomb interaction: $U_{\boldsymbol{r}\boldsymbol{r}'} = e^2/\kappa|\boldsymbol{r} - \boldsymbol{r}'|$
  (NB: interaction of pre-formed Cooper pairs!)

Analytic approach and approximations

- Replica trick:

$$\overline{\ln Z} = \lim_{n\to 0} \frac{\partial}{\partial n} \overline{Z^n}$$

- Plasmon field (Hubbard-Stratanovich):

$$\exp\left(-\frac{\beta}{2}\hat{\sigma}^z \hat{U} \hat{\sigma}^z\right) = \int \mathcal{D}\varphi \exp\left(-\frac{\beta}{2}\varphi \hat{U}^{-1}\varphi + i\beta\varphi\hat{\sigma}^z\right)$$

- RPA approximation for $\mathcal{G}(\boldsymbol{r},\boldsymbol{r}') = \langle \varphi_{\boldsymbol{r}} \varphi_{\boldsymbol{r}'} \rangle$:

$$\hat{\mathcal{G}}^{-1} = \hat{U}^{-1} + \hat{Q}, \quad Q_{ab}(\boldsymbol{r},\boldsymbol{r}') = \left\langle\left\langle \hat{\sigma}^z_{\boldsymbol{r},a} \hat{\sigma}^z_{\boldsymbol{r}',b} \right\rangle\right\rangle$$

- Local approximation (huge disorder):

$$Q_{ab} \approx \delta_{\boldsymbol{r}\boldsymbol{r}'} \cdot \langle\langle \hat{\sigma}^z_a \hat{\sigma}^z_b \rangle\rangle_{\text{loc}}, \quad -\beta n \hat{H}_{\text{loc}} = \frac{\beta^2}{2} \sum_{ab} \hat{\sigma}^z_a \left(W^2 \mathcal{I}_{ab} + G_{ab}\right) \hat{\sigma}^z_b$$

- Self-consistency equation (cactus diagrams): $G_{ab} = -\mathcal{G}_{ab}(\boldsymbol{r} = \boldsymbol{r}')$

Mean-field superconductivity

BCS (neglecting Coulomb)

- Order parameter: $\Delta_i = \langle \hat{a}_{i,\uparrow} \hat{a}_{i,\downarrow} \rangle = \frac{1}{2} \left\langle \hat{\sigma}_i^- \right\rangle$
- Mean-field approximation:

$$\hat{H}_{\text{SC}}^{(\text{MF})} \approx \sum_{\boldsymbol{r}} \left(u_{\boldsymbol{r}} \hat{\sigma}_{\boldsymbol{r}}^z - \frac{1}{2} (\Delta_{\boldsymbol{r}} \hat{\sigma}_{\boldsymbol{r}}^+ + h.c.) \right)$$

- Spatially homogeneous approximation $\Delta_{\boldsymbol{r}} \approx \Delta = \text{const}$ (not quite true — see talk by Anton Khvalyuk):

$$1 = J \int_0^{\omega_D} d\varepsilon \nu(\varepsilon) \frac{\tanh\left(\beta \sqrt{\varepsilon^2 + |\Delta|^2}\right)}{\sqrt{\varepsilon^2 + |\Delta|^2}}$$

$$\boxed{T_c = \frac{4e^\gamma}{\pi} \omega_D e^{-1/\lambda}, \quad \Delta_0 = 2\omega_D e^{-1/\lambda} \quad \lambda = \nu_0 J}$$

- Coulomb gap: suppression of $\nu(\varepsilon)$ at Fermi energy!

Coulomb effect (neglecting glass)

- Dynamical Debye screening:

$$\langle\langle \hat{\sigma}^z \hat{\sigma}^z \rangle\rangle_\Omega = \frac{|\Delta|^2}{\sqrt{|\Delta|^2 + \varepsilon^2}\,(|\Delta|^2 + \varepsilon^2 + (\Omega/2)^2)}$$

$$\widetilde{Q}(\Omega) = \overline{\langle\langle \hat{\sigma}^z \hat{\sigma}^z \rangle\rangle_\Omega} = 2\nu_0 \frac{\theta}{\sinh\theta}, \quad \sinh\frac{\theta}{2} = \frac{\Omega}{2\Delta}$$

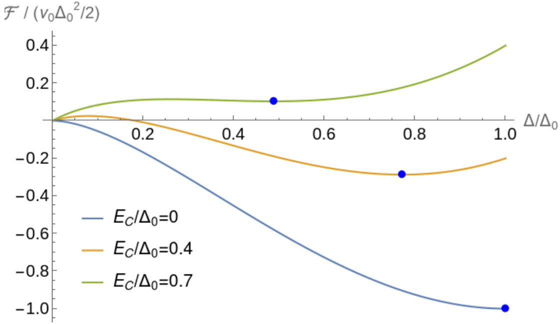
- Gives rise to non-analytic in Δ contribution to energy (RPA, $T = 0$):

$$\mathcal{F} \approx \text{const} \cdot \nu_0 |\Delta| E_C - \frac{1}{2}\nu_0 |\Delta|^2 \ln\frac{e\Delta_0^2}{|\Delta|^2}, \quad \text{const} \approx 1.9467$$

where $E_C = e^3\sqrt{2\pi\nu_0/\kappa^3}$

BCS + Coulomb ($T = 0$)

It is possible to get SC state with positive energy!

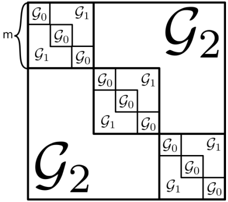


Coulomb glass

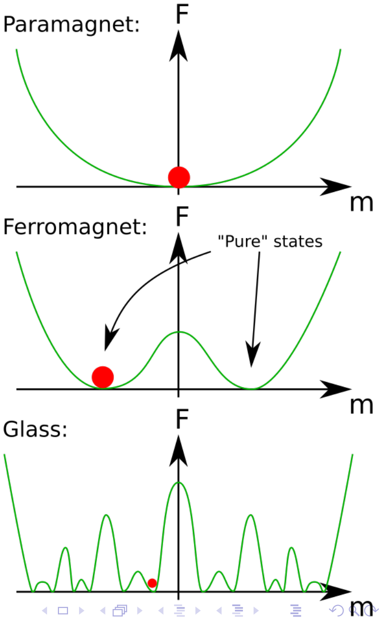
Based on:
M. Mueller, L.B. Ioffe, PRL **93**, 256403 (2004),
M. Mueller, S. Pankov, PRB **75**, 144201 (2007)

Replica symmetry breaking: Parisi ansatz

- RSB ansatz:



- Mathematically: spontaneous symmetry breaking $S_n \mapsto (S_m)^{n/m} \otimes S_{n/m}$.
- Infinite order hierarchical structure: $Q_{ab} \mapsto Q(x \in [0,1])$
- Breaking of ergodicity!



RSB description

- Parisi equations:

$$-\partial_x m = \frac{1}{2} G'(x) \left(\partial_\varepsilon^2 m + \beta x \partial_\varepsilon (m^2) \right), \quad m(x=1,\varepsilon) = \tanh \beta\varepsilon$$

$$\partial_x \nu = \frac{1}{2} G'(x) \left(\partial_\varepsilon^2 \nu - 2\beta x \partial_\varepsilon (\nu m) \right), \quad \nu(x=0,\varepsilon) = \nu_0(\varepsilon) \approx \text{const}$$

- Self-consistency equations:

$$\hat{G} = 2E_C \sqrt{\hat{Q}/2\nu_0},$$

$$Q(x) = \beta \int d\varepsilon \nu(x,\varepsilon) m^2(x,\varepsilon)$$

- Non-trivial solution appears when ("marginal stability"):

$$\frac{E_C}{2\nu_0} \int d\varepsilon \nu_0(\varepsilon) \left(\frac{\partial m(x=1,\varepsilon)}{\partial \varepsilon} \right)^2 = 1 \quad \Rightarrow \quad T_G = \frac{2}{3} E_C$$

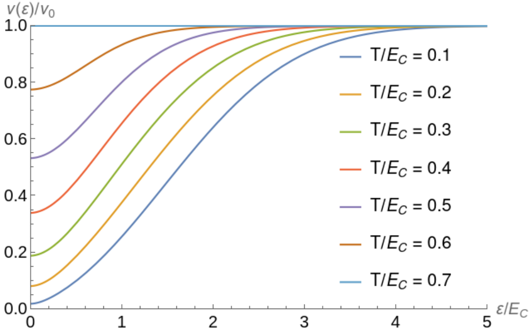
- Competition with superconductivity!

RSB: solution (Coulomb gap)

- Soft gap at Fermi energy:

$$\nu(T \ll |\varepsilon - \mu| \ll E_C) = \text{const} \times \nu_0 \left(\frac{\varepsilon - \mu}{E_C}\right)^2, \quad \text{const} \approx 0.164$$

- Numerical solution of Parisi equations ($T_G = \frac{2}{3} E_C$):



Competition between Coulomb glass and superconductivity

Stability

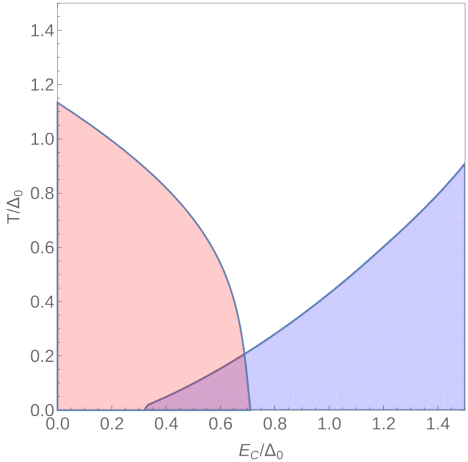
- Q: where is glass stable w.r.t. appearance of SC order parameter?
- A: Cooper stability:

$$\frac{1}{\nu_0}\int_0^\infty \mathrm{d}\varepsilon \ln\frac{2\varepsilon}{\Delta_0}\frac{\partial}{\partial\varepsilon}\left(\nu(\varepsilon)\tanh\beta\varepsilon\right) > 0$$

- Q: where is superconductor stable w.r.t. appearance of glass order parameter?
- A: marginal stability:

$$\frac{E_C}{2}\int d\varepsilon\left(\frac{\partial m}{\partial\varepsilon}\right)^2 < 1, \quad m(\varepsilon) = \frac{\varepsilon}{\sqrt{\varepsilon^2 + |\Delta(T)|^2}}\tanh\beta\sqrt{\varepsilon + |\Delta(T)|^2}$$

Sketch of the phase diagram



- Glass phase stable w.r.t. superconductivity
- Superconducting phase stable w.r.t. glass

Conclusions

- 1^{st} order PT between Coulomb glass and superconductor
- Relevant energy scales: $\Delta_0 \sim E_C = e^2\sqrt{2\pi\nu_0/\kappa^3}$
- Estimates: $\kappa \sim 1000$, $\nu_0 \sim 5 \cdot 10^{32}$ erg^{-1} cm^{-3}
- $E_C \sim 1.5\ K$ (reasonable)

Thank you for your attention!