Disorder in Andreev Reflection of a Quantum Hall Edge State

L.I. Glazman

Yale University

Vlad Kurilovich, Zach Raines

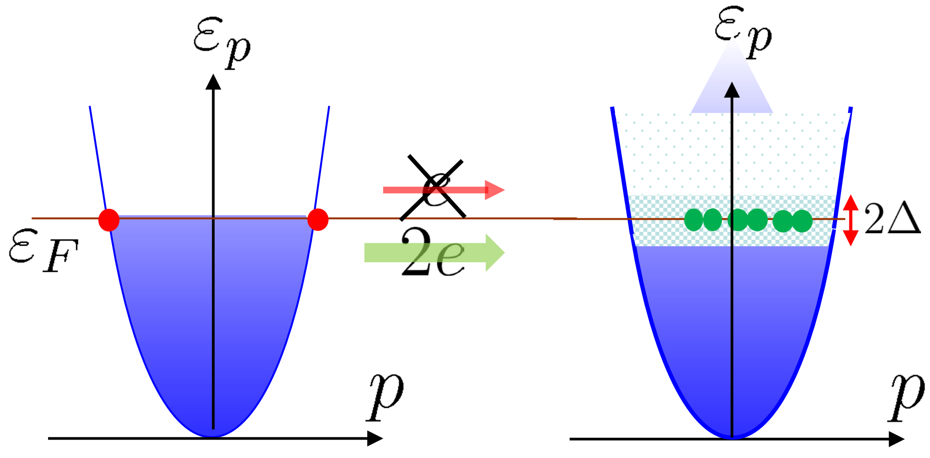
Vlad Kurilovich, Zach Raines, LG -- Nature Comm. (2023)**14**:237
Vlad Kurilovich, LG -- arXiv:2209.12932

Les Houches June 2023

Outline

- Notion of Andreev reflection (AR)
- Andreev reflection off an “ideal” superconductor
- Disorder-induced randomness in Andreev reflection, single ν=2 edge
- Effect of magnetic vortices in superconductor
- Conductance distribution function and parametric correlations, ν=2 single edge
- Quantum criticality of counter-propagating n=1 edges coupled by “dirty” supercond.
- Conclusions

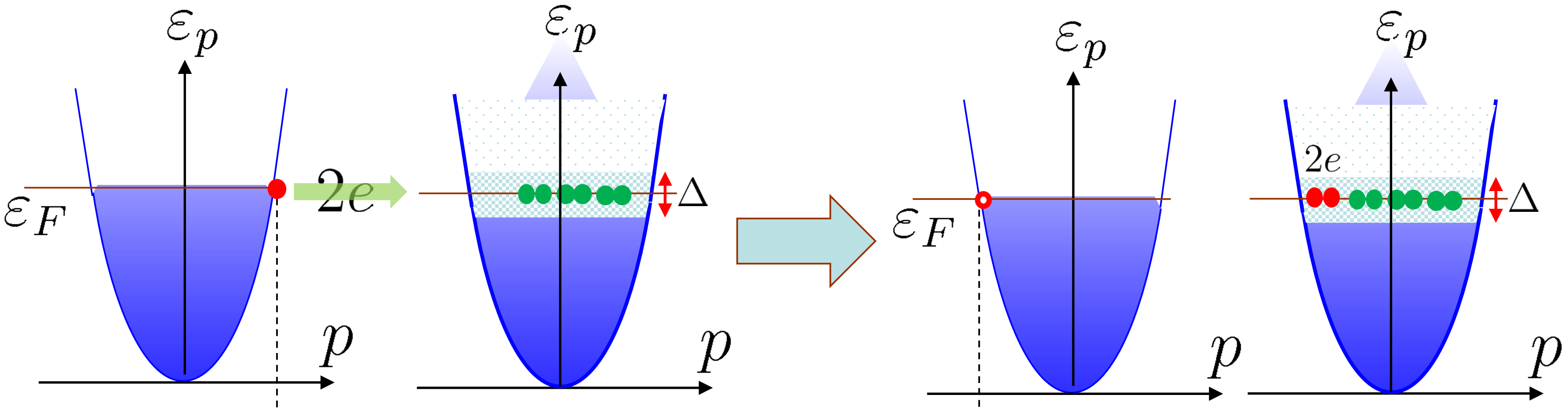
Notion of Andreev reflection



Cooper pair

gap for single particles, but
pairs are allowed to enter

normal conductor in contact with superconductor



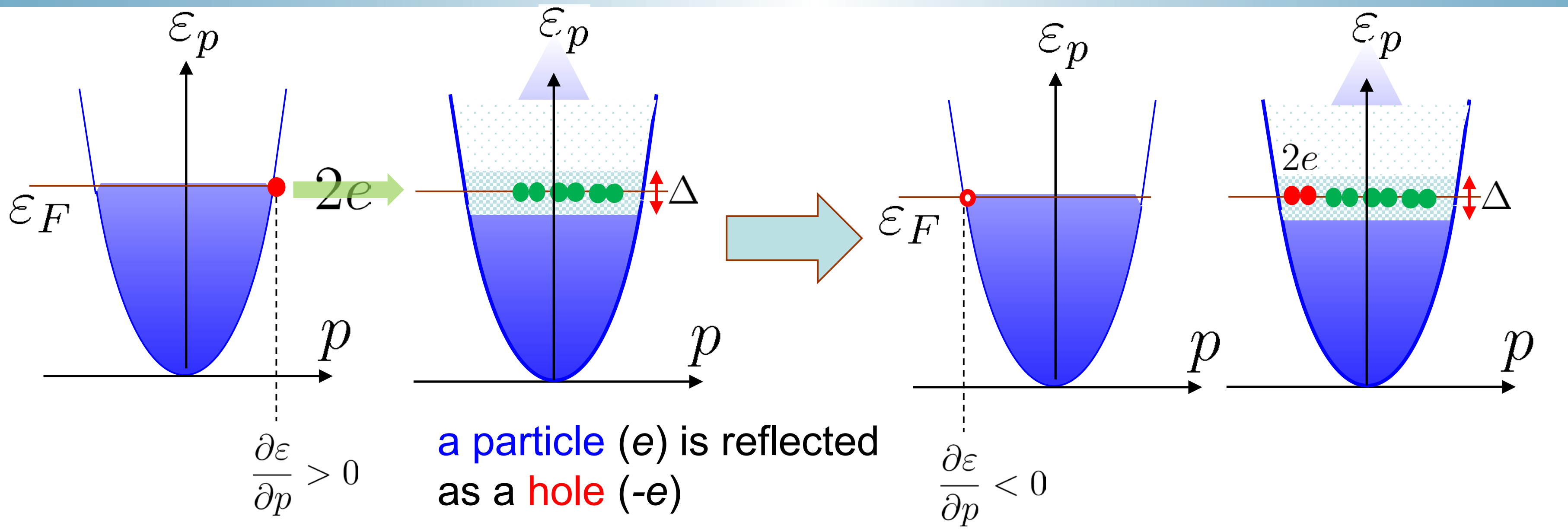
$\frac{\partial \varepsilon}{\partial p} > 0$

a particle (*e*) is reflected
as a hole (-*e*)

$\frac{\partial \varepsilon}{\partial p} < 0$

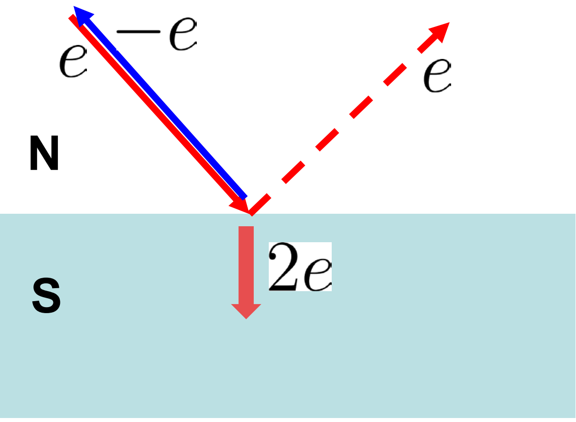
A.F. Andreev (1964) [in the context of the intermediate state properties of a type-I superconductor]

Andreev reflection off a flat interface



a particle (e) is reflected as a hole (-e)

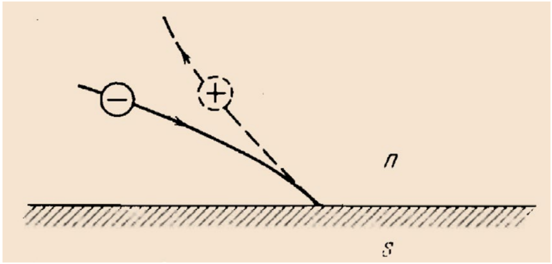
Semiclassical picture of AR: real-space trajectories



$e \to -e$

$\vec{v} \to -\vec{v}$

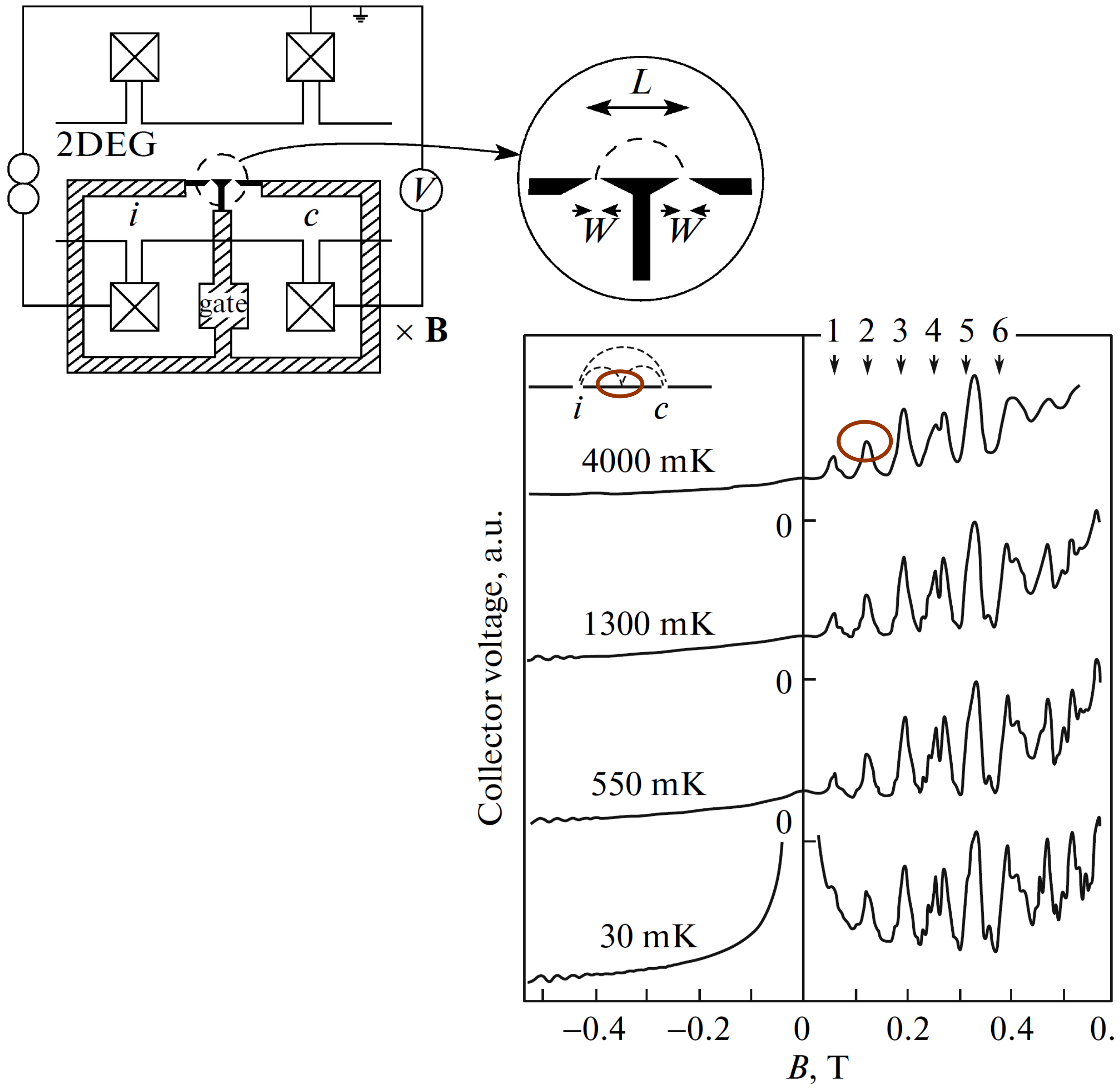
at $\vec{B} \neq 0$



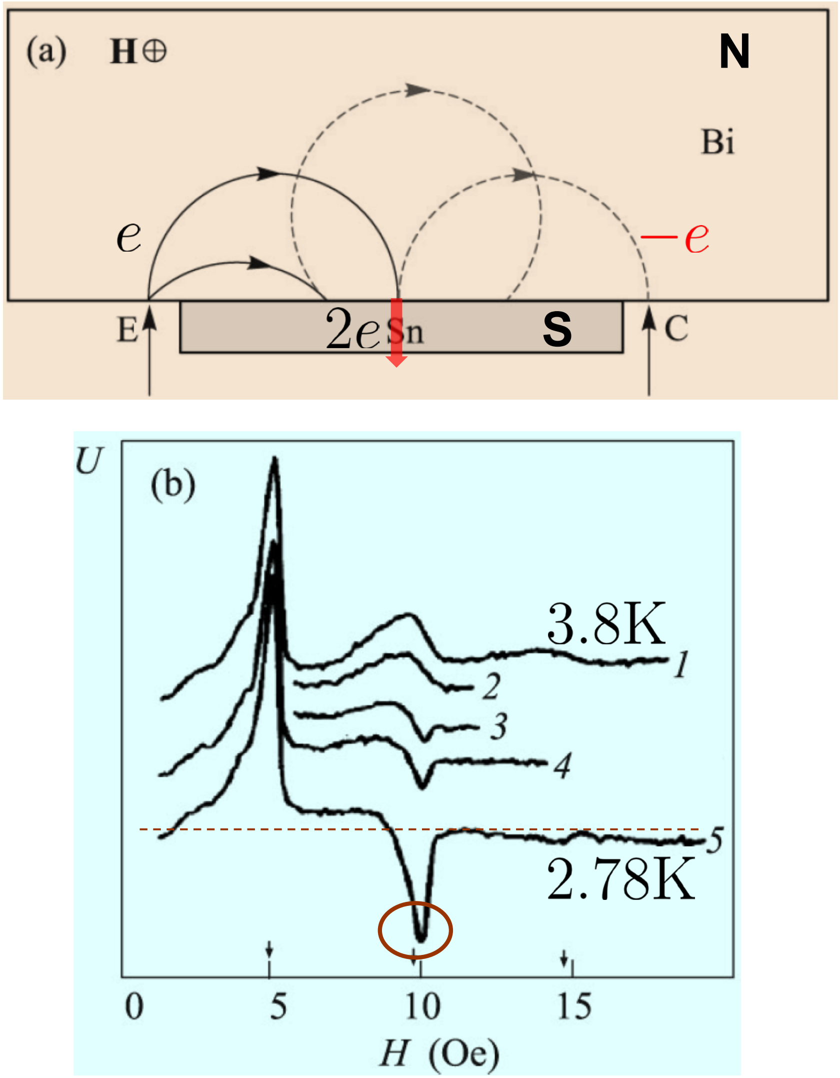
From: Krylov and Sharvin, JETP 1973

Visualizing semiclassical trajectories by magnetic focusing

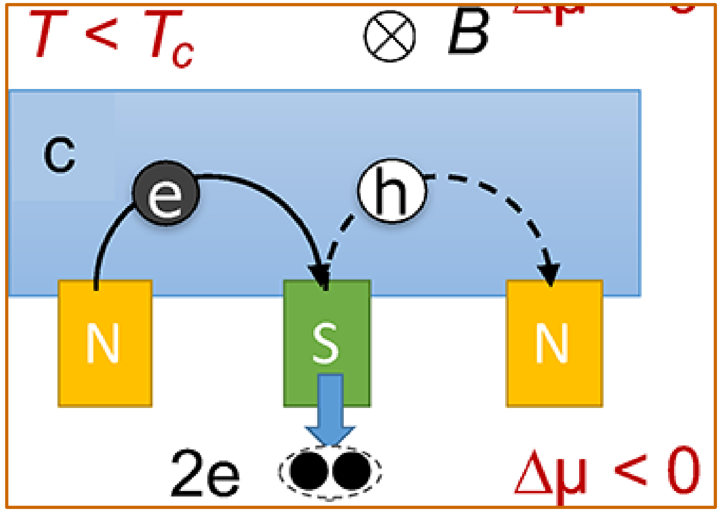
From: van Houten et al, Europhys Lett 1988

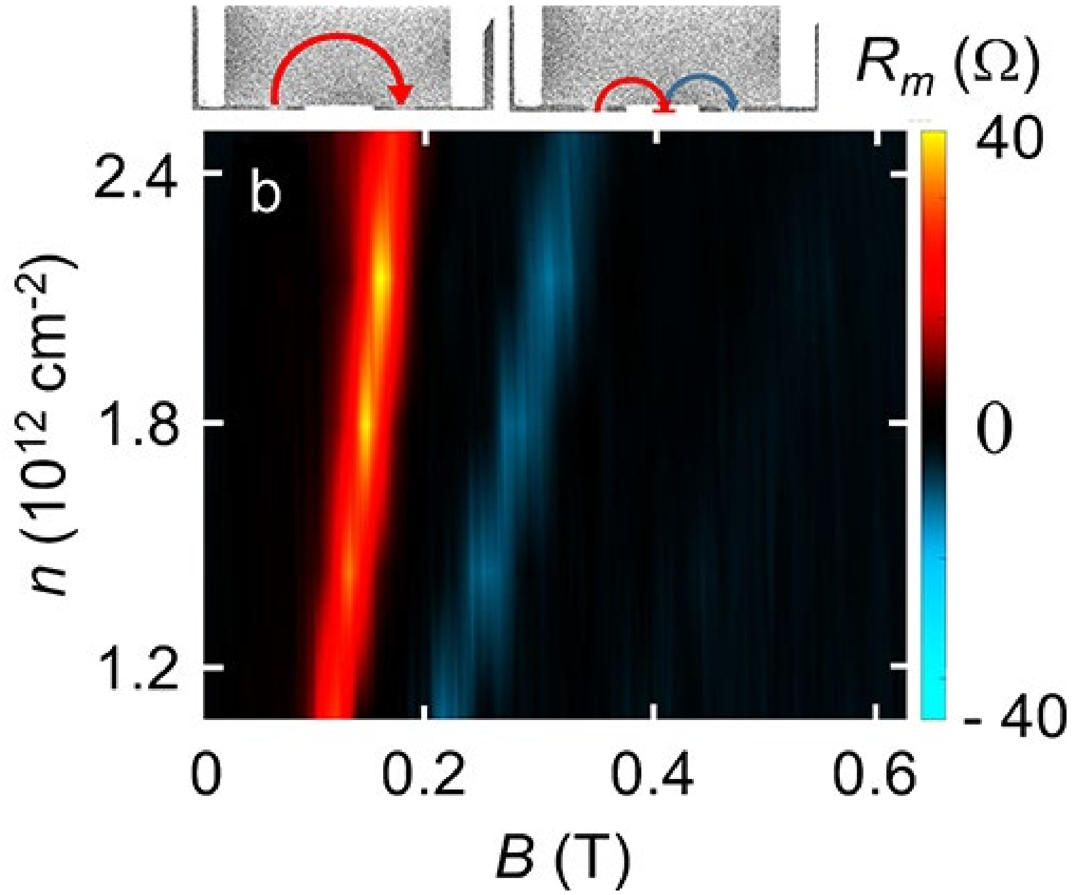


Bozhko, Tsoi, Yakovlev, JETP Lett 1983



Magnetic focusing + Andreev reflection in graphene

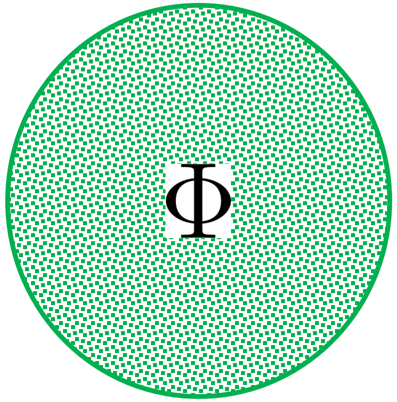




From: Bhandari,…, Kim, Westervelt, NanoLett 2020

Orbits quantization, no disorder

$\vec{B} \neq 0$ Integer number of flux quanta encircled by trajectory



2D bulk

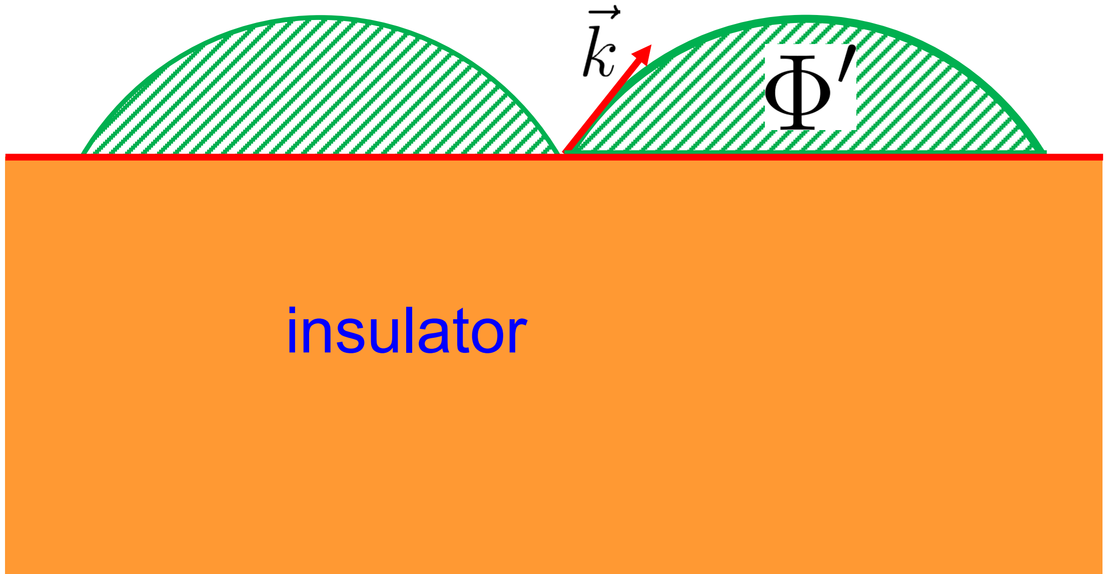
$$\Phi = n\Phi_0 \qquad \varepsilon_n = \hbar\omega_c\left(n + \frac{1}{2}\right)$$

Discrete energy spectrum of Landau levels

edge of 2D: Skipping orbits

$$\Phi' = n\Phi_0$$

$$\varepsilon_n \equiv \varepsilon_n(k_x)$$



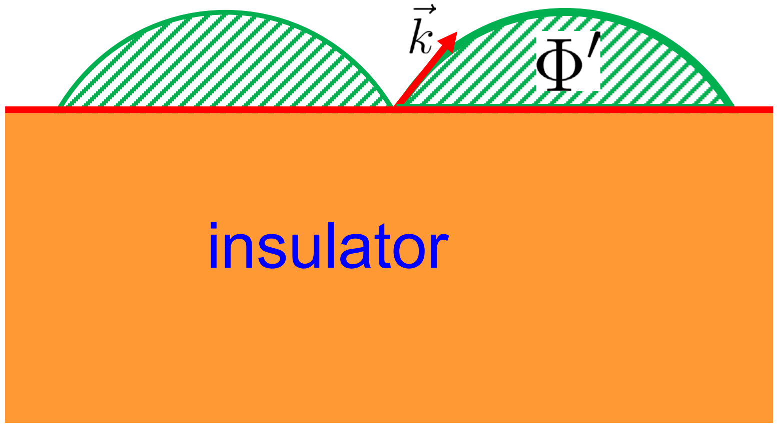
set of 1D spectra (edge states)

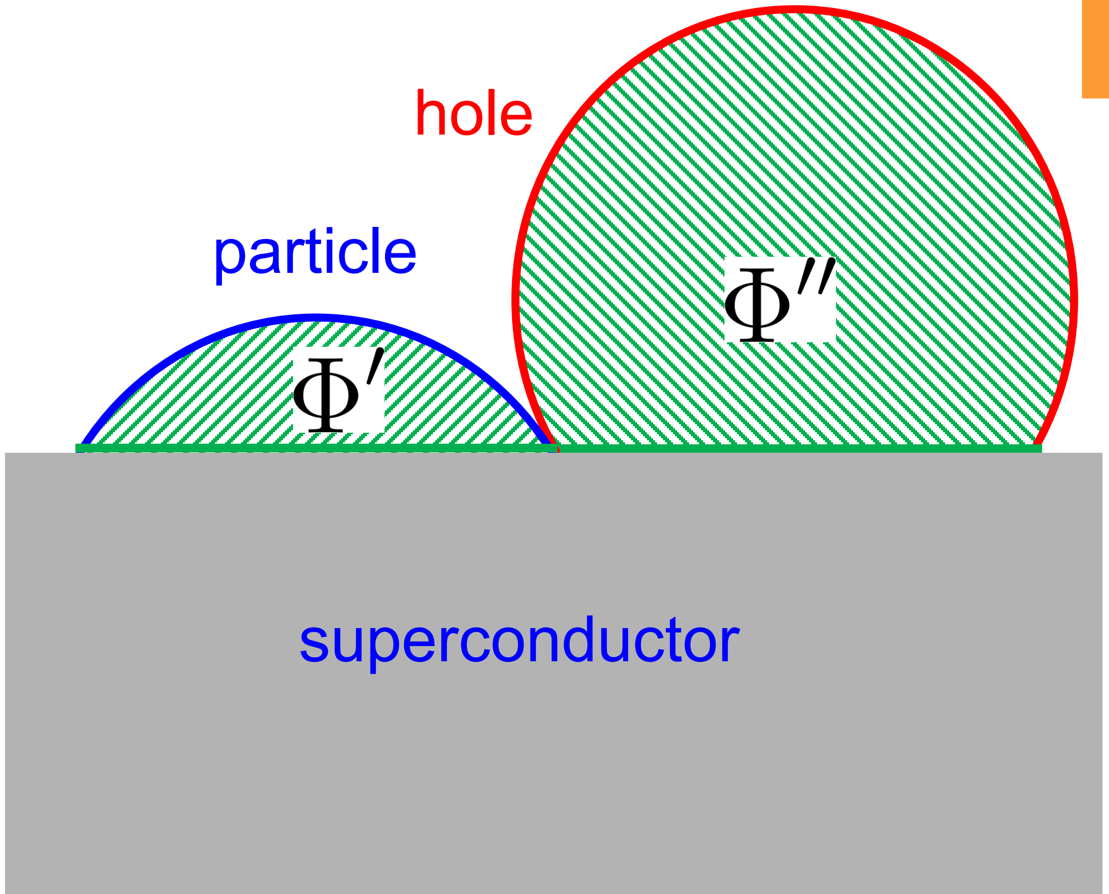
Quantization of particle-hole orbits, no disorder

set of 1D spectra

$$\Phi' = n\Phi_0$$

$$\varepsilon_n \equiv \varepsilon_n(k_x)$$





$$\Phi' + \Phi'' = n\Phi_0$$

$$\Phi' + \Phi'' = \Phi$$

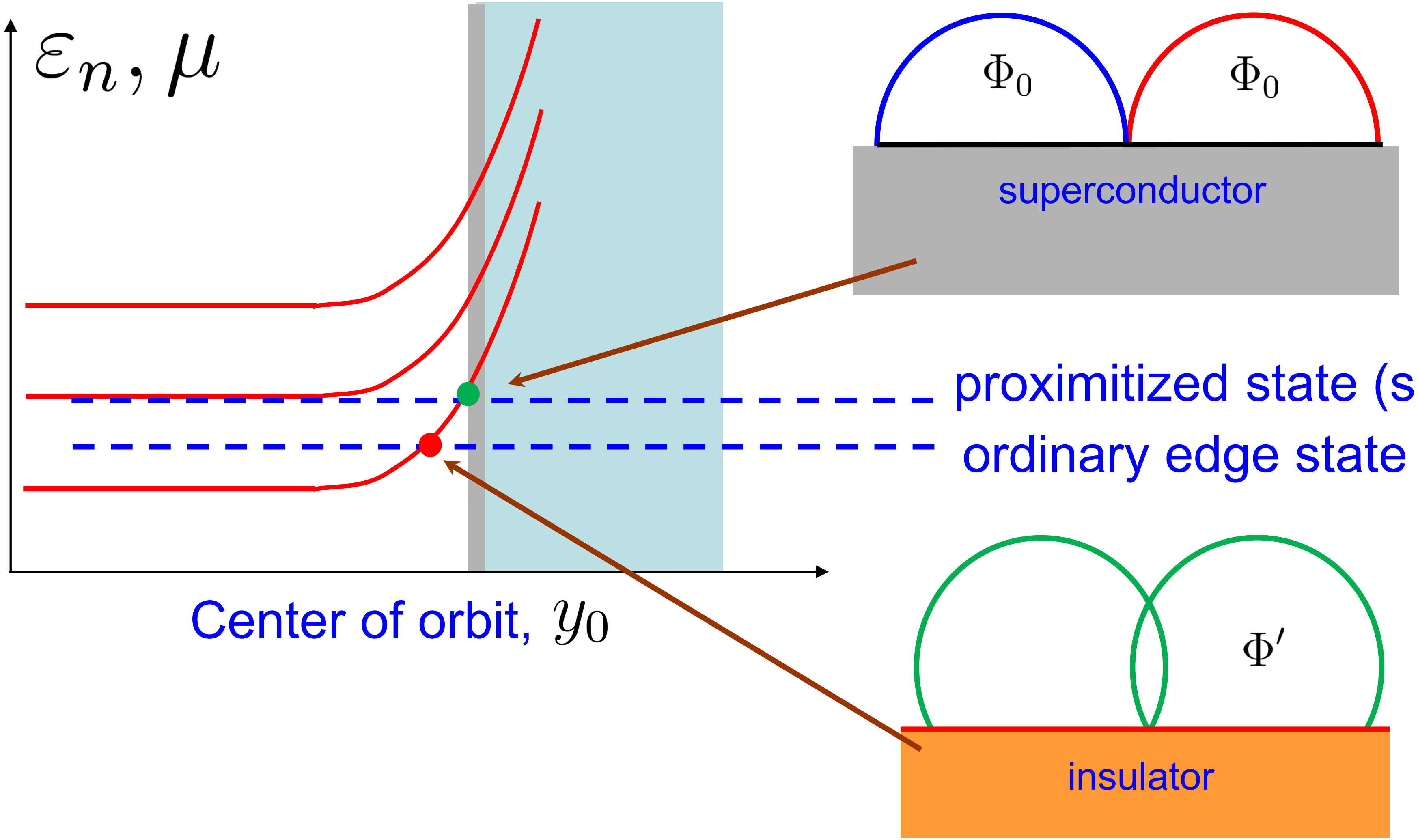
$$\Phi = n\Phi_0$$

$$\varepsilon_n = \hbar\omega_c\left(n + \frac{1}{2}\right)$$

Discrete spectrum!

An effective proximitization would need a fine-tuning

… of the chemical potential (μ)



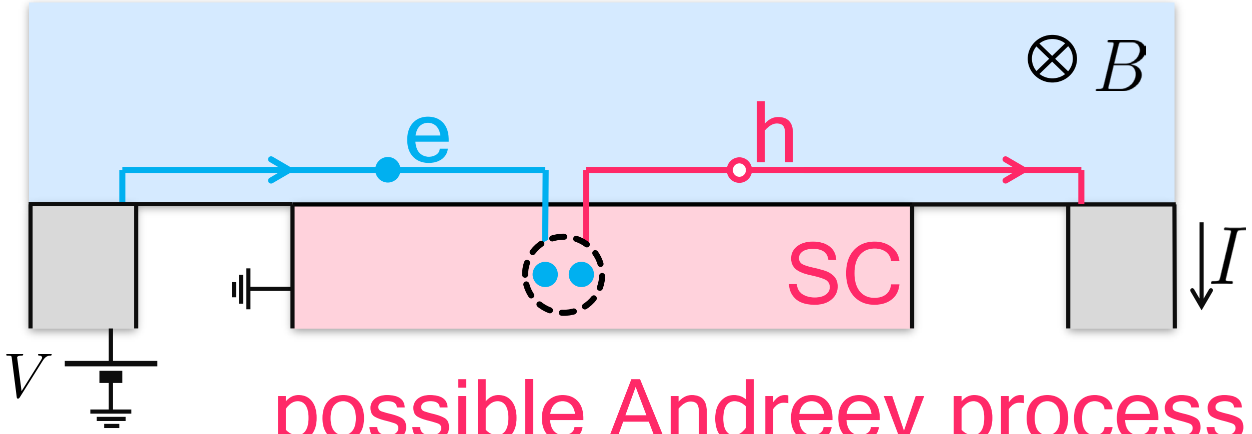
Solution of the Schrödinger equation: same-order e and h amplitudes only if

$$|y_0| \lesssim l_B \frac{\Delta}{\hbar\omega_c}$$

$$\delta n \lesssim \frac{1}{l_B^2}\frac{\Delta}{\hbar\omega_c} \ll \frac{1}{l_B^2}$$

Experiment: no fine-tuning needed for Andreev refl. (AR)

Schematics of the experiment, Zhao et al, Nat Phys 2020



possible Andreev process(es)

Conductance

$$G = \frac{I}{V}$$

Normal state

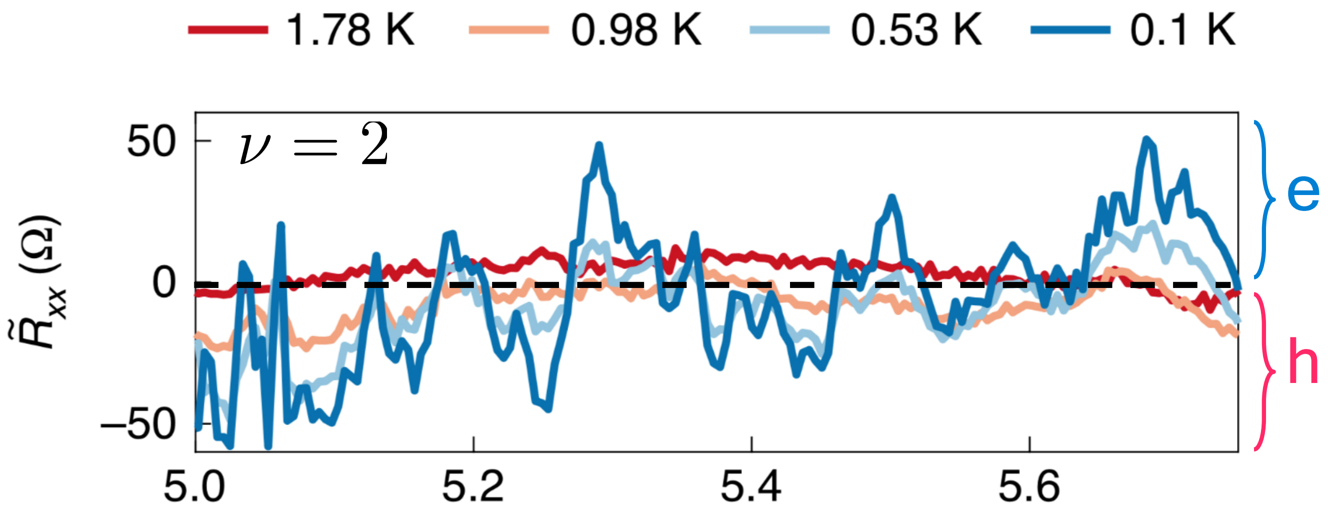
$$G = 0$$

Superconducting state

$G < 0$ hole

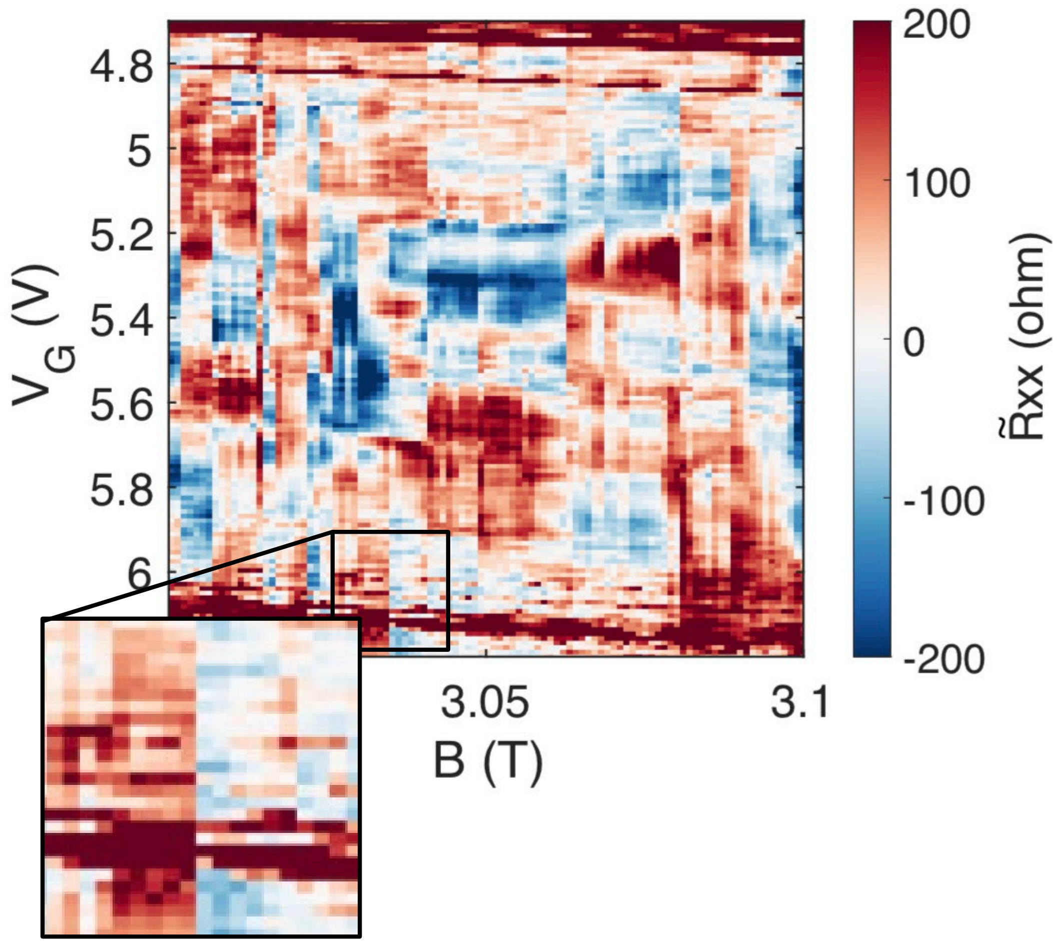
$G > 0$ electron

Experiment: no fine-tuning needed for AR



[L. Zhao et al., 2020]

V_G (V) ∝ electron density



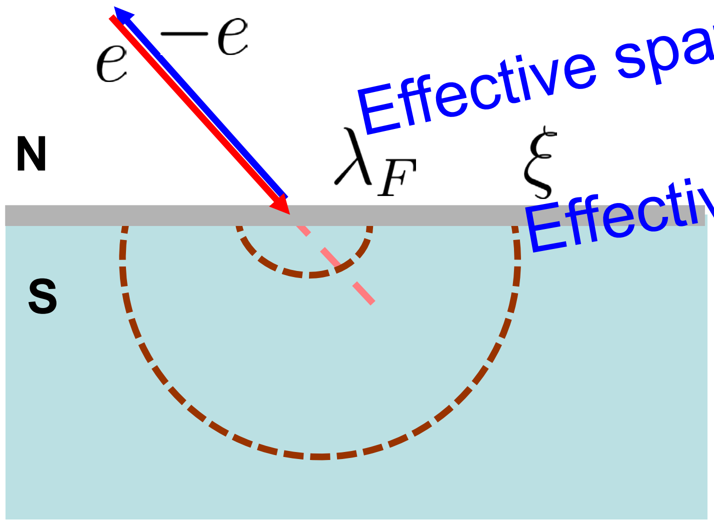
- Sign-alternating signal at low *T*
- Mesoscopic fluctuations, ~ 50/50
- Switches in the fluctuations pattern with *B*

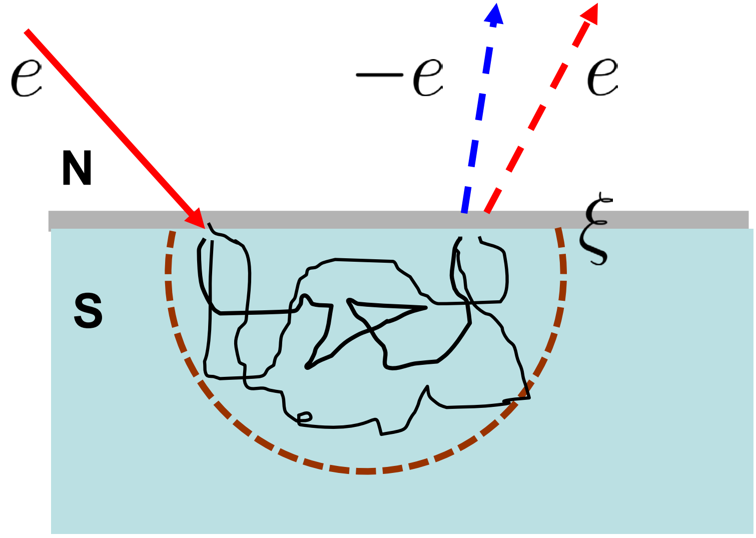
Amplitude of AR off a disordered superconductor

Length scales: $\xi \gg l \gg \lambda_F$

Coherence length in "dirty" limit: $\xi = \sqrt{D/\Delta}$

$D \sim v_F l$

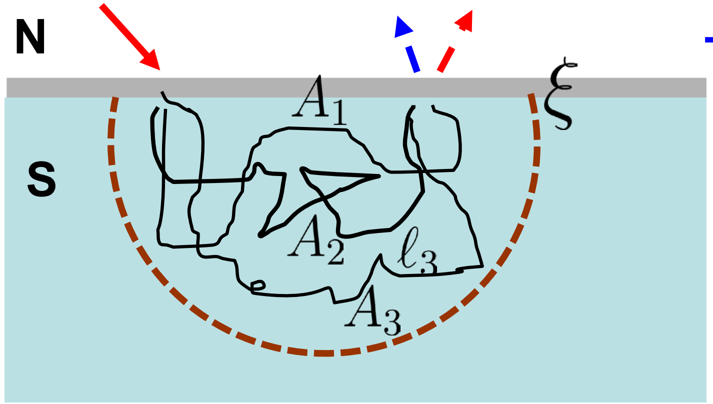




Amplitude=Sum over random diffusive trajectories; random outcome (p-h) and random phase

Randomness of the scattering amplitude

Amplitude=Sum over random diffusive trajectories; random outcome (p-h) and random phase of the amplitude



Tunnel barrier (conductance ***g*** per unit length)

$$A_{\rm tot} = \sum_i A_i \,, \quad A_i \sim t^2 e^{ik_F \ell_i} \sim g e^{ik_F \ell_i}$$ random

$$\langle A_{\rm tot} \rangle = \sum_i \langle A_i \rangle = 0$$

but $$\langle |A_{\rm tot}|^2 \rangle = \sum_{ij} \langle A_i A_j^* \rangle = \sum_i \langle |A_i|^2 \rangle \neq 0$$

The dirtier, the better (more trajectories return to region $\sim \xi$ of the interface)

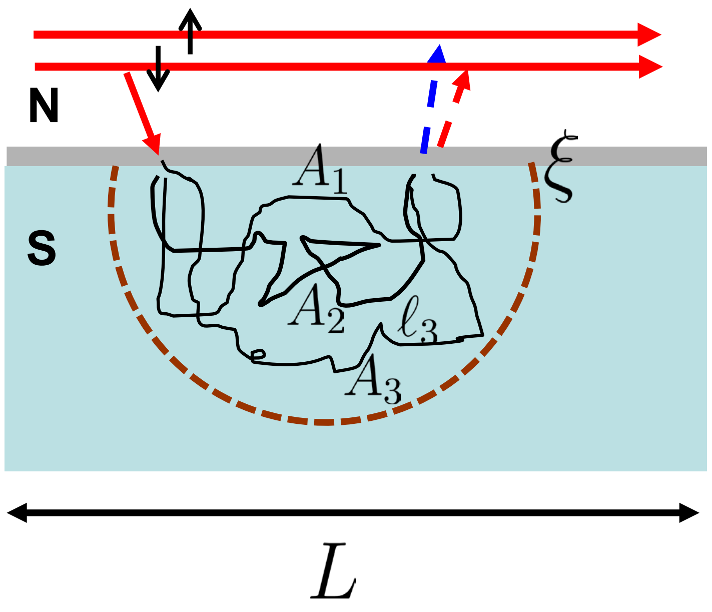
Hekking, Nazarov, PRB 1993

Andreev reflection of Hall edge state: a short segment

Length scales: $\xi \gg l \gg \lambda_F$

Coherence length in "dirty" limit: $\xi = \sqrt{D/\Delta}$

$D \sim v_F l$



Tunnel barrier (conductance g per unit length)

$$A_i \sim t^2 e^{ik_F \ell_i} \sim g e^{ik_F \ell_i}$$ random

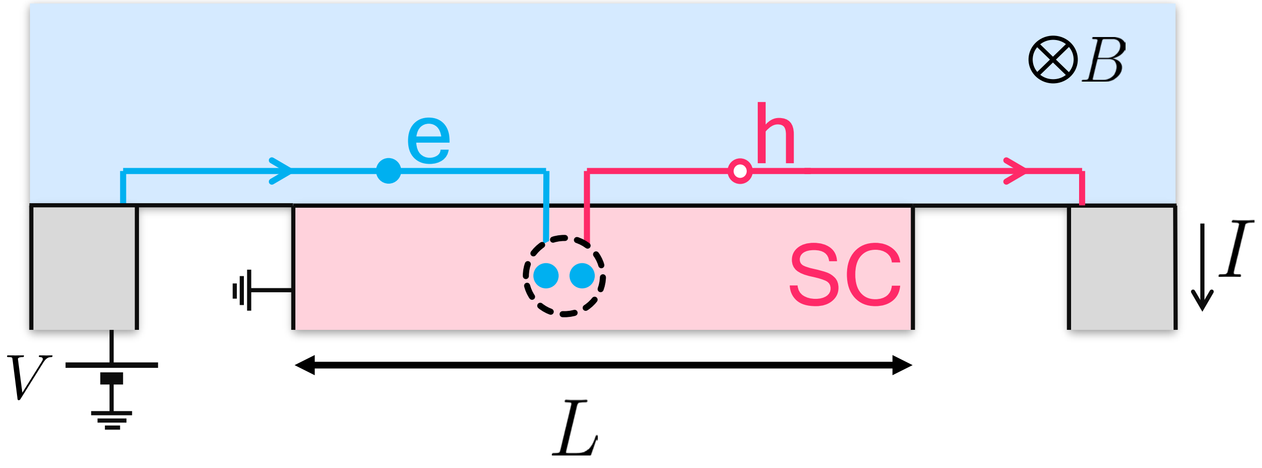
$$\langle |A_{\rm tot}|^2 \rangle = \sum_{ij} \langle A_i A_j^* \rangle = \sum_i \langle |A_i|^2 \rangle \neq 0$$

$$\langle |A_{eh}|^2 \rangle = \frac{L}{\ell_A}\,; \quad \boxed{\frac{1}{\ell_A} = \frac{4\pi g^2}{G_Q \sigma} \ln \frac{\xi}{l}} \qquad \ell_A \gg L \gg \xi$$

$G_Q = 2e^2/h$, σ : normal-state conductivity of the superconductor

$$g = 2\pi G_Q t^2 (\partial_y \Phi)^2 \nu p_F / (\hbar^2 v_{\rm edge})$$

Conductance of a short Hall edge segment



$\ell_A \gg L \gg \xi$

$$\frac{1}{\ell_A} = \frac{4\pi g^2}{G_Q \sigma} \ln \frac{\xi}{l}$$

$$G_Q = 2e^2/h\,,$$

σ : normal-state conductivity of the superconductor

$$g = 2\pi G_Q t^2 (\partial_y \Phi)^2 \nu p_F / (\hbar^2 v_{\text{edge}})$$

In general,

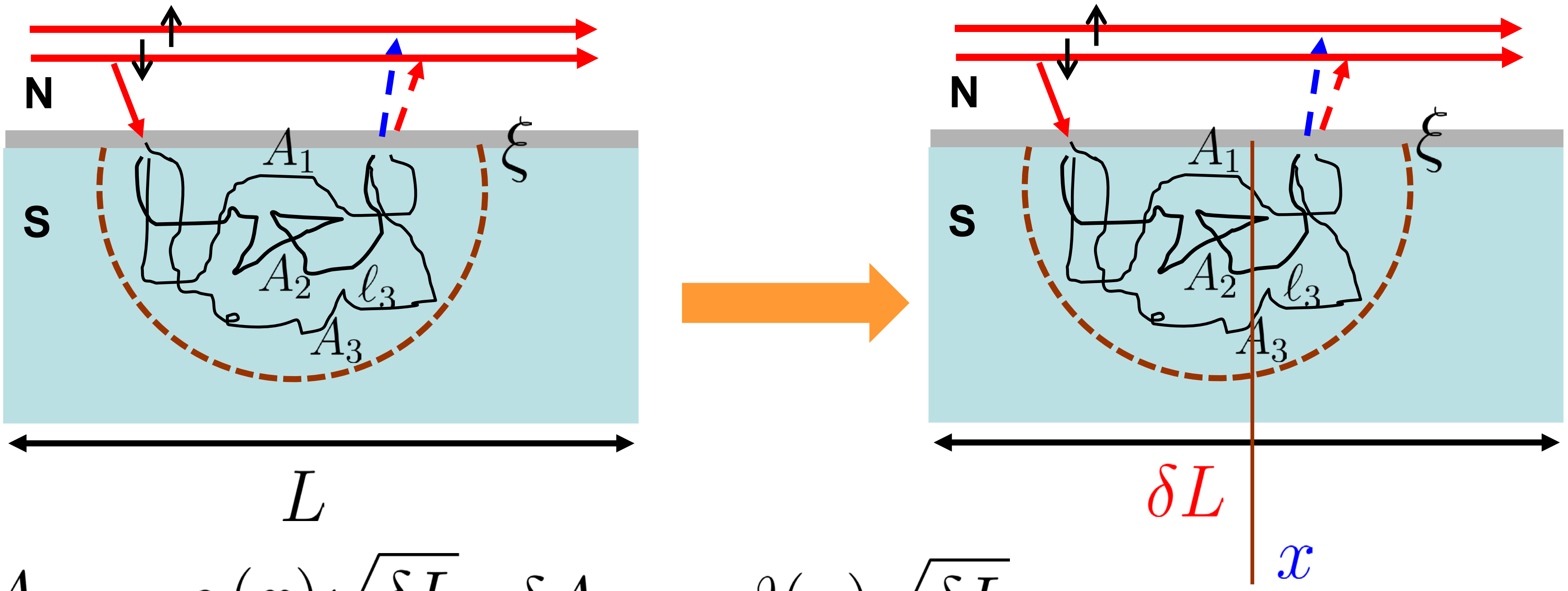
$$G = G_Q(|A_{ee}|^2 - |A_{eh}|^2) = G_Q(1 - 2|A_{eh}|^2)$$

$$|A_{ee}|^2 + |A_{eh}|^2 = 1$$

$$\langle G \rangle = G_Q(1 - 2L/\ell_A)$$

$$\langle G^2 \rangle - \langle G \rangle^2 = 4G_Q^2 L^2/\ell_A^2$$

Conductance of a long edge



$$\frac{1}{\ell_A} = \frac{4\pi g^2}{G_Q \sigma} \ln \frac{\xi}{l}$$

$$L \gg \ell_A$$

$$\delta A_{eh} = \alpha(x)\sqrt{\delta L}\,,\ \delta A_{ee} = \vartheta(x)\sqrt{\delta L}$$

$$\langle \alpha(x)\alpha^\star(x')\rangle = \langle \vartheta(x)\vartheta(x')\rangle = \frac{1}{\ell_A}\delta(x-x')$$

Evolution of the wave function along the edge:

$$i\frac{\partial}{\partial x}\begin{pmatrix} a_{\mathrm{e}}(x) \\ a_{\mathrm{h}}(x)\end{pmatrix} = \begin{pmatrix} -\vartheta(x) & \alpha^\star(x) \\ \alpha(x) & \vartheta(x)\end{pmatrix}\begin{pmatrix} a_{\mathrm{e}}(x) \\ a_{\mathrm{h}}(x)\end{pmatrix}$$

Mapping on a random walk over a Bloch sphere

Evolution of the wave function along the edge:

$$i\frac{\partial}{\partial x}\begin{pmatrix} a_{\mathrm{e}}(x) \\ a_{\mathrm{h}}(x) \end{pmatrix} = \begin{pmatrix} -\vartheta(x) & \alpha^{\star}(x) \\ \alpha(x) & \vartheta(x) \end{pmatrix} \begin{pmatrix} a_{\mathrm{e}}(x) \\ a_{\mathrm{h}}(x) \end{pmatrix}$$

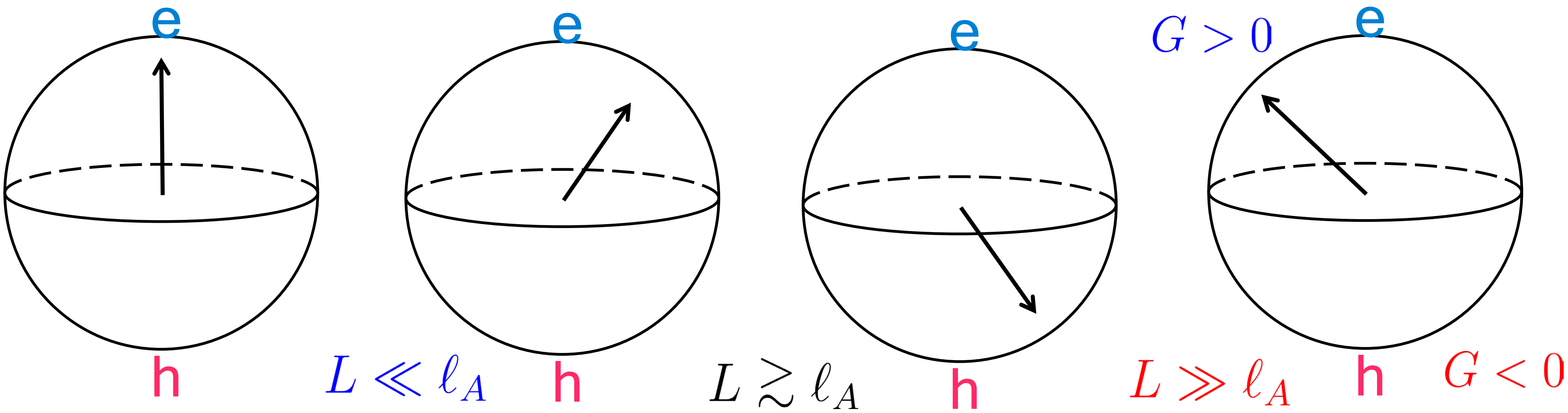
Parametrization by coordinates of a “spin” on a Bloch sphere:

$$a_{\mathrm{e}}(x) = \cos(\theta(x)/2)$$

$$a_{\mathrm{h}}(x) = e^{i\phi(x)}\sin(\theta(x)/2)$$

$$\langle \alpha(x)\alpha^{\star}(x')\rangle = \langle \vartheta(x)\vartheta(x')\rangle = \frac{1}{\ell_A}\delta(x-x')$$

← stochastic “magnetic field”



Conductance distribution function

Evolution of the wave function along the edge:

$$i\frac{\partial}{\partial x}\begin{pmatrix} a_{\mathrm{e}}(x) \\ a_{\mathrm{h}}(x) \end{pmatrix} = \begin{pmatrix} -\vartheta(x) & \alpha^\star(x) \\ \alpha(x) & \vartheta(x) \end{pmatrix}\begin{pmatrix} a_{\mathrm{e}}(x) \\ a_{\mathrm{h}}(x) \end{pmatrix}$$

$$\langle\alpha(x)\alpha^\star(x')\rangle = \langle\vartheta(x)\vartheta(x')\rangle = \frac{1}{\ell_A}\delta(x-x')$$

← stochastic "magnetic field"

Parametrization by coordinates of a "spin" on a Bloch sphere:

$$a_{\mathrm{e}}(x) = \cos(\theta(x)/2)$$

$$a_{\mathrm{h}}(x) = e^{i\phi(x)}\sin(\theta(x)/2)$$

$$G = G_Q\cos\theta(L)$$

$$\langle G\rangle = G_Q e^{-2L/\ell_A}$$

$$\frac{\partial\mathcal{P}(\theta,\phi|x)}{\partial x} = \frac{1}{\ell_A}\left(\Delta_{\theta,\phi} + \partial_\phi^2\right)\mathcal{P}(\theta,\phi|x)$$

(Fokker-Planck eq.)

Flat distribution function ↦

$$L/\ell_A \gg 1$$

$$\langle G\rangle = 0\,,\ \langle\langle G^2\rangle\rangle = G_Q^2/3$$

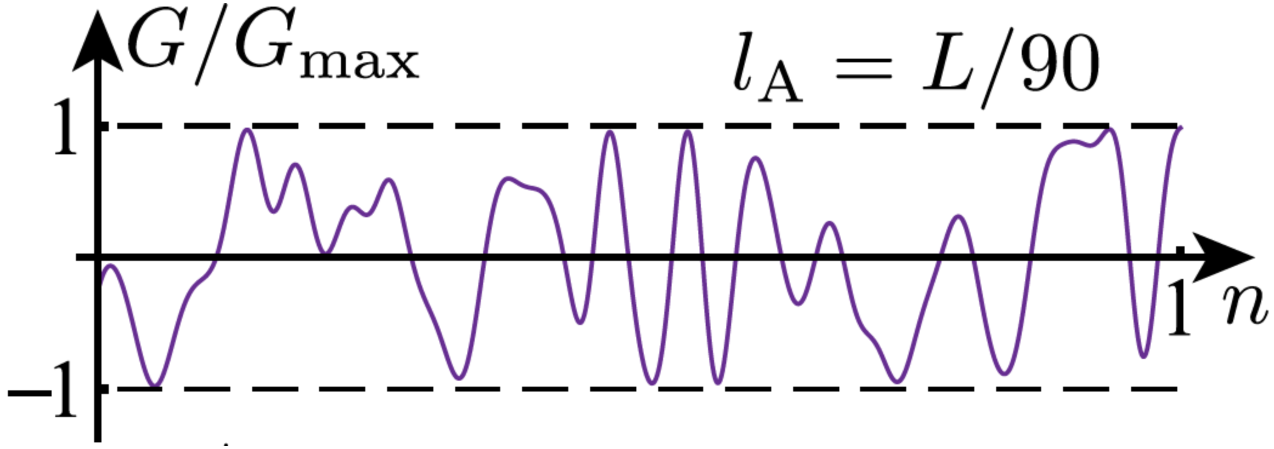
$$-G_Q < G < G_Q$$

Conductance distribution function

$L/\ell_A \gg 1$

$\langle G \rangle = 0\,, \langle\langle G^2 \rangle\rangle = G_Q^2/3$

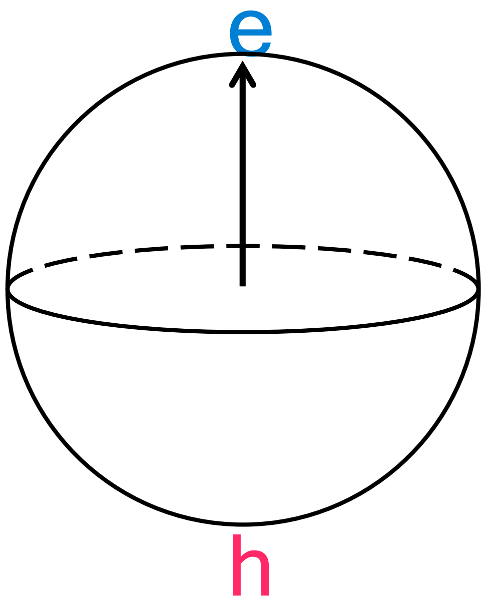
$-G_Q < G < G_Q$



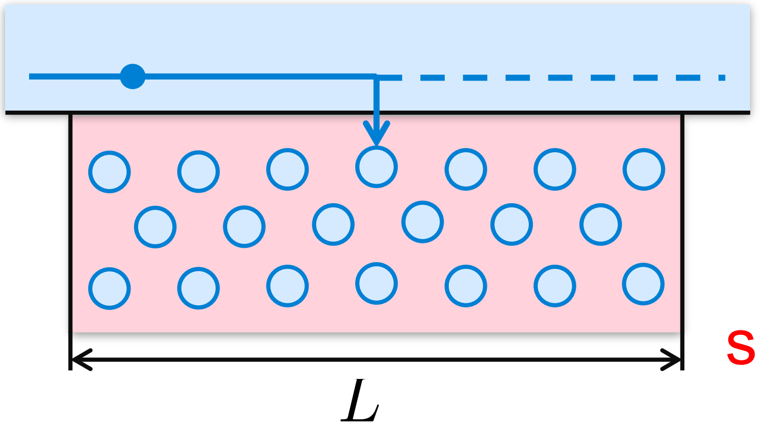
Trace of conductance vs. 2DEG density for a fixed realization of disorder

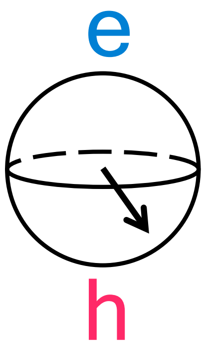
$G_{\mathrm{max}} = G_Q$ in the absence of vortices

Effect of electron loss to vortex cores



$$P_{\rm surv} = \exp\left[-\frac{gL}{G_Q} f\left(\frac{B}{H_{c2}}\right)\right]$$





shrinking Bloch sphere

crude estimate:

$$f\left(\frac{B}{H_{c2}}\right) \sim \sqrt{\frac{B}{H_{c2}}}$$

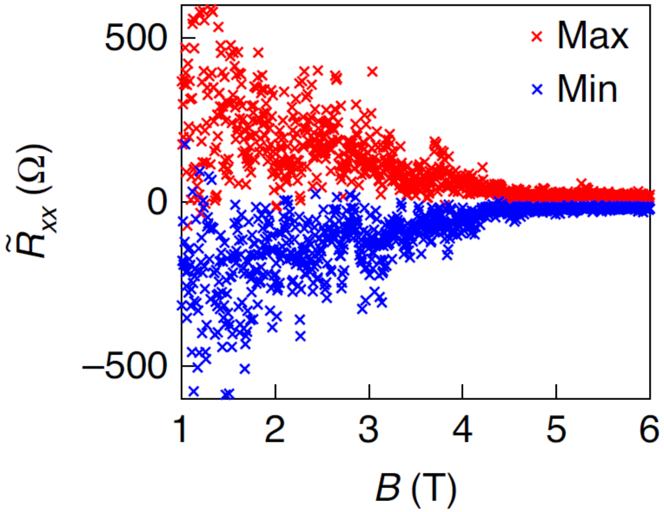
from the Fermi-level DOS, accessible by tunneling the from Hall edge into vortex cores

in the presence of vortices

$$G_{\rm max} = G_Q P_{\rm surv}(B)\,,$$

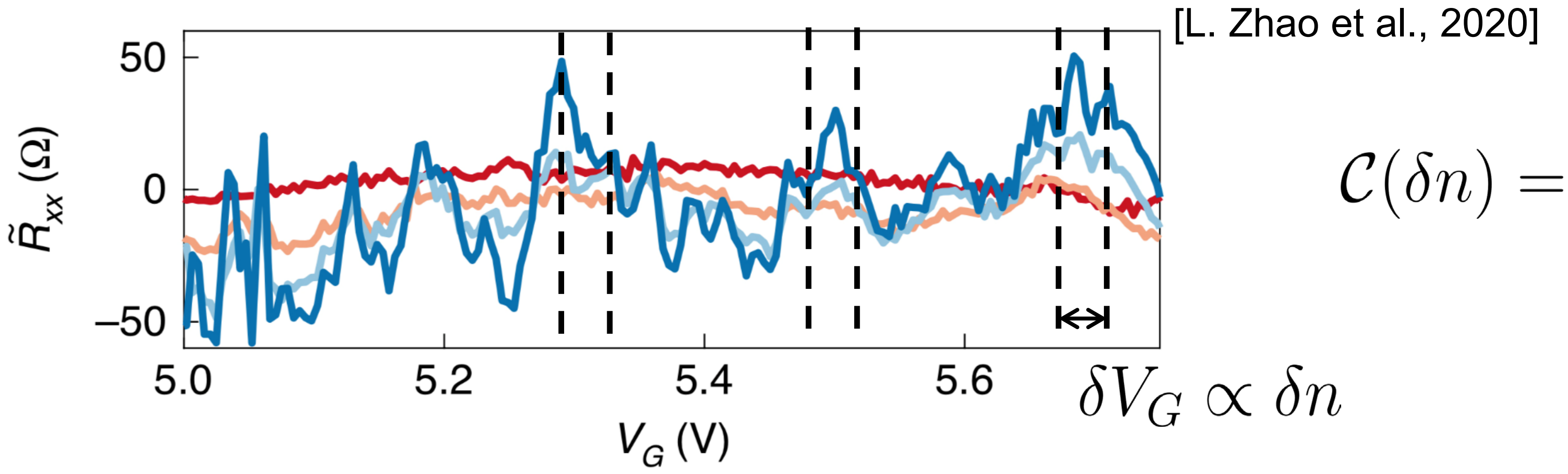
$$\langle\langle G^2\rangle\rangle / G_{\rm max}^2 = 1/3$$

Flat distribution function



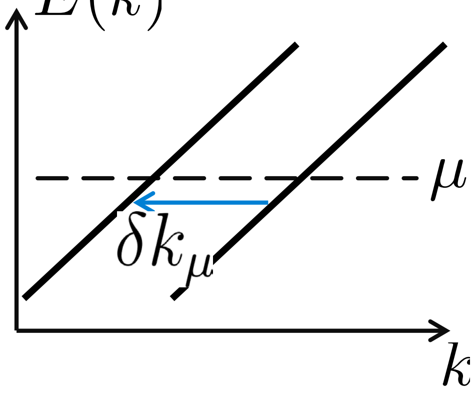
[L. Zhao et al., 2020]

Parametric correlations of conductance



$\delta V_G \propto \delta n$

$$\mathcal{C}(\delta n) = \langle\langle G(n) \cdot G(n+\delta n)\rangle\rangle_n$$



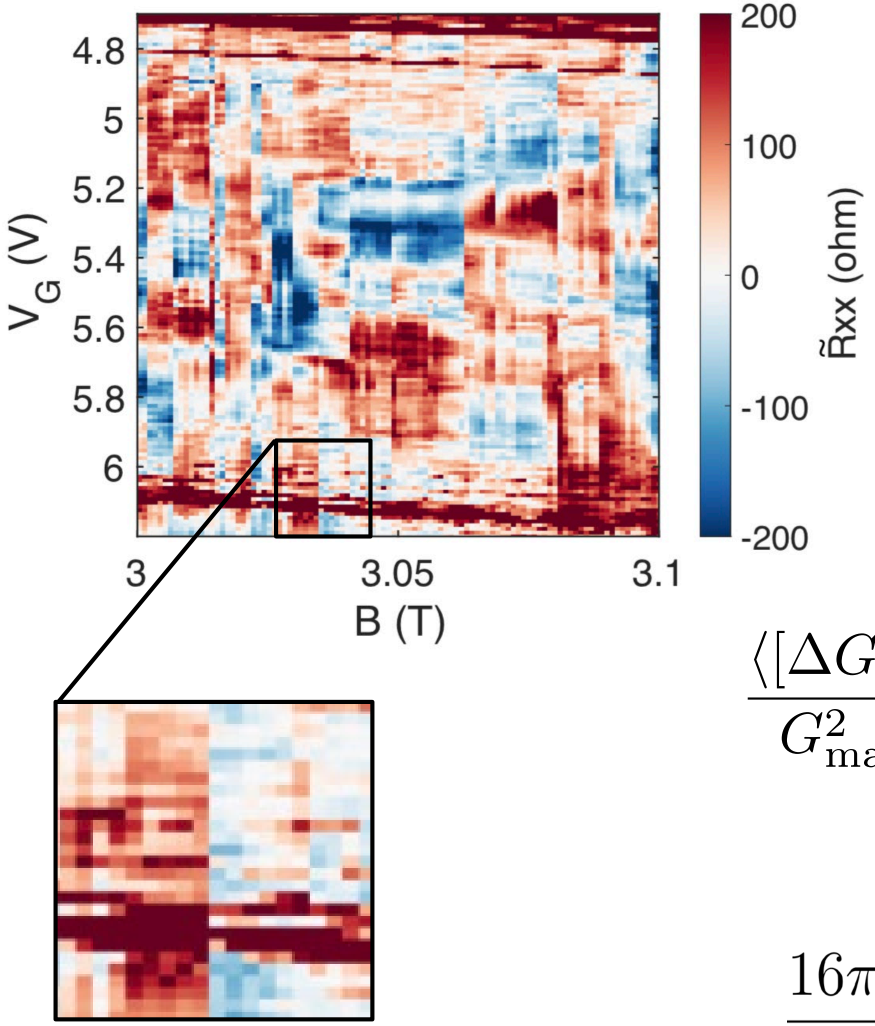
$$i\frac{\partial}{\partial x}\begin{pmatrix} a_{\rm e}(x) \\ a_{\rm h}(x)\end{pmatrix} = \begin{pmatrix} -\vartheta(x) & \alpha^\star(x) \\ \alpha(x) & \vartheta(x)\end{pmatrix}\begin{pmatrix} a_{\rm e}(x) \\ a_{\rm h}(x)\end{pmatrix}$$

$$\alpha(x) \to \alpha(x) \cdot e^{2i\delta k_\mu x}$$

$$\mathcal{C}(\delta n) = \langle\langle G^2\rangle\rangle \cdot \exp\left[-\frac{4}{3}\left(\frac{\delta n}{n_{\rm cor}}\right)^2\right], \qquad n_{\rm cor} = \frac{\partial n}{\partial \mu}\frac{\hbar v_{\rm edge}}{\sqrt{\ell_A L}}$$

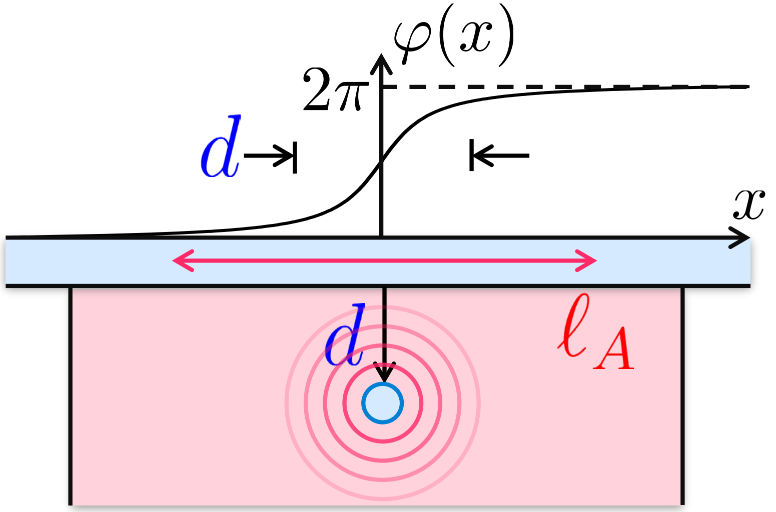
Similar to motional narrowing in NMR

Random jumps of conductance due to vortex entrance



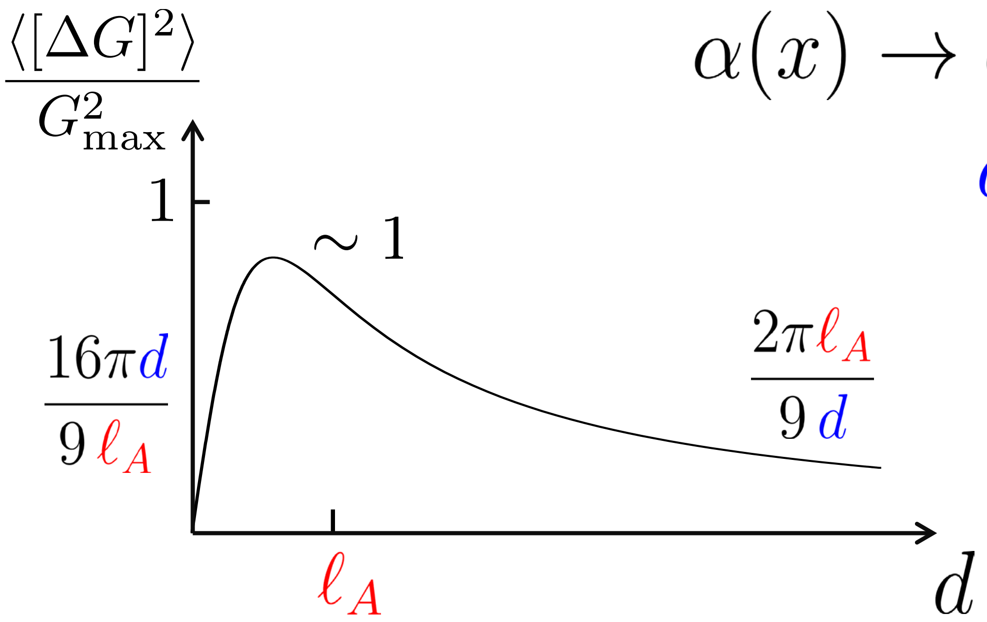
[L. Zhao et al., 2020]

Vortex entrance induces a phase in the Andreev reflection amplitude

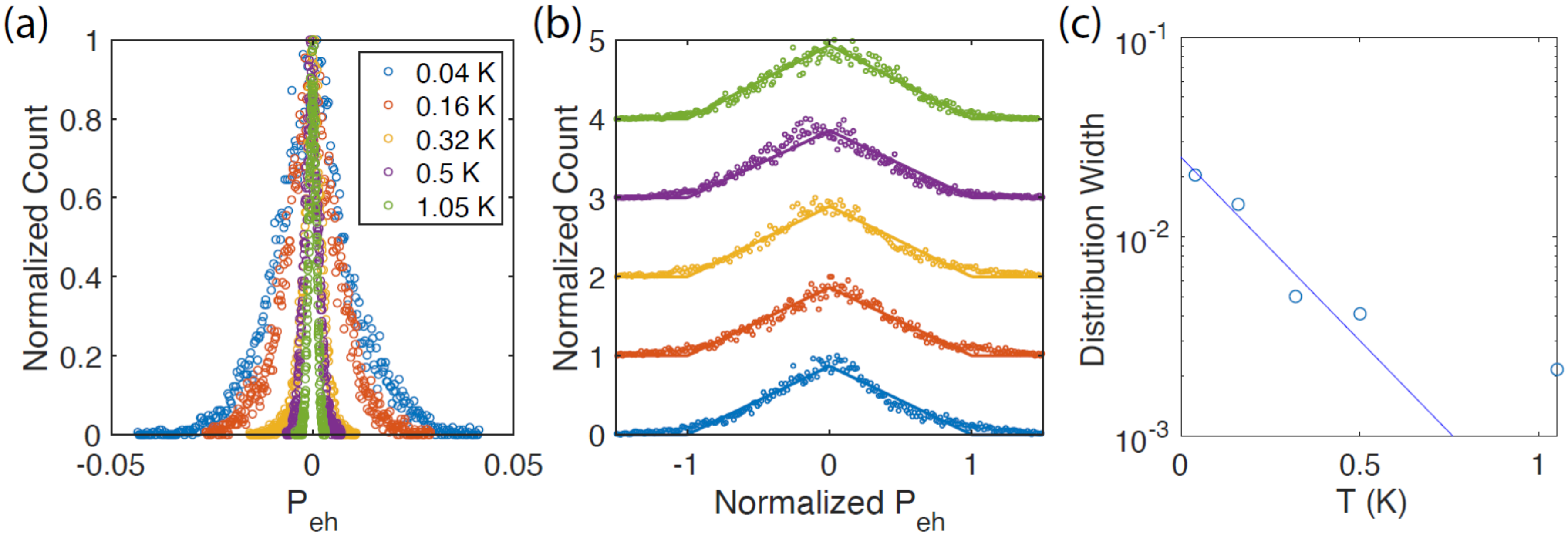


$$\alpha(x) \to \alpha(x) \cdot e^{i\varphi(x)}$$

d vs. ℓ_A



Newer experiment, arXiv:2210.0482



$$\sigma(P_{\rm eh}) \propto \exp(-L/l_{\rm in})$$

$$\frac{1}{l_{\rm in}} \propto \frac{B}{B_0} + \frac{T}{T_0}$$

“OK” ?

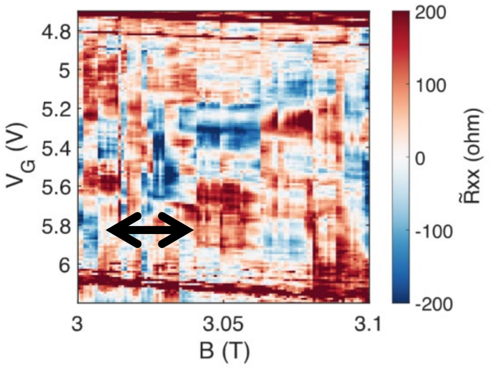
$$\frac{t_{\rm flight}}{\tau_\varphi} \to \frac{T}{T_0}\,; \quad \frac{1}{\tau_\varphi} \propto T g \ell_A$$

Figure 4: **Distribution function of P_{eh}.** (a) Histograms of P_{eh} collected in a 100 mT window around 1.45 T on the $\nu = 2$ plateau at various temperatures. Here, $L = 1\ \mu$m. (b) Replot of (a) with triangular distribution fits (lines). The y-axis is offset by 1 for each temperature

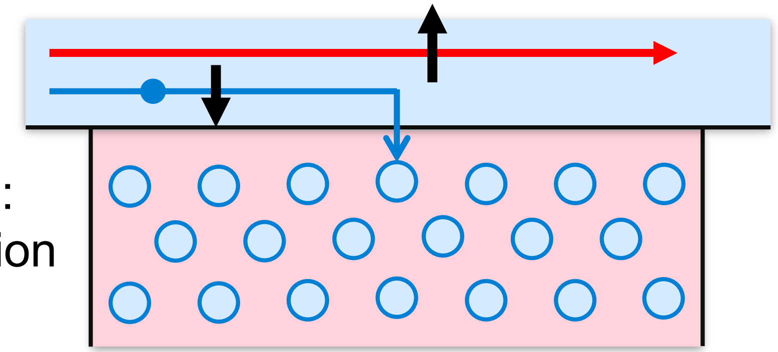
Instead of **rectangular** (flat) in theory

Possible reasons for discrepancy:

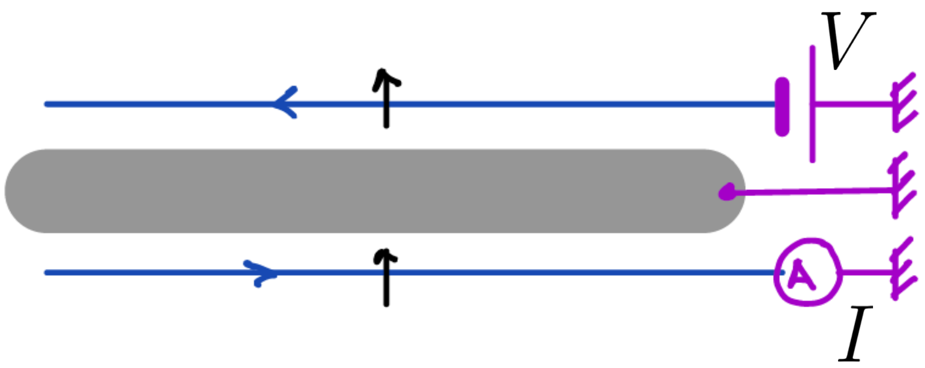
Mundane: vortex re-ordering



“Sophisticated”: spatial separation of edge states



Proximity-coupled counter-propagating ν=1 edges



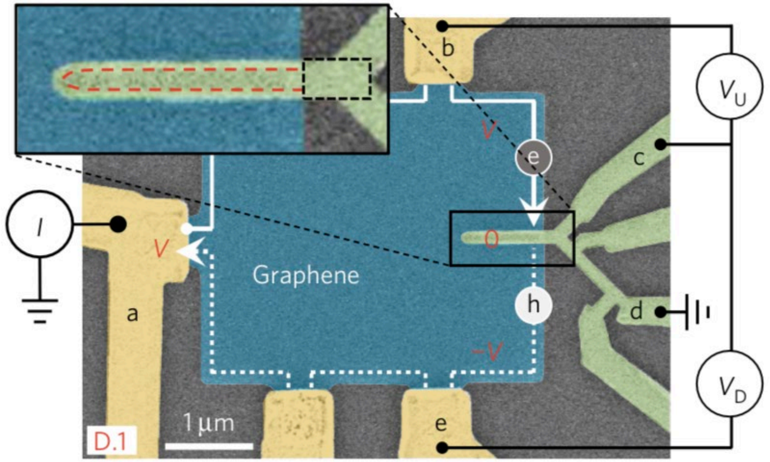
Why bother?

A promise of topological superconductivity!
[Kitaev, Fu-Kane (2008), …]

Edge states:
$$H_{\rm QH} = -iv\int dx\left(\psi_R^\dagger\partial_x\psi_R - \psi_L^\dagger\partial_x\psi_L\right)$$

The ideal coupling term:
$$H_{\rm CAR} = \int dx\left(\Delta\psi_R^\dagger\psi_L^\dagger + {\rm h.c.}\right)$$
crossed Andreev reflection, CAR

Great motivation for experiment!



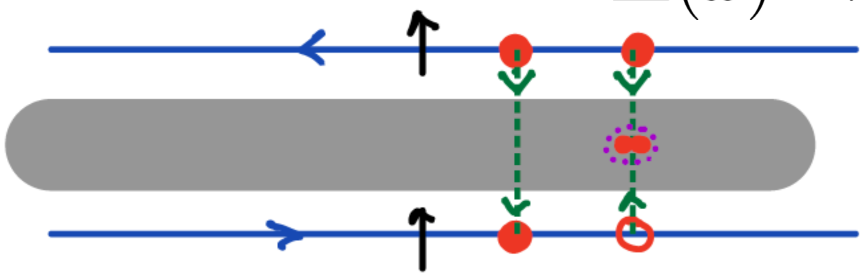
[Gil-Ho Lee et al., 2017]

Proximity-coupled counter-propagating ν=1 edges

The reality is different: (1) normal tunneling along with CAR; (2) disorder.

$$H_{\mathrm{N}} = \int dx \left(t_N(x) \psi_R^\dagger \psi_L + \mathrm{h.c.} \right) \; ; \qquad H_{\mathrm{CAR}} = \int dx \left(\Delta(x) \psi_R^\dagger \psi_L^\dagger + \mathrm{h.c.} \right)$$

$\Delta(x) \rightarrow \delta A_{\mathrm{A}}$



$t_N(x) \rightarrow \delta A_{\mathrm{N}}$

Only $\delta A_{\mathrm{A}}(x)$ ↦ top. superconductor

Only $\delta A_{\mathrm{N}}(x)$ ↦ insulator (Anderson localization)

$$\langle |\delta A_{\mathrm{A/N}}|^2 \rangle = \frac{\delta L}{l_{\mathrm{A/N}}};$$

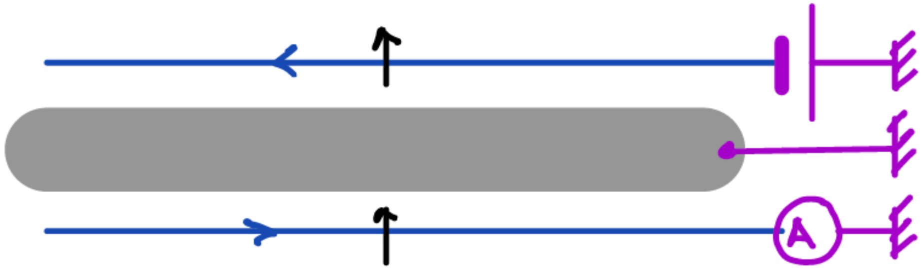
in general, $\frac{1}{l_{\mathrm{N}}} \geq \frac{1}{l_{\mathrm{A}}}$ (insulator wins)

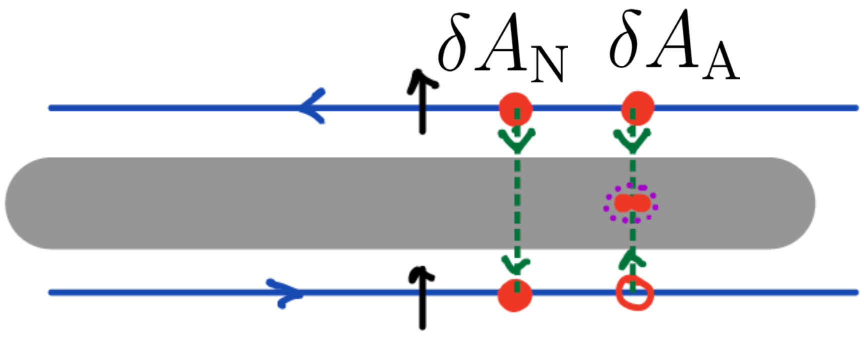
$\frac{1}{l_{\mathrm{N}}} = \frac{1}{l_{\mathrm{A}}}$ at $\tau_{\mathrm{so}} \Delta \ll 1$ and $E \rightarrow 0$ (critical state)

Proximity-coupled counter-propagating ν=1 edges

$$\langle|\delta A_{A/N}|^2\rangle = \frac{\delta L}{l_{A/N}};\qquad \frac{1}{l_N} = \frac{1}{l_A} \text{ at } \tau_{so}\Delta \ll 1 \text{ and } E \to 0$$

Microscopic calculation: $\frac{1}{l_N} = \frac{1}{l_A} \equiv \frac{1}{2l_0};\ \frac{1}{2l_0} = \frac{4\pi g^2}{G_Q\sigma}\sqrt{\frac{\pi\xi}{2d}}e^{-d/\xi}$





Transport problem: scattering matrix

$$i\frac{dS}{dx} = -\frac{2(E+i\Gamma)}{v}S + \mathcal{U} + S\,\mathcal{U}^\dagger S + \ldots$$

Γ : electron loss to vortex cores

$\mathcal{U}$: 2x2 matrix (Nambu space), linear in random amplitudes $\delta A_N, \delta A_A$

A particularly convenient parameterization of the S-matrix for analyzing Eq. (9) is

$$S = \frac{1}{2}\begin{pmatrix} F_+(w_1,w_2)e^{i\alpha} & F_-(w_1,w_2)e^{i\phi} \\ F_-(w_1,w_2)e^{-i\phi} & F_+(w_1,w_2)e^{-i\alpha} \end{pmatrix}, \tag{11a}$$

$$F_\pm(w_1,w_2) = -\tanh w_1 + \frac{i}{\cosh w_1} \pm \operatorname{sign}(w_1-w_2)\left[-\tanh w_2 + \frac{i}{\cosh w_2}\right]. \tag{11b}$$

Using this parameterization in Eq. (9), we obtain a system of equations governing the evolution of w_1, w_2, α, and ϕ:

$$\frac{dw_1}{dx} = \frac{2E}{v}\cosh w_1 + \eta_{\mathrm{N}x}\sin\alpha - \eta_{\mathrm{N}y}\cos\alpha + \eta_{\mathrm{A}x}\sin\phi - \eta_{\mathrm{A}y}\cos\phi, \tag{12a}$$

$$\frac{dw_2}{dx} = \frac{2E}{v}\cosh w_2 + \big(\eta_{\mathrm{N}x}\sin\alpha - \eta_{\mathrm{N}y}\cos\alpha - \eta_{\mathrm{A}x}\sin\phi + \eta_{\mathrm{A}y}\cos\phi\big)\operatorname{sign}(w_1-w_2), \tag{12b}$$

$$\frac{d\alpha}{dx} = \vartheta_{\mathrm{R}} + \vartheta_{\mathrm{L}} + q(w_1,w_2)\big(\eta_{\mathrm{N}x}\cos\alpha + \eta_{\mathrm{N}y}\sin\alpha\big), \tag{12c}$$

$$\frac{d\phi}{dx} = \vartheta_{\mathrm{R}} - \vartheta_{\mathrm{L}} - q(w_2,w_1)\operatorname{sign}(w_1-w_2) \times \big(\eta_{\mathrm{A}x}\cos\phi + \eta_{\mathrm{A}y}\sin\phi\big). \tag{12d}$$

First two equations can be reduced to Langevin-like equations

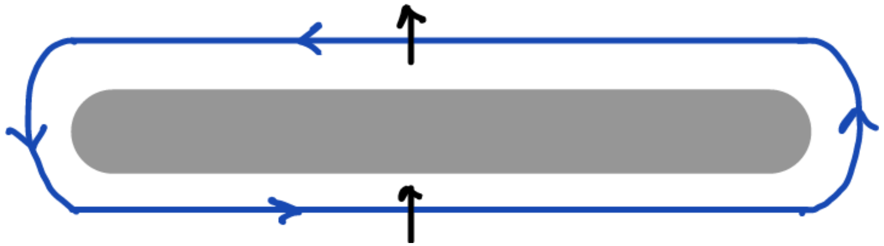
$$\frac{dw_i}{dx} = -\frac{\partial U(w_1,w_2)}{\partial w_i} + \tilde{\eta}_i(x)$$

Competition of $\delta A_{\mathrm{N}}, \delta A_{\mathrm{A}}$, signature of criticality in DOS

$$i\frac{dS}{dx} = -\frac{2(E+i\Gamma)}{v}S + \mathcal{U} + S\mathcal{U}^{\dagger}S + \ldots$$

set $\Gamma = 0$

consider a closed geometry:



Integrated DOS:

$$N(E) = \frac{1}{L}\frac{1}{2\pi i}\ln\det S(E,L)$$

$\mathcal{U}$: 2x2 matrix (Nambu space), linear in random amplitudes $\delta A_{\mathrm{N}}, \delta A_{\mathrm{A}}$

$$\langle|\delta A_{\mathrm{A/N}}|^2\rangle = \frac{\delta L}{l_{\mathrm{A/N}}};$$ allow for $l_{\mathrm{A/N}} = 2l_0(1\pm\lambda);\ |\lambda| \ll 1 \ \rightarrow \frac{dN}{dE} \equiv \nu(E) \propto \frac{1}{E^{1-2|\lambda|}}$

$$\nu(E) \propto \frac{1}{E\ln^3(v/El_0)}$$

Dyson singularity, indicative of the "infinite randomness" critical point at λ=0

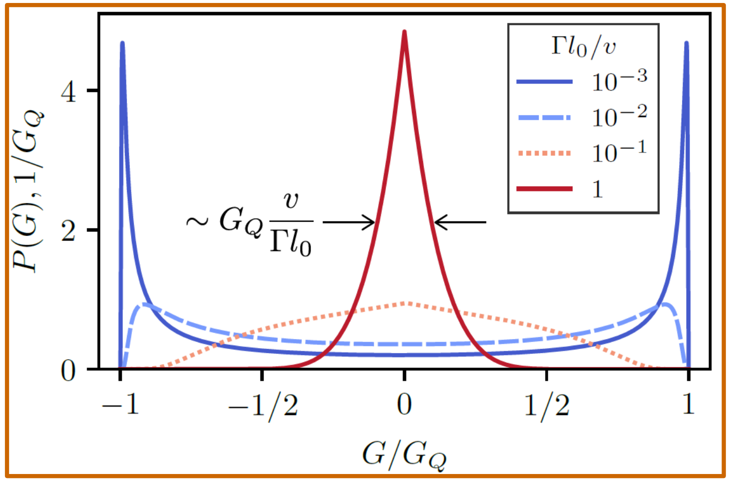
Brouwer et al, 2011; Motrunich et al, 2001

Zero-bias conductance at the critical point (λ=0)

at $\Gamma = 0$ conductance distribution function $P(G) = \frac{1}{2}\left[\delta(G+G_Q) + \delta(G-G_Q)\right]$

in the practical case (vortices present) $\Gamma \gg v/l_0$

$$P(G) = \frac{1}{G_Q}\frac{4\Gamma l_0}{v}\exp\left[-\frac{8\Gamma l_0}{v}\frac{|G|}{G_Q}\right]$$

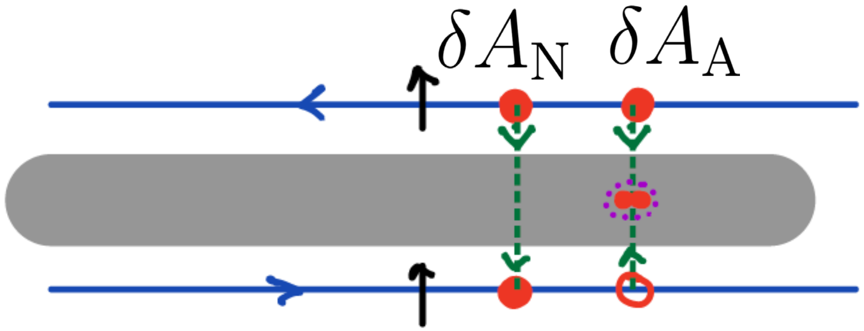


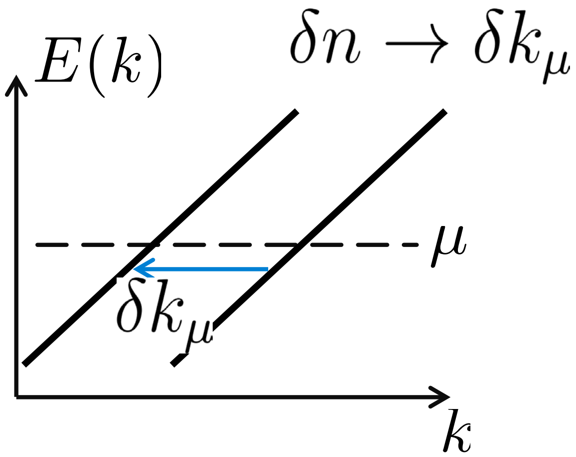
Other cool stuff, which was left out from this talk:

--Predictions for finite-bias conductance

--Predictions for conductance correlation functions

Parametric correlations of conductance





$$\delta A_{\rm N}(x) \to \delta A_{\rm N}(x) e^{-2i\delta k_\mu x}$$

$(L \to R)$

$$\delta A_{\rm A}(x) \to \delta A_{\rm A}(x)$$

$(L \to R + p \to h)$

in the approximation of "vertical" tunneling

$$\frac{\langle G(n+\delta n) G(n) \rangle}{\langle G^2 \rangle} = \frac{1}{2} + \frac{1}{2} \frac{1}{1 + (\delta n / n_{\rm c,N})^2}$$

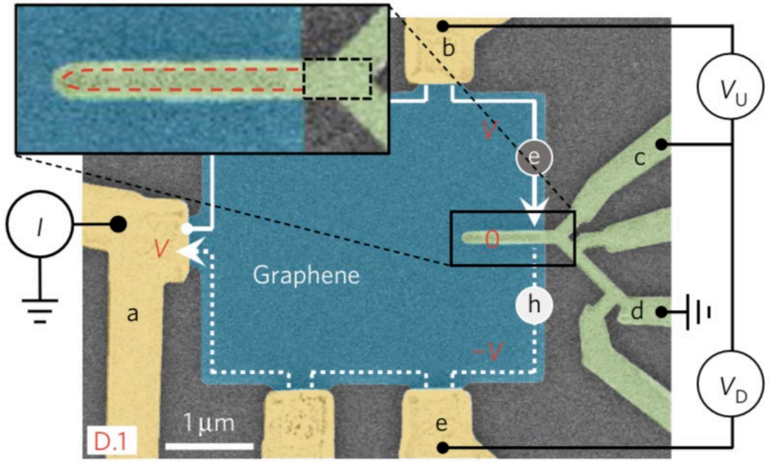
$$n_{\rm c,N} = 2\Gamma(\partial n / \partial \mu)$$

Conductance correlation function does not decay to zero

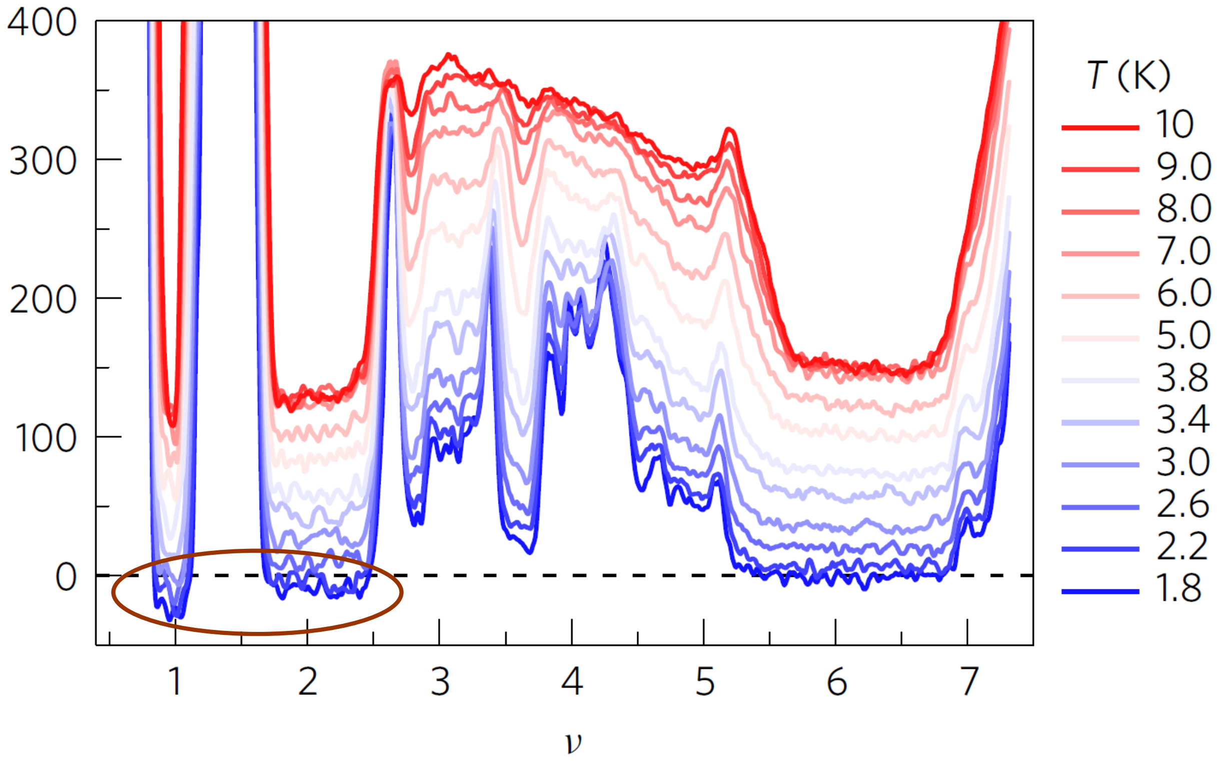
Negative value of conductance may be "robust" wrt to the 2D electron density variation

Parametric correlations of conductance

Negative value of conductance “robust” wrt to the 2D electron density variation may (?) explain experiment



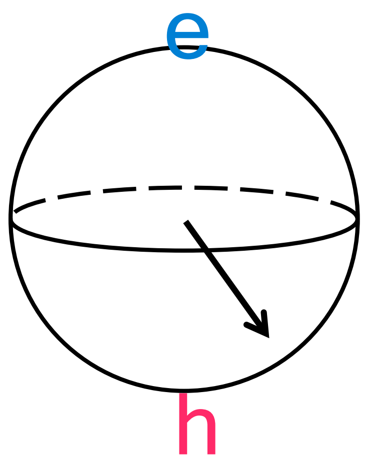
[Gil-Ho Lee et al., 2017]



Conclusions

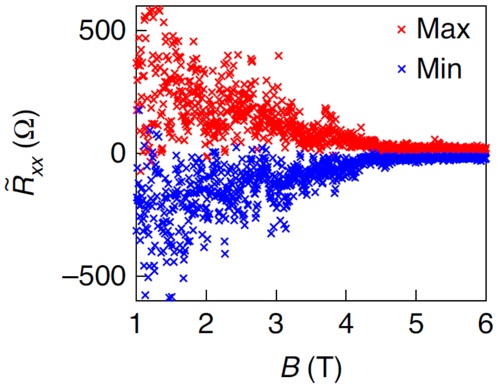
Characteristic length of **random** electron-hole conversion

$$\frac{1}{\ell_A} = \frac{4\pi g^2}{G_Q \sigma} \ln \frac{\xi}{l}$$



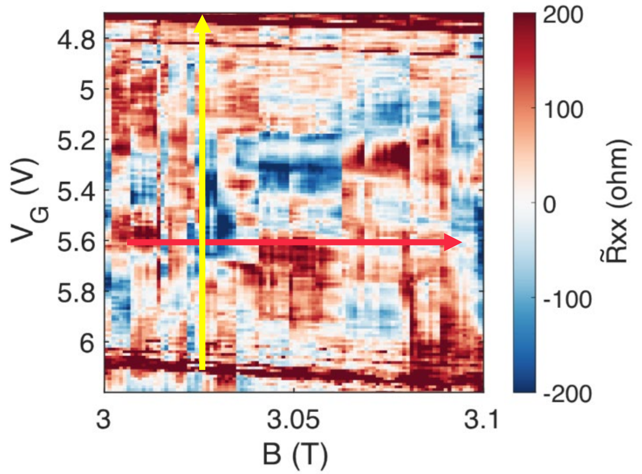
Single edge: conductance values defined by random walk on a Bloch sphere

Dual role of vortices:
(1) shrinking the Bloch sphere – qualitatively explains the experimental observation, but the form of the conductance distr. function remains unexplained



Conclusions

Dual role of vortices:
(2) Conductance jumps upon a vortex entrance

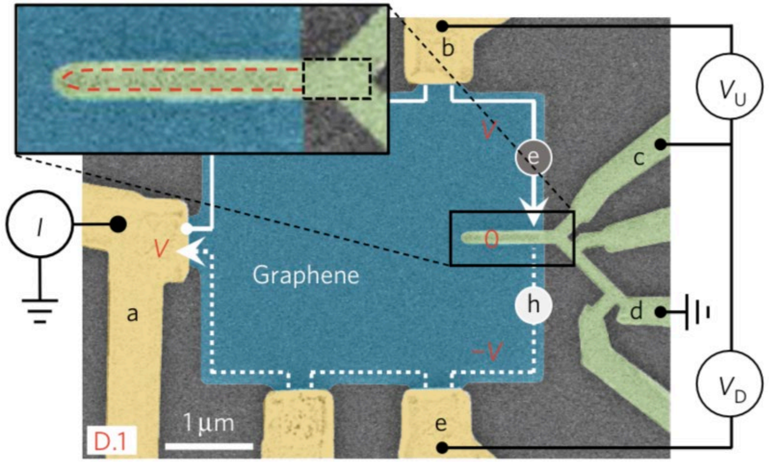


Parametric correlations of conductance with varying 2DEG density and magnetic field

Proximity-coupled counter-propagating $\nu = 1$ edges are **naturally tuned to the quantum critical point** between a topological superconductor and insulator;

A **possible** explanation of a stable G<0, calls for new experiments!!!

We make **quantitative predictions for the linear and nonlinear conductance**



[Gil-Ho Lee et al., 2017]