The Abdus Salam
International Centre for Theoretical Physics

QuanDi-2023,
Les Huche,
05 June 2023

ELECTRON-PHONON COOLING POWER IN "MULTIFRACTAL" ANDERSON INSULATORS:

V.E.Kravtsov

ICTP, Trieste

Collaboration:

M.V.Feigelman

Phys. Rev. B v.99, 125415 (2019)

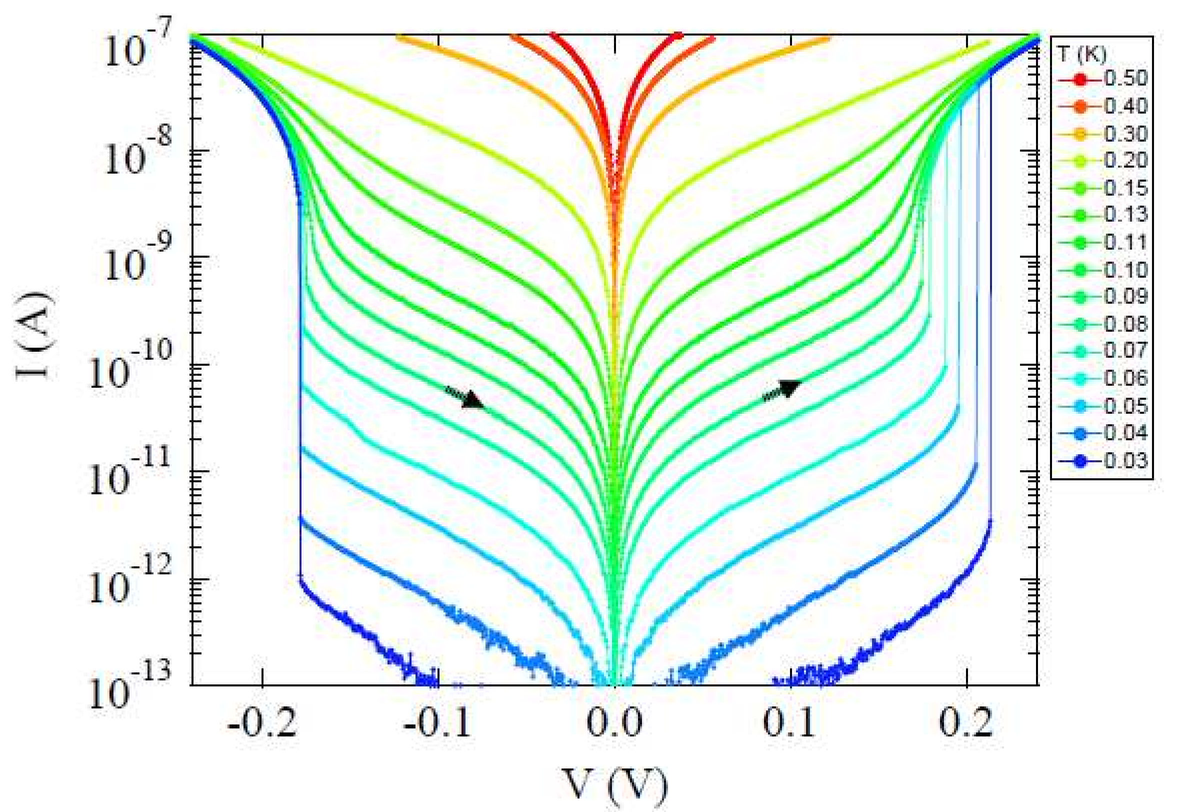
Jumps in I-V

Sambandamurthy, Engel, Johansson, Peled, Shahar, ’05
Baturina, Mironov, Vinokur, Baklanov, Strunk,’07

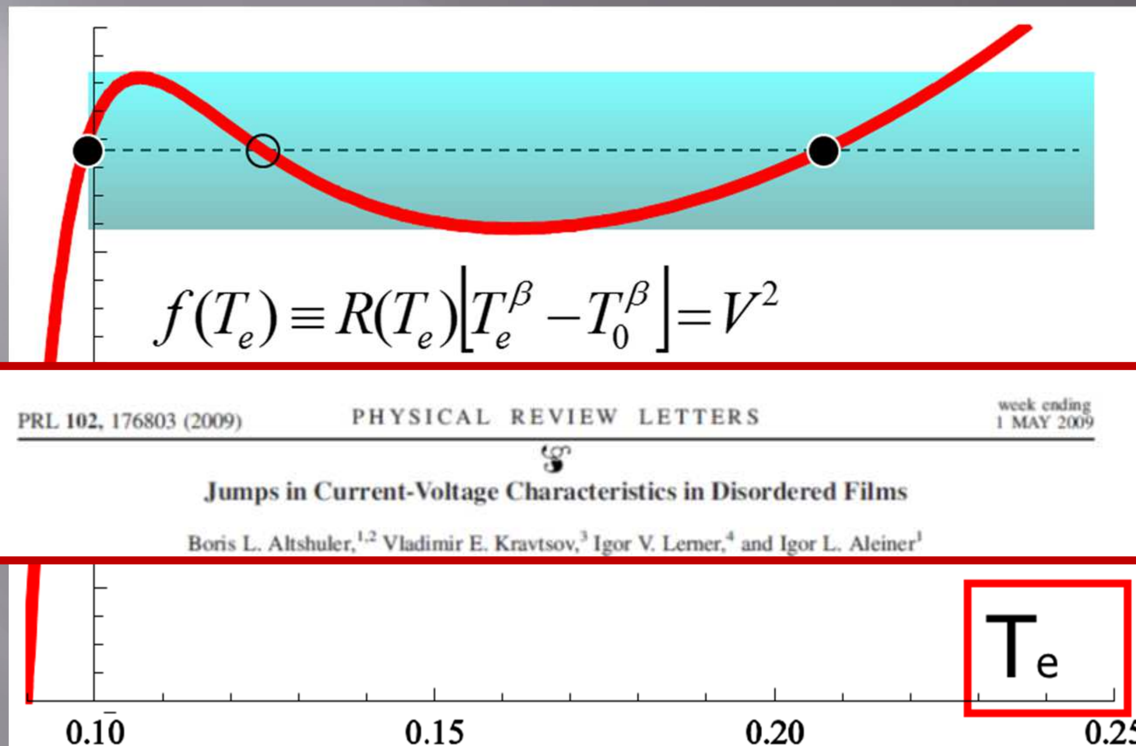
PRL **102**, 176802 (2009) PHYSICAL REVIEW LETTERS week ending 1 MAY 2009

Electron-Phonon Decoupling in Disordered Insulators

M. Ovadia, B. Sacépé,* and D. Shahar
Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel
(Received 28 January 2009; published 28 April 2009)



Thermal bistability



PRL 102, 176803 (2009) PHYSICAL REVIEW LETTERS week ending 1 MAY 2009

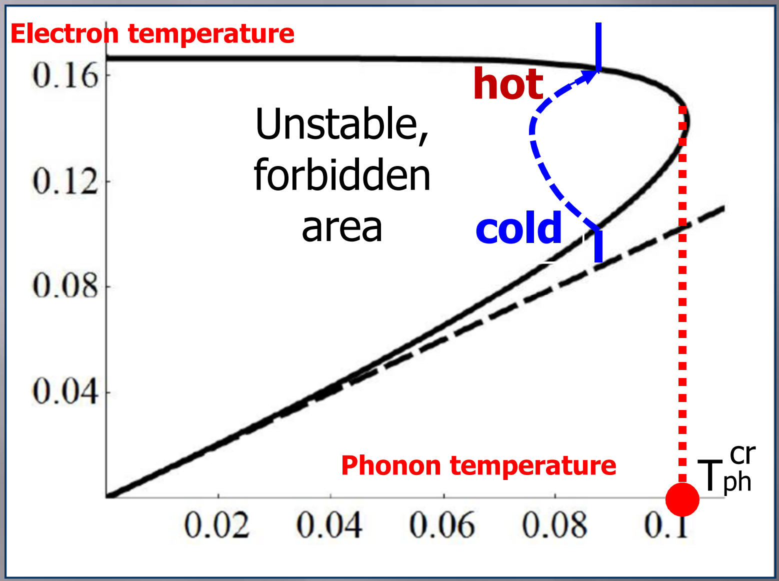
Jumps in Current-Voltage Characteristics in Disordered Films

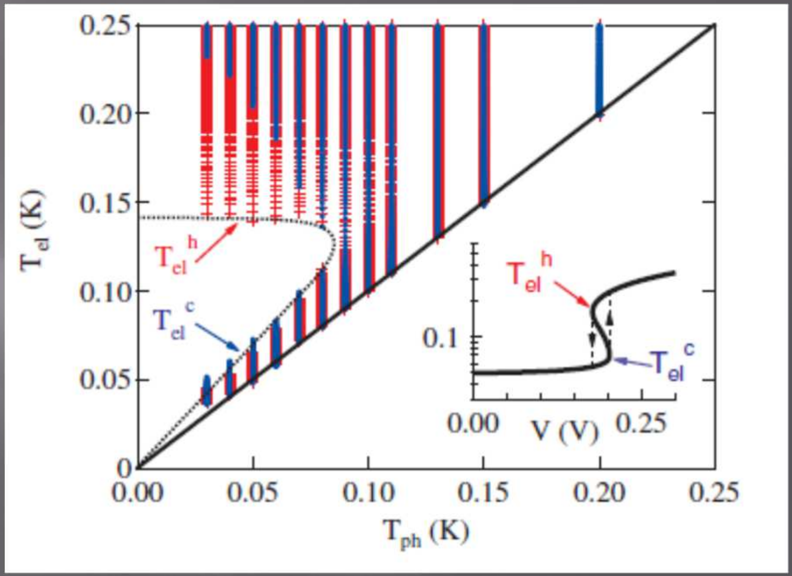
Boris L. Altshuler,[1,2] Vladimir E. Kravtsov,[3] Igor V. Lerner,[4] and Igor L. Aleiner[1]

*Joule heating (T_e, V) = **Phonon Cooling** (T_e, T_0)*

$$\frac{V^2}{R(T_e)} = W(T_e) - W(T_0)$$

Comparison of theory and experiment



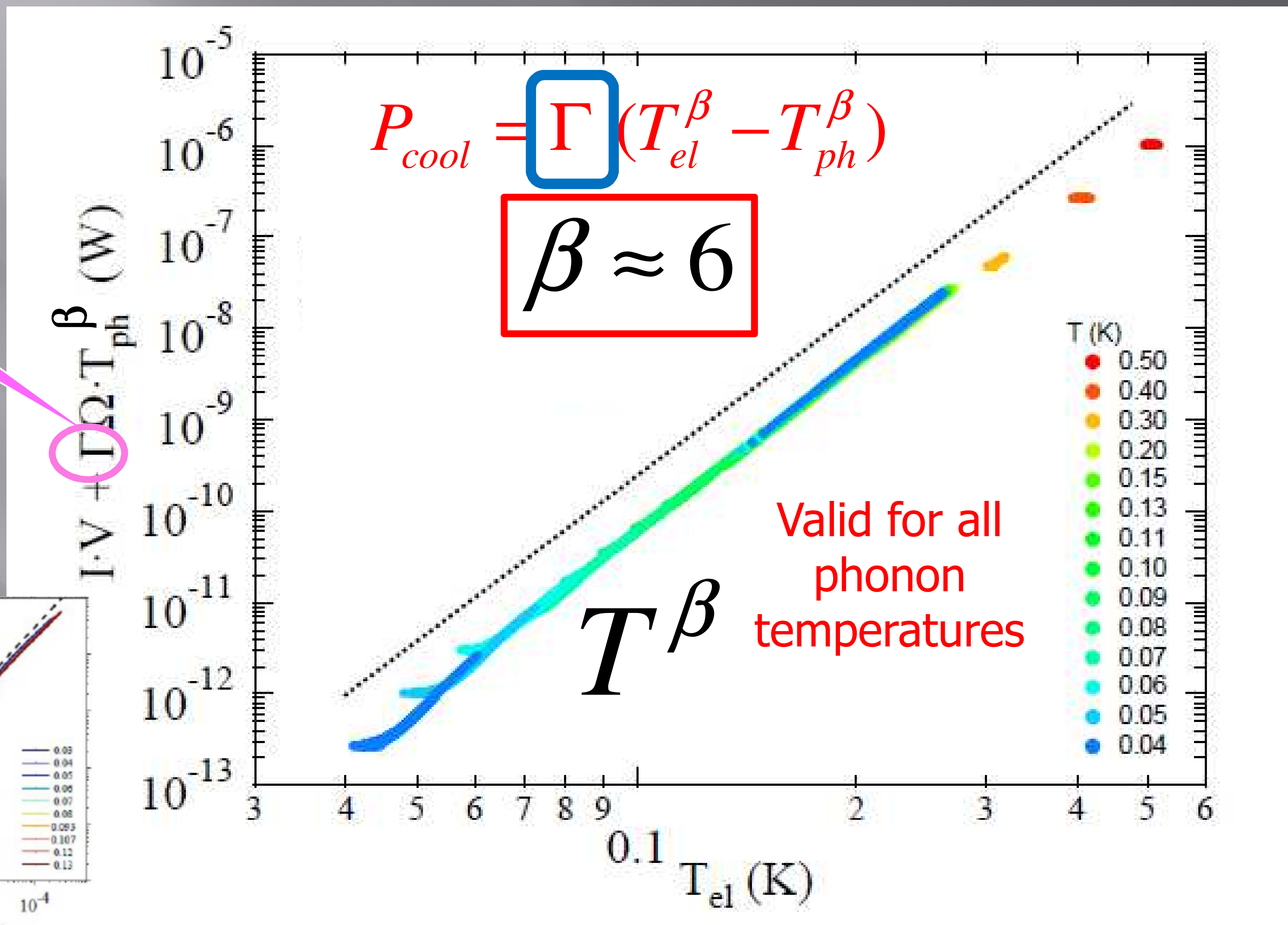


Theory: Altshuler, VEK, Lerner, Aleiner, 2009

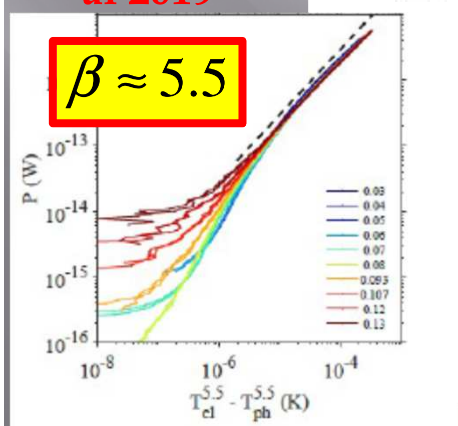
Experiment: Ovadia, Sacepe, Shahar, 2009

Experimental cooling rate

Fitting const.



Levinson et al 2019



$$P_{cool} = \Gamma \, (T_{el}^6 - T_{ph}^6)$$

experiment:

$$\Gamma = 1.7 \times 10^3 \; W \; cm^{-3} \; K^{-6}$$

Theory: Dirty electrons interacting with phonons, but NO LOCALIZATION

$$\Gamma_{\text{theor}} = \frac{(\mathrm{k_F}\ell)\, \mathrm{n_{el}}}{\hbar^4 \rho_i v_s^5}$$

$$\Gamma_{\text{theor}} \sim 1 \; W \; cm^{-3} \; K^{-6}$$

- Why Γ is so large?

Other similar discrepancies

$$\Gamma_{theor} = \frac{(k_F \ell)\, n_{el}}{\hbar^4 \rho_i v_s^5}$$

Metallic Ti with

M.Gershenzon, Appl. Phys. Lett., 2000

$$n_{el} \approx 6 * 10^{22}\, cm^{-3},\ k_F l \sim 6$$

Weakly insulating $Nb_x Si_{1-x}$ with

S. Marnieros, L. Bergé, A. Juillard, and L. Dumoulin, Phys. Rev. Lett. 84, 2469 (2000).

$$n_{el} \approx 6 * 10^{21}\, cm^{-3},\ k_F l \sim 0.3$$

Cooling power in Ti should be 200 times stronger than in NbSi but it is 3 times weaker at T=100 mK

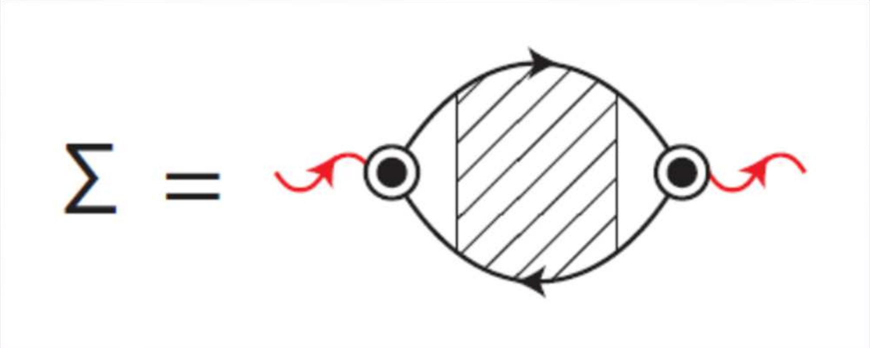
Electron-phonon cooling rate

$$P_{cool}(T_{el},T_{ph}) = \int_0^\infty d\omega\,\omega\,\nu_{ph}(\omega)\,\frac{B_{ph}(\omega)}{\tau_{ph}(\omega)}$$

$$B_{ph}(\omega) = \frac{1}{2}[\coth(\omega/2T) - 1]$$

$$\frac{1}{\tau_{ph}} = \frac{1}{2\rho_i\,\omega}\,\mathrm{Im}\left(\Sigma_\omega^R - \Sigma_\omega^A\right)$$

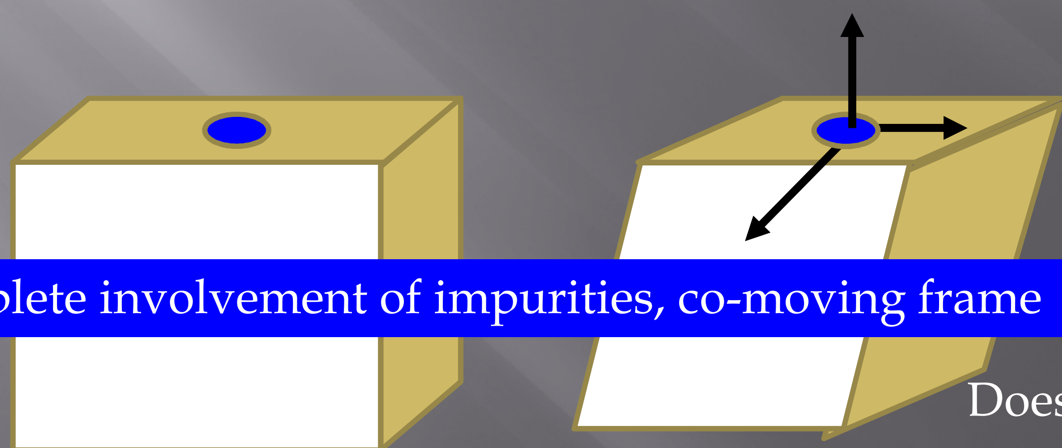
Measurement of ultrasound attenuation



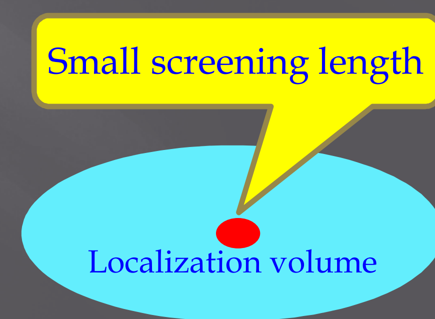
Expression in terms of wave functions

$$\frac{1}{\tau_{\rm ph}^{(1)}} = \pi \frac{q_\beta q_\delta}{m^2} e_\alpha e_\gamma \frac{1}{\rho_i}\int d^d\mathbf{R}\, e^{i\mathbf{qR}}\, K_{\alpha\beta\gamma\delta}(\mathbf{R},\omega)$$

$$K_{\alpha\beta\gamma\delta}(\mathbf{R};\omega) = \Big\langle \sum_{nm} [\partial_\alpha\psi_m^*(\mathbf{r})][\partial_\beta\psi_n(\mathbf{r})][\partial_\gamma\psi_n^*(\mathbf{r}')][\partial_\delta\psi_m(\mathbf{r}')] \times \delta(E-E_n)\,\delta(E'-E_m)\Big\rangle.$$

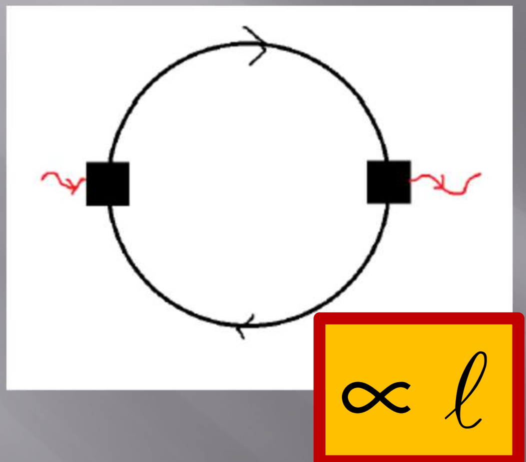


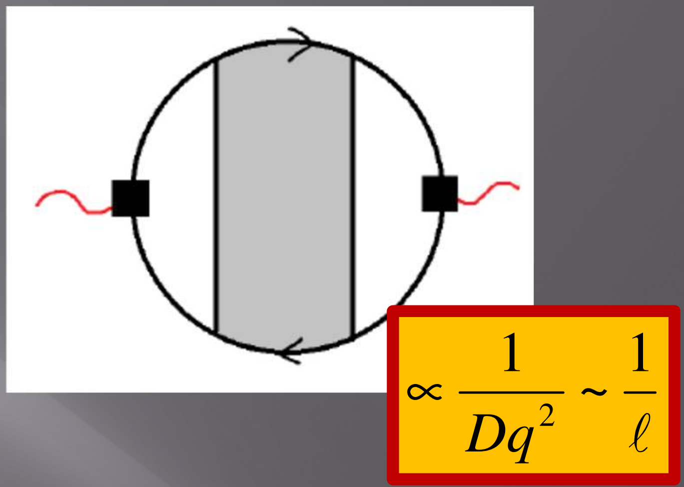
Complete involvement of impurities, co-moving frame

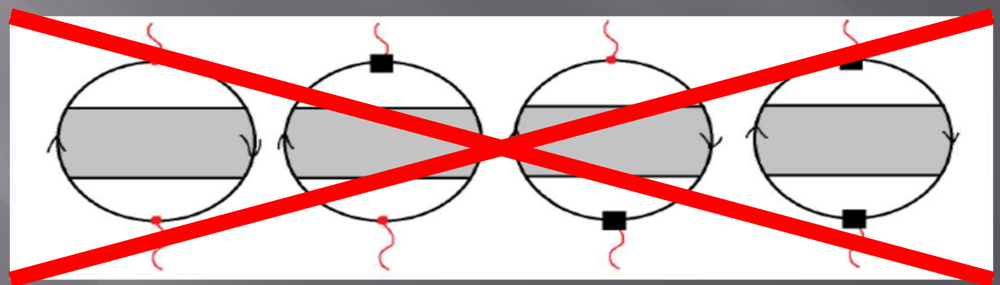


Does not work for strong insulators with small carrier density

Local and non-local effects







Non-local diagrams cancel out due to electro-neutrality at complete screening

Enhancement factor

Local contribution to phonon relaxation rate (|r-r′|< l):

$$\frac{1}{\tau_{ph}} = \left(\frac{1}{\tau_{ph}}\right)_{\substack{dirty\\metal}} \times K(\omega)$$

LDOS corr. function

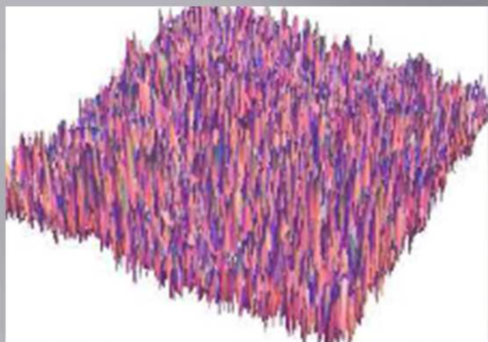
$$K(\omega) = \Delta^2 \left\langle \sum_{n,m} \phi_n^2(\mathbf{r})\, \phi_m^2(\mathbf{r})\, \delta(E - E_n)\delta(E + \omega - E_m) \right\rangle$$

$\phi_m(r)$ is the envelope of wave function

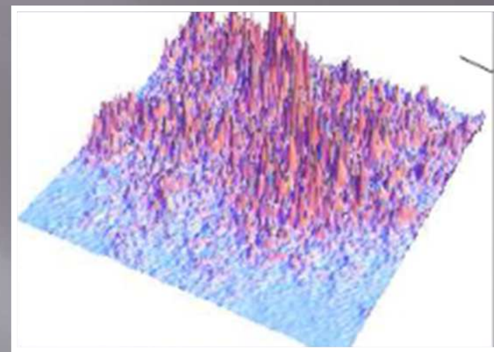
$$\psi_n(r) = \varphi_{diff}(r) \times \phi(r)$$

$\phi_m(r)$ is the envelope of wave function that contains the localization effects

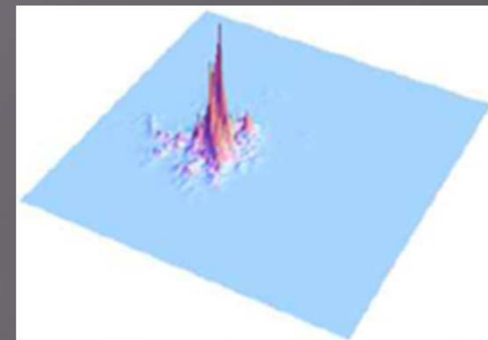
Multifractal metal and insulator



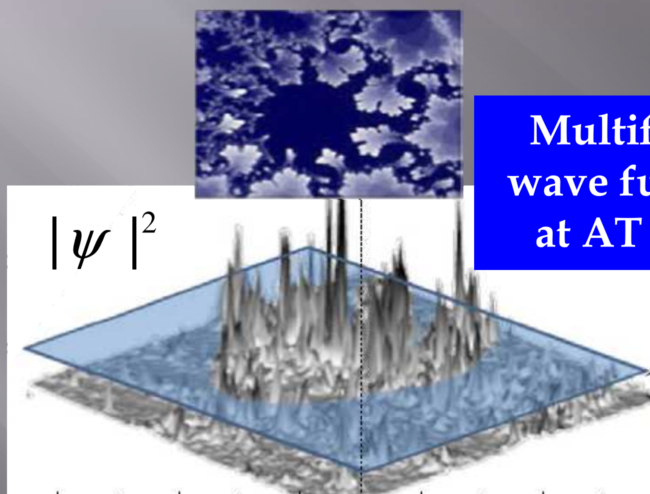
Good metal



AT point or 2D

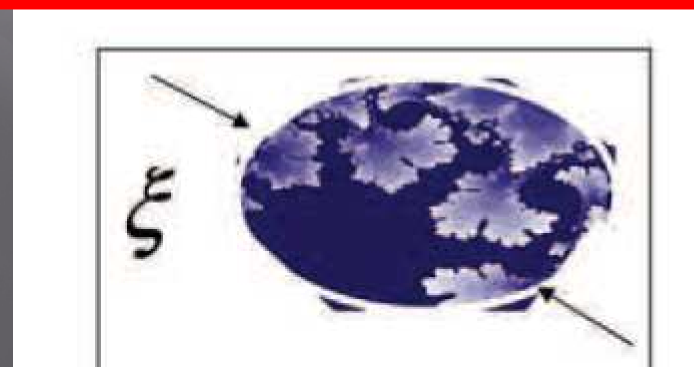


Strong insulator



Multifractal wave function at AT point

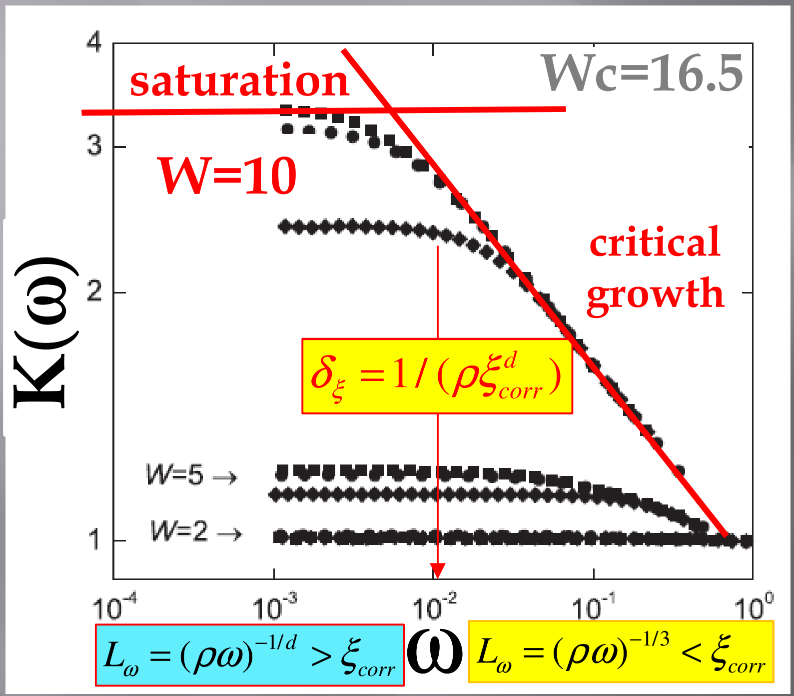
Disordered 2D system is a weakly multifractal insulator



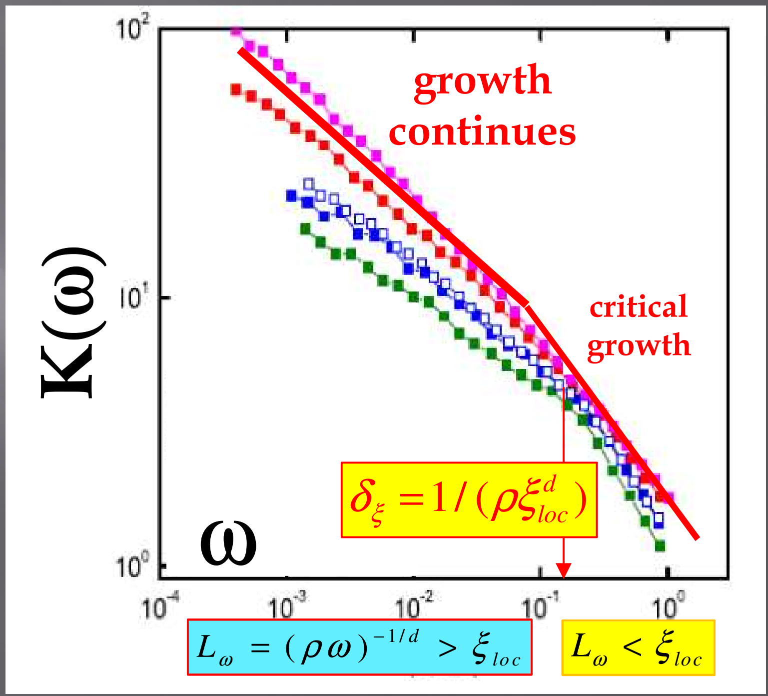
Multifractal insulator: fractal structure persists inside localization volume

LDoS correlation function in multifractal metal and insulator

E. Cuevas, V.E.K. Phys. Rev. B v. 76, 235119 (2007)

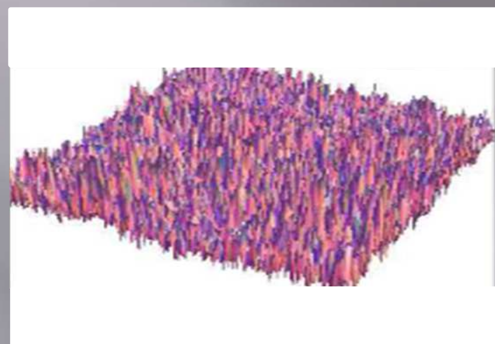


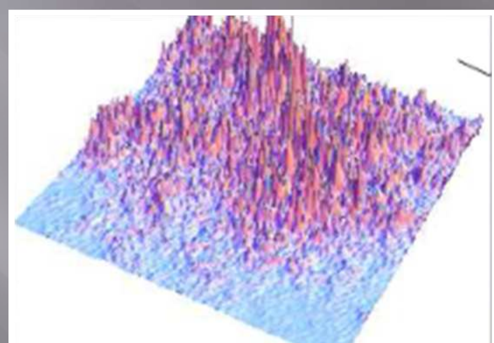
Multifractal metal:
slightly below AT

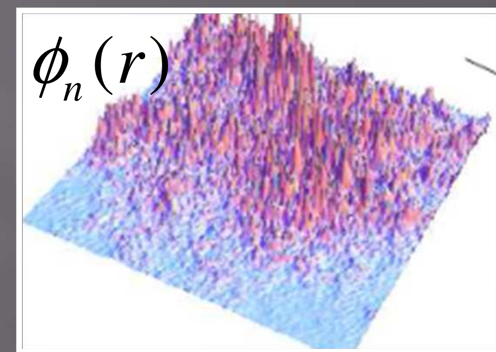


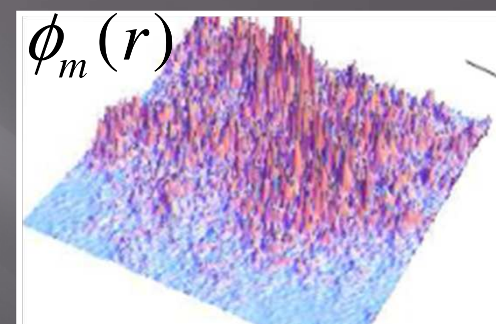
Multifractal insulator:
slightly above AT

Why critical enhancement?







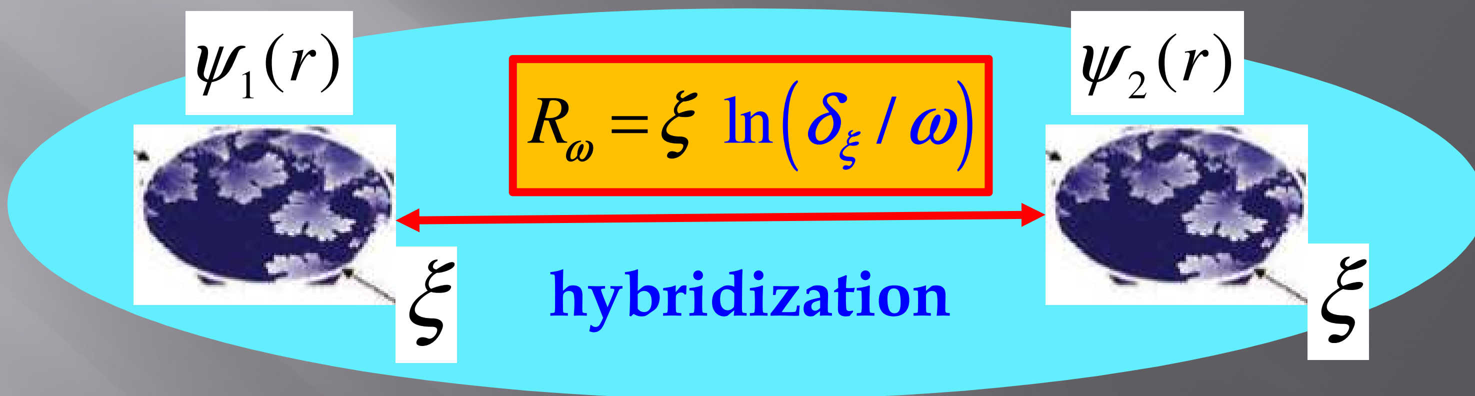


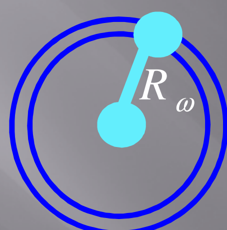
Large amplitude at a small number of “occupied” sites due to normalization constraint

Highly correlated amplitudes for different wave functions

Why K(ω) grows in insulator at $\omega<\delta_\xi$?

Mott's resonant pairs





$$\Psi_m(r) = (\psi_1(r) - \psi_2(r))/\sqrt{2}$$

$$\Psi_n(r) = (\psi_1(r) + \psi_2(r))/\sqrt{2}$$

E. Cuevas, V.E.K. Phys. Rev. B v. 76, 235119 (2007)

cf. Berezinskii's -Mott's:

$\sigma(\omega) \propto \omega^2 \ln^{d+1}(\delta_\xi / \omega)$

All matrix elements are enhanced by the factor $[R(\omega)/\xi]^{d-1} = \ln^{d-1}(\delta_\xi/\omega)$

Interpolating formula for K(ω) in multifractal insulator and critical regime

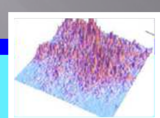
M. V. Feigelman, L. B. Ioffe, V. E. Kravtsov, and E. Cuevas, Ann. Phys. **325**, 1390 (2010).

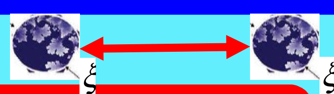
$$K(\omega) = \frac{(E_0/\delta_\xi)^\gamma \ \ln^2(\delta_\xi/\omega)}{c + (\omega/\delta_\xi)^\gamma \ \ln^2(\delta_\xi/\omega)} \qquad (\omega < \delta_\xi < E_0)$$

$3D: \ \gamma = 1 - d_2/3 \approx 0.6$

$2D: \ \gamma \sim 1/g, \ \ln^{d-1} \to \ln^1$

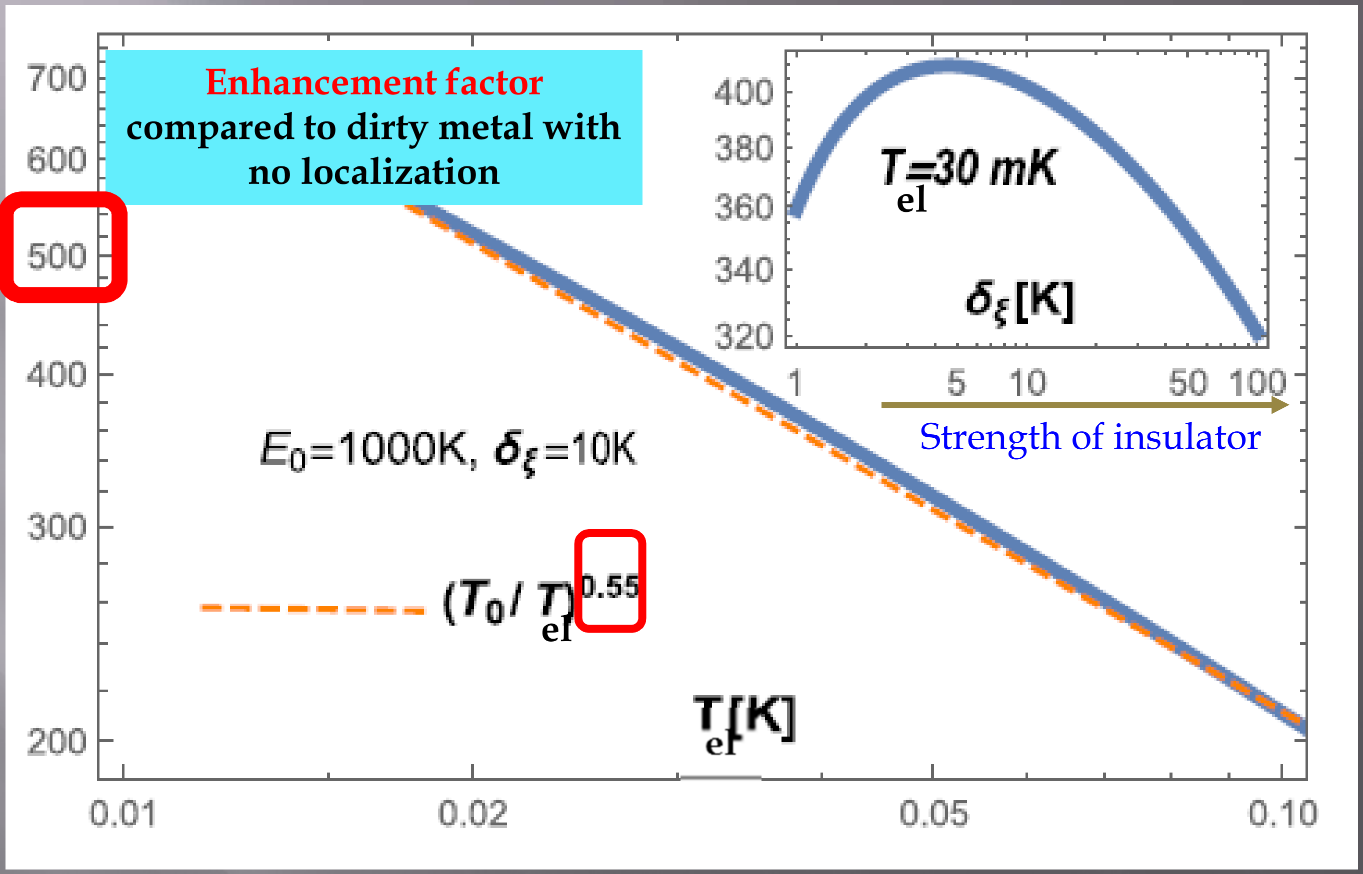
Valid in 3D multifractal insulator and critical regime





$$\mathbf{K(\omega)} = \begin{cases} (E_0/\delta_\xi)^\gamma \ln^2(\delta_\xi/\omega), & \omega \sim T_e < \delta_\xi < E_0 \ \text{(weak insulator)} \\ (E_0/\omega)^\gamma, & \delta_\xi < \omega \sim T_e < E_0 \ \text{(critical)} \\ (E_0/\delta_\xi)^\gamma, & \omega \sim T_e < \delta_\xi < E_0 \ \text{(multifractal metal)} \\ 1, & \omega \sim T_e < E_0 < \delta_\xi \ \text{(good metal)} \end{cases}$$

Enhancement factor in insulator

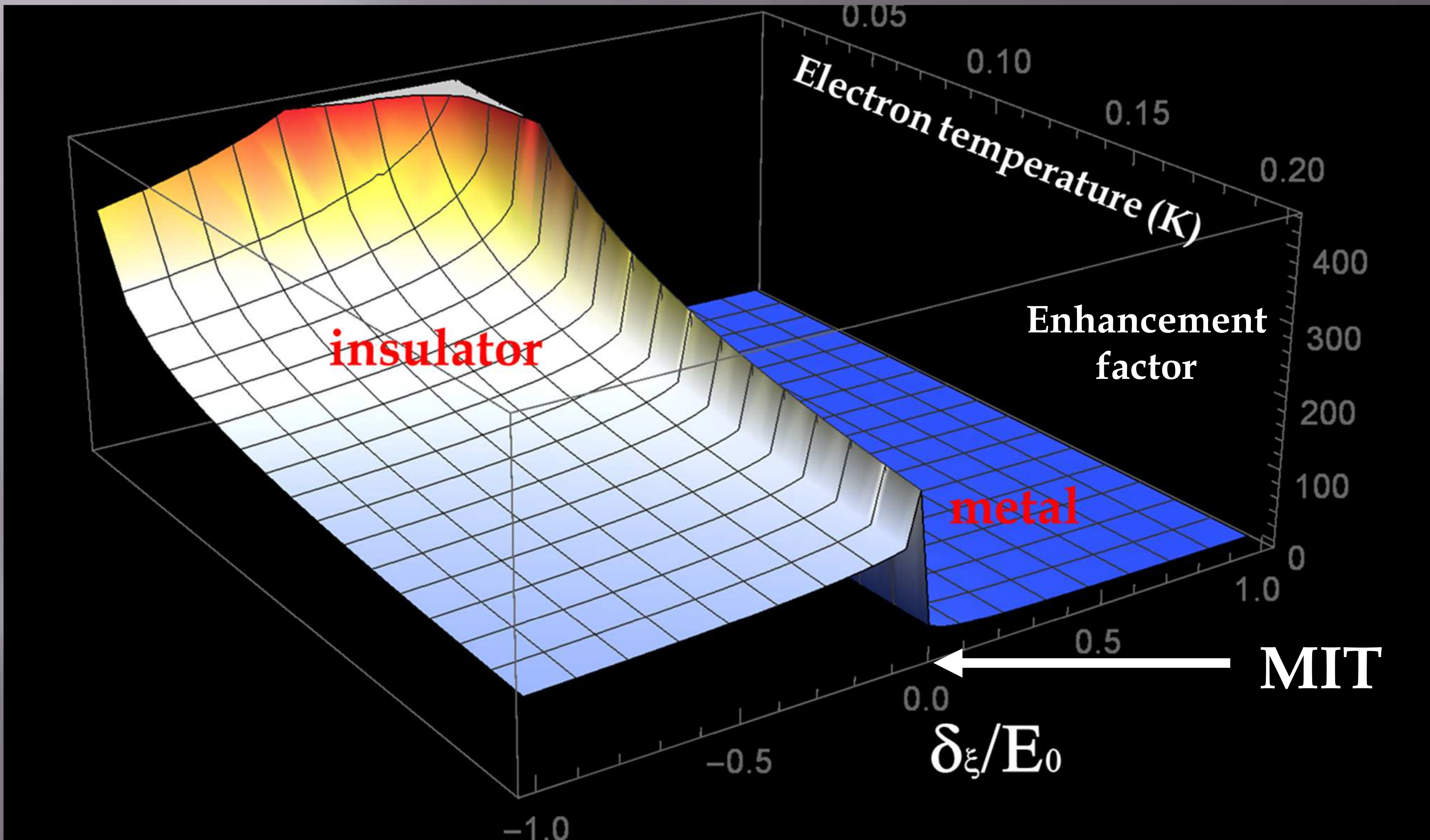


Explains:

(i) the power 5.5 (instead of 6 in dirty metal) and

(ii) enhancement ~500 in broad range of parameters

Conclusion: enhancement of cooling rate in weak insulator



Conditions for self-averaging

$p = (N_\xi / N) \ln^2(\delta_\xi / T).$

$\#\text{states involved} = (T/\delta)^2.$

Condition for self-averaging: $p \times (\#\text{states involved}) \geq 1.$

$(T\nu(0)a^3)^2 \ln^2(\delta_\xi / T) > N^{-1}.$

Always guaranteed for bulk samples