

# Analyzing LDOS and energy gap fluctuations: quantifying the interplay between electron-electron interactions and disorder in 2D superconductors

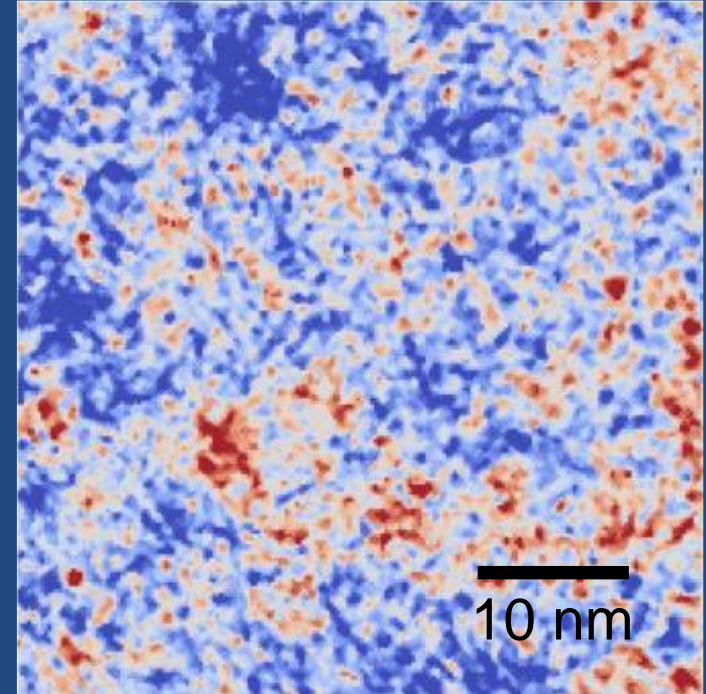
Christophe Brun

*Institute of Nanosciences of Paris*

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*Sorbonne University*

*Paris, FRANCE*



# Motivation

Effects produced by the interplay between disorder  
and electron-electron interactions in 2D  
superconductors

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superconductors

2D means  $d < (<) \xi$

Structure matters !!



Thin film top view

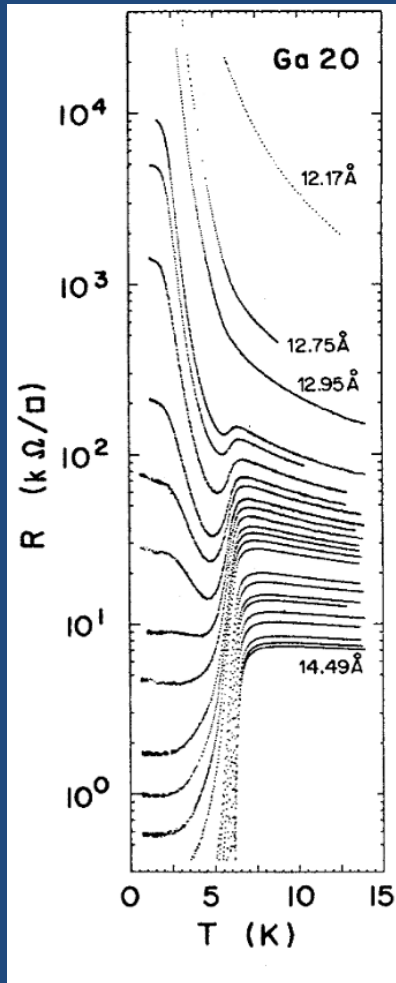
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Effects produced by the interplay between disorder  
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$$\Delta(\vec{r})e^{i\varphi(\vec{r})}$$

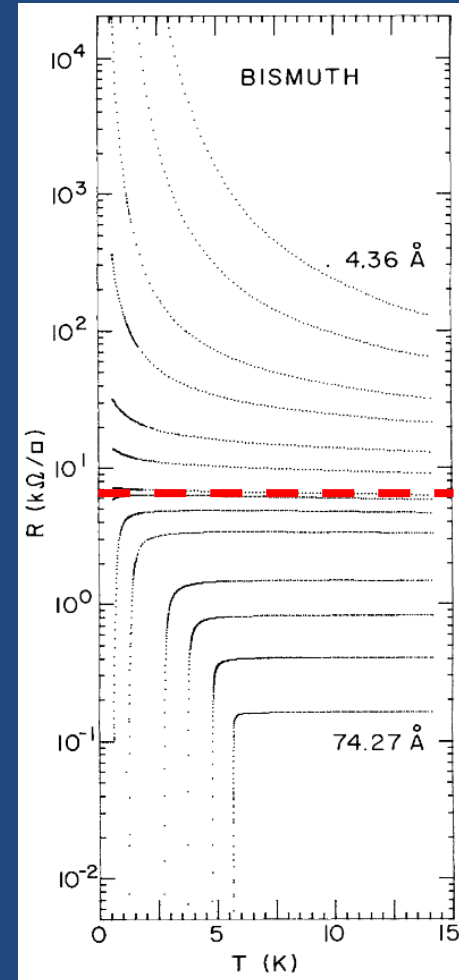
# Disordered superconducting ultrathin films

## Granular thin films



Jaeger et al. PRB 40, 182 (1989)

## Homogeneous thin films

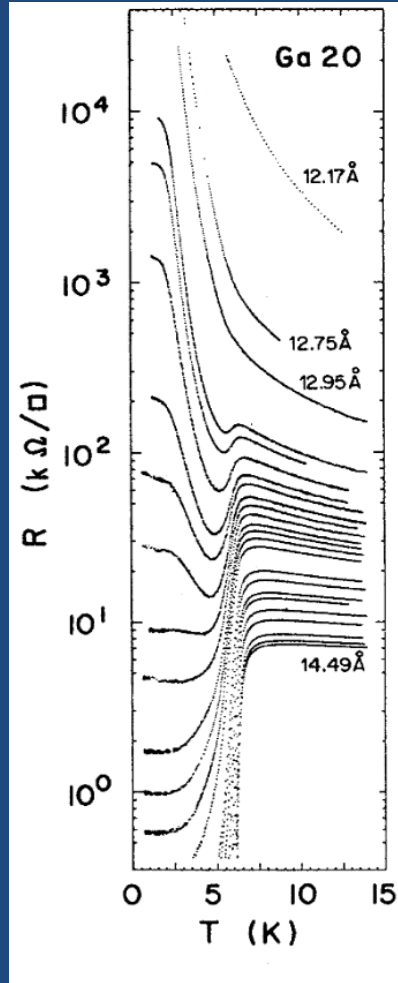


$$\frac{h}{4e^2} = 6.45\text{ k}\Omega$$

Haviland et al. PRL 62, 2180 (1989)

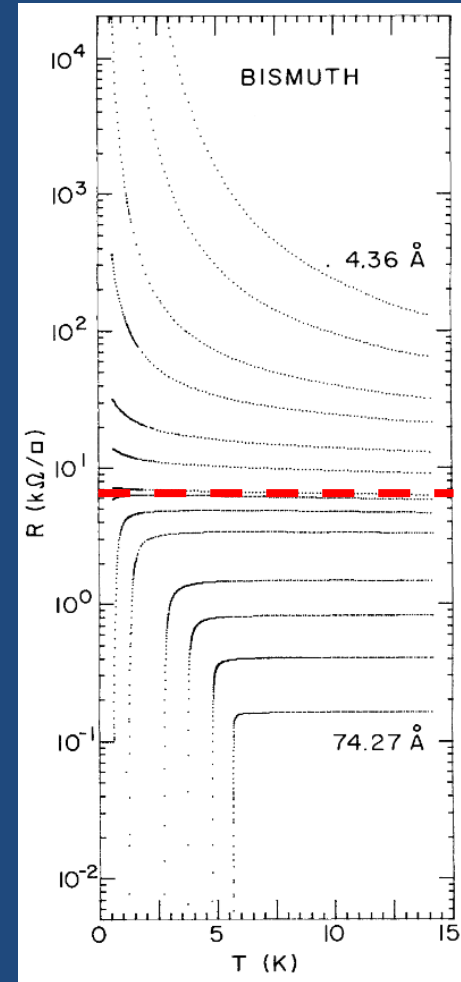
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# Homogeneous thin films

$$\Delta(\vec{r})e^{i\varphi(\vec{r})}$$

Fermionic scenario

$$k_F l \rightarrow 1$$

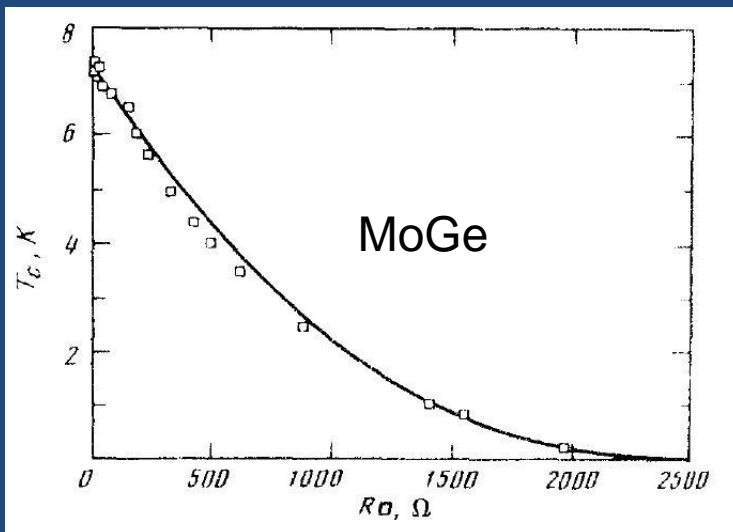
Bosonic scenario

Finkelstein mechanism

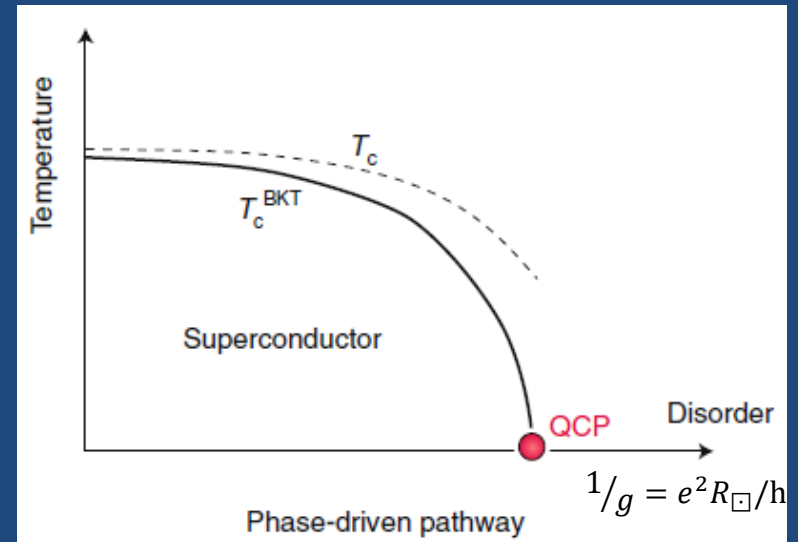
increased Coulomb repulsion:  
poor electronic screening reduces  $\lambda$

Anderson localization

local phase fluctuations



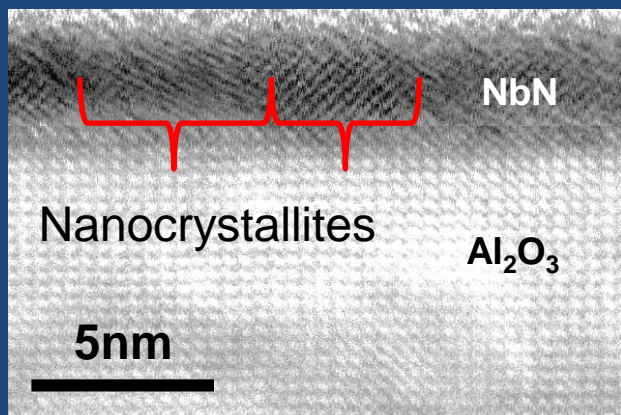
Finkelstein Pis'ma Zh. Ekip. Teor. Fis. 1987



Review: Sacépé et al Nat. Phys. 2020

# Local scale picture of nominally homogeneous thin films

## Structure of NbN on $\text{Al}_2\text{O}_3$ (ex situ growth)

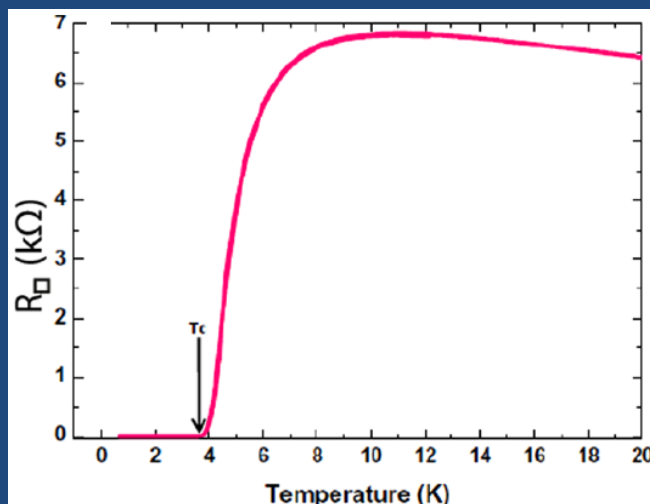


TEM image

$$d_{\text{NbN}} = 2.33 \text{ nm}$$
$$l_{\text{grain}} = 2 - 8 \text{ nm}$$



STM and in-situ 4 probe  $R(T)$



In situ  $R(T)$

Moderate disorder

$$k_F \ell_e \approx 2 - 3$$

$$T_c \approx 0.3 T_{c-\text{bulk}}$$

$$R_{\square} \approx 7 \text{ k}\Omega$$

Carbillet et al PRB 93, 144509 (2016)



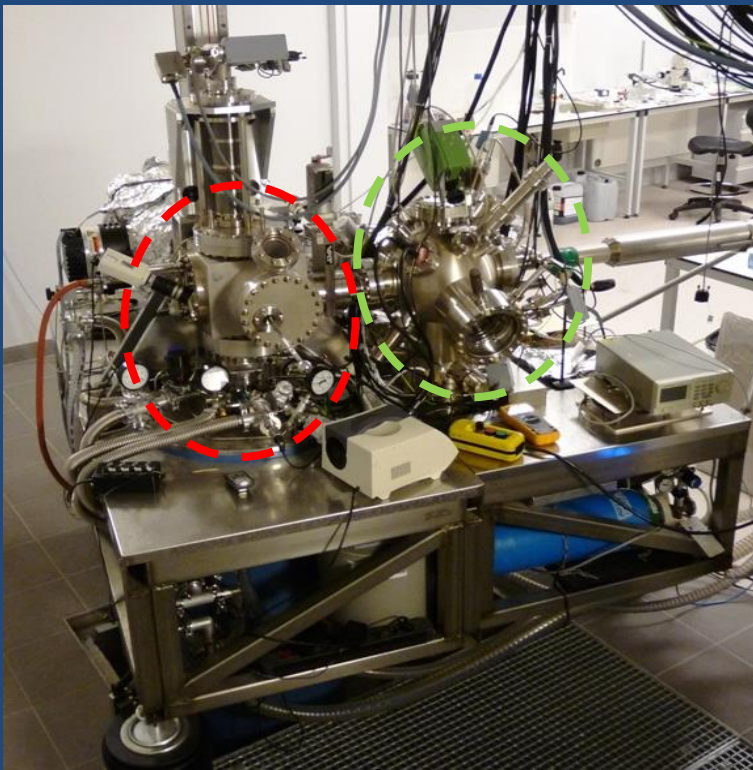
# STM/STS

UHV :  $P < 5 \times 10^{-11}$  mbar

*In situ* growth @  $P < 1 \times 10^{-10}$  mbar

Base  $T^\circ$  300 mK (He<sub>3</sub> single shot)  
 $T_{\text{electrons}} \sim 380$  mK (55 hours)

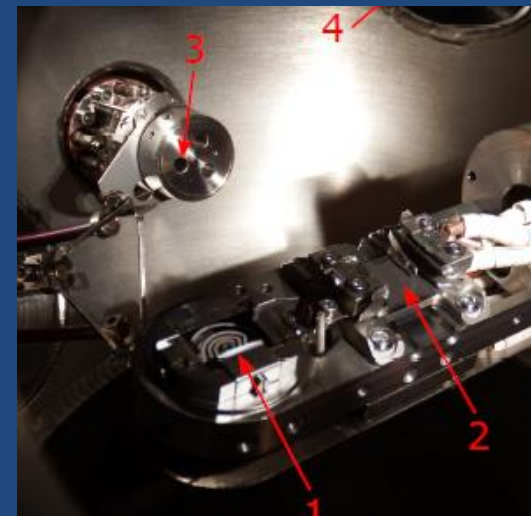
Magnetic Field: 0 – 10 T



Home-made apparatus

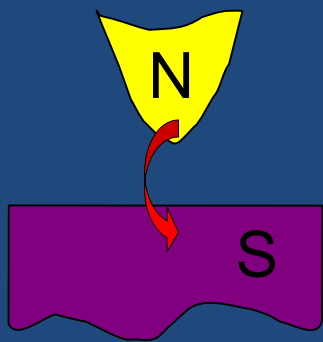


STM head



Preparation chamber  
e-beam evaporators

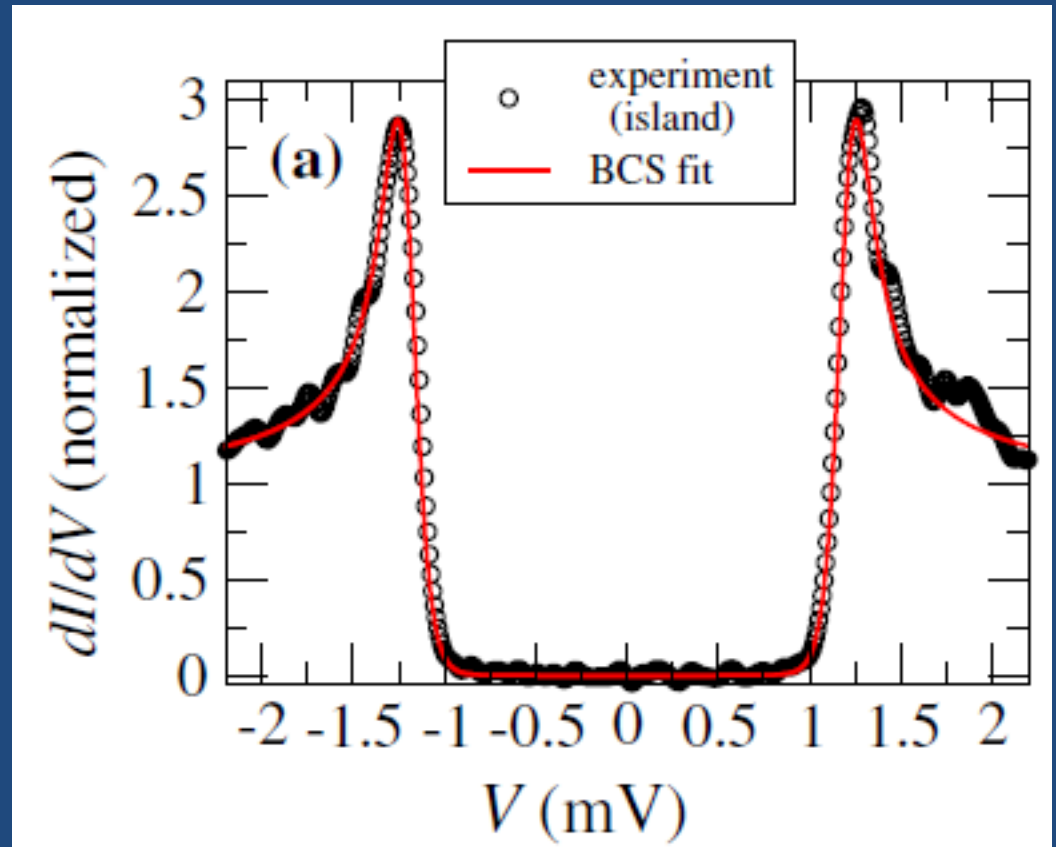
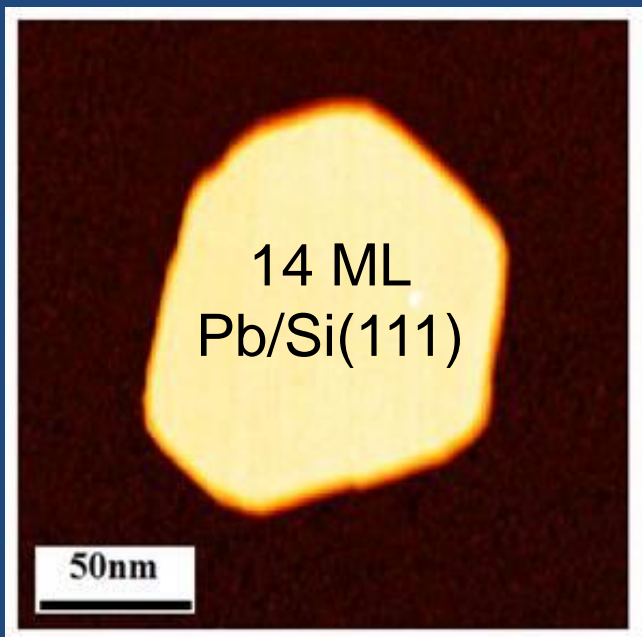
# Recap: tunneling spectroscopy of superconductors



$$dI / dV(\mathbf{r}) = \int_{-\infty}^{\infty} N_s(E, \mathbf{r}) \left[ \frac{-\partial f(E + eV)}{\partial(eV)} \right] dE$$

## BCS DOS

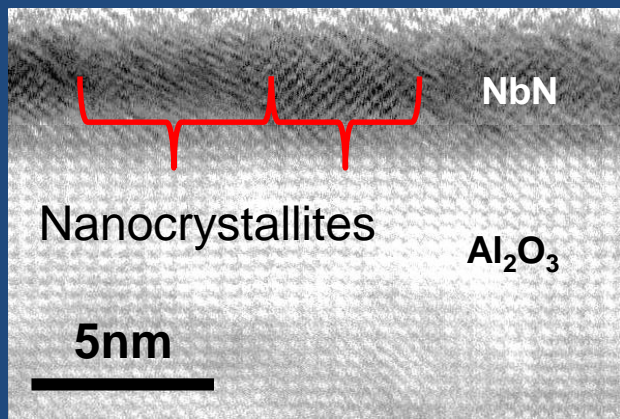
$$N_S(E) = N_N(E) \frac{E}{\sqrt{E^2 - \Delta^2}}$$



$$\Delta = 1.20 \text{ meV } T_{\text{eff}} = 0.38 \text{ K}$$

# Local scale picture of nominally homogeneous thin films

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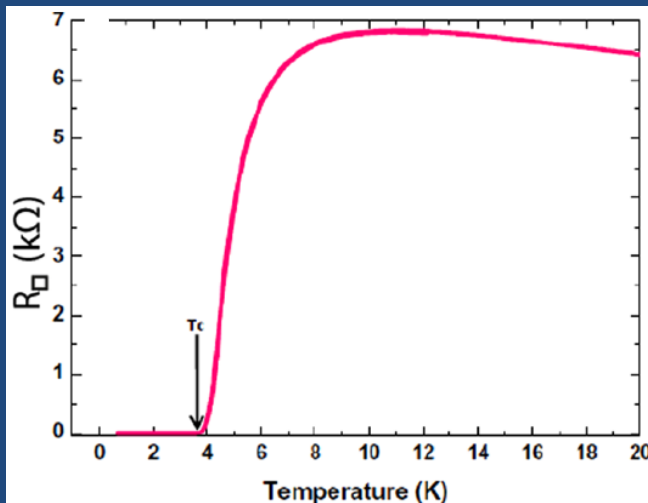


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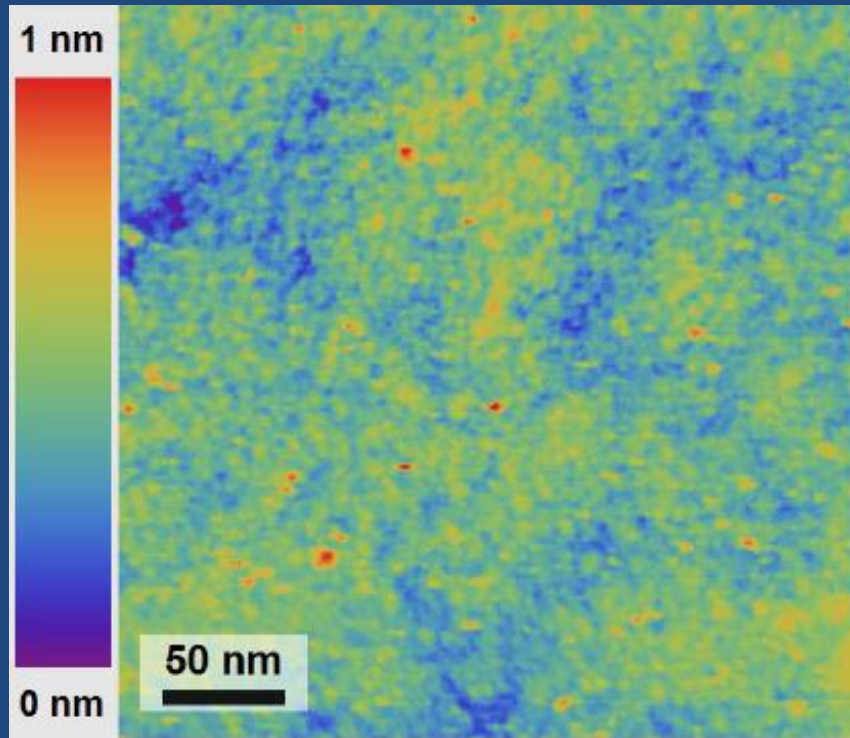
Carbillet et al PRB 93, 144509 (2016)



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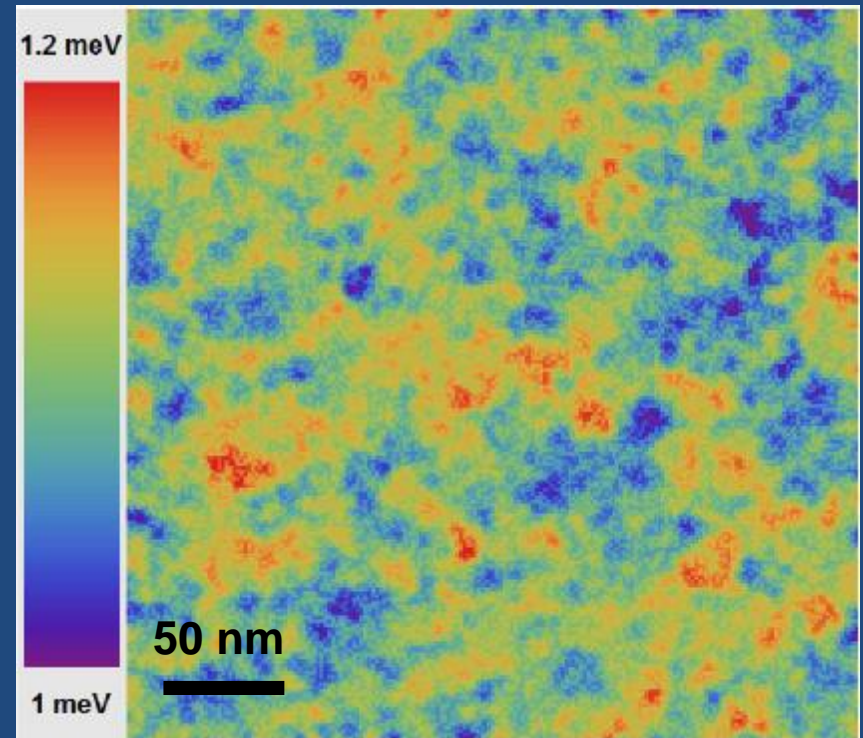
➔ Emerging electronic and superconducting inhomogeneities

STM topography



$$z(\vec{r})$$

Gap map



$$|\Delta|(\vec{r})$$

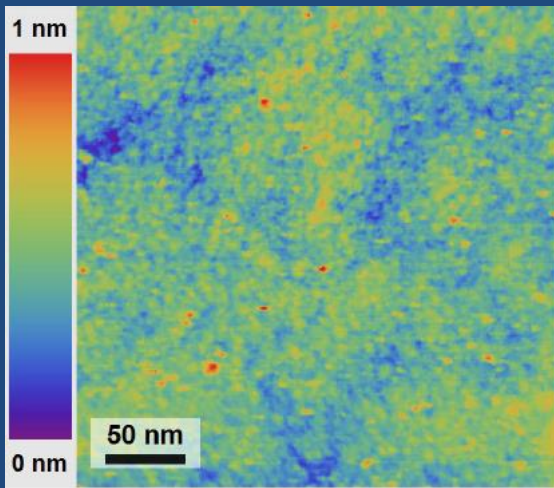
Carbillet et al PRB 93, 144509 (2016)

See also: Sacepe et al PRL 2008, Chand et al PRLB 2012, Noat et al PRB 2013...

# Local scale picture of nominally homogeneous thin films

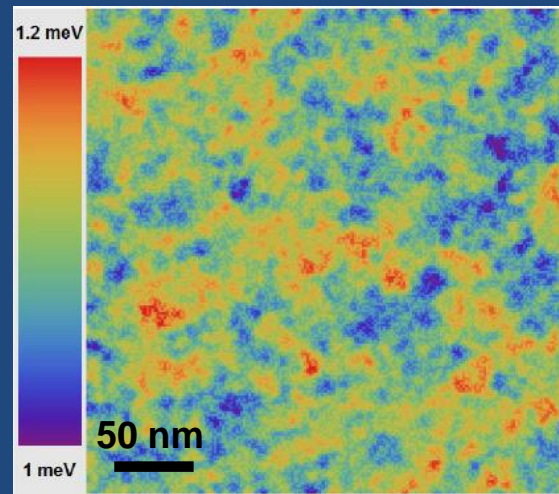
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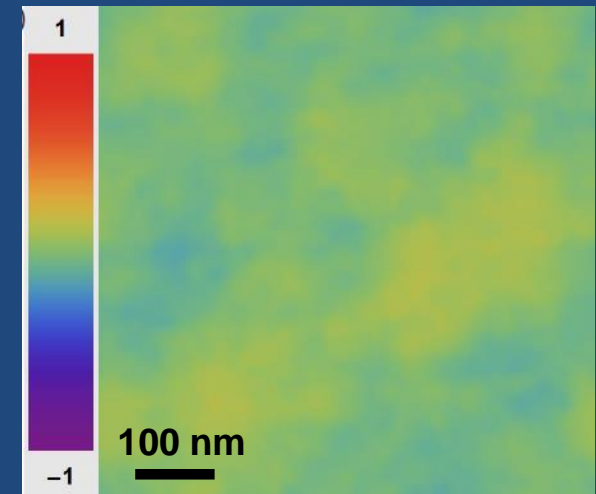
$$z(\vec{r})$$

gap map



$$|\Delta|(\vec{r})$$

cross correlation



$$\rho_{cross}(\vec{r}) = z * |\Delta|(\vec{r})$$

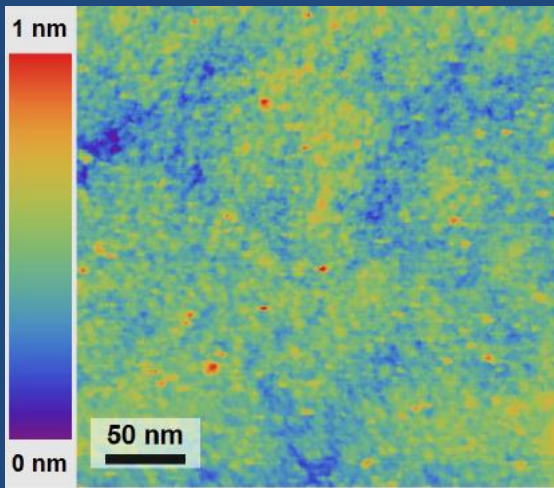


No local cross-correlation between  
the grain structure and gap inhomogeneities

# Local scale picture of nominally homogeneous thin films

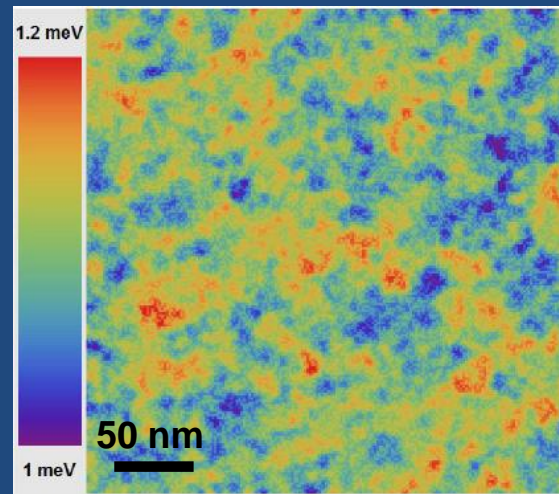
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STM topography



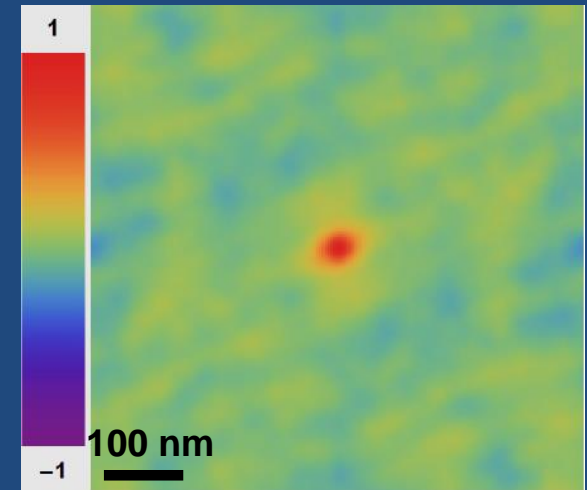
$$z(\vec{r})$$

gap map



$$|\Delta|(\vec{r})$$

auto correlation



$$\rho_{|\Delta|}(\vec{r}) = |\Delta| * |\Delta|(\vec{r})$$



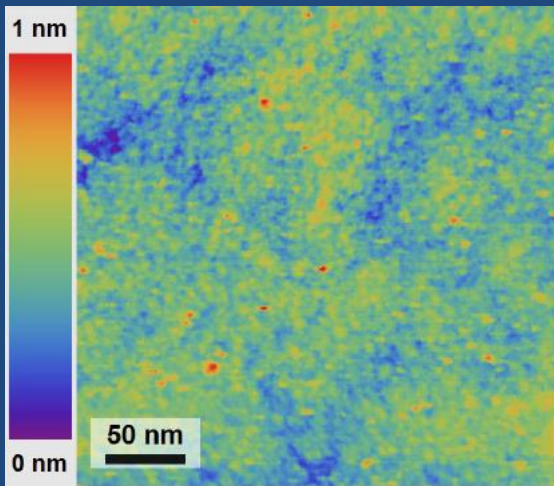
Superconducting puddles much larger than NbN nanocrystals



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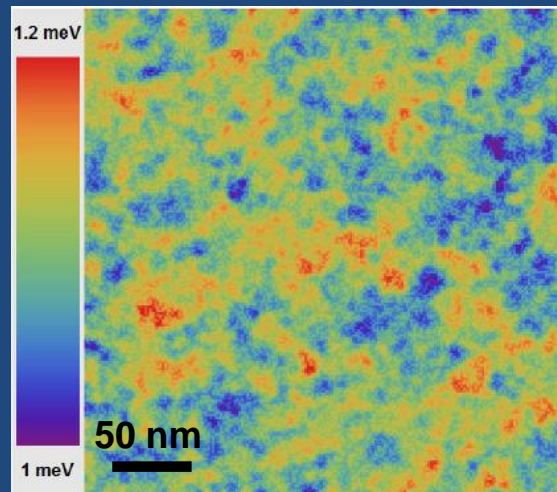
Emerging electronic and superconducting inhomogeneities

STM topography



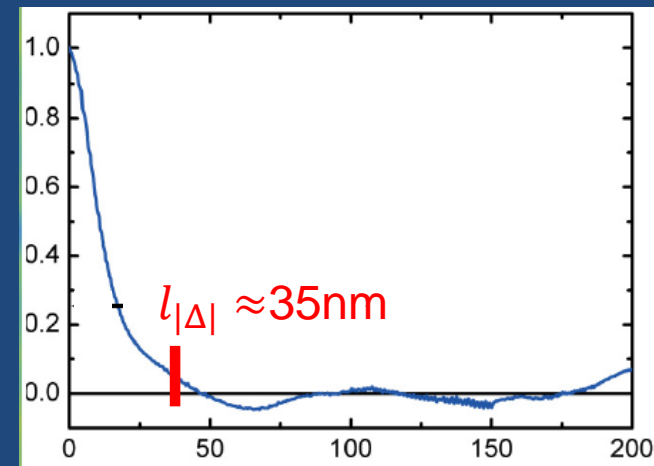
$z(\vec{r})$

gap map



$|\Delta|(\vec{r})$

auto correlation



$\rho_{|\Delta|}(\vec{r}) = |\Delta| * |\Delta|(\vec{r})$

$\longrightarrow l_{|\Delta|} \approx 35 \text{ nm} \gg \xi \approx 5 \text{ nm} \approx l_{\text{grain}}$

Carbillet et al PRB 93, 144509 (2016)

# Content

Revealing quantitatively the interplay between disorder and electron-electron interactions in 2D superconductors from LDOS analysis

## 1. Case of moderate disorder in NbN thin films

PHYSICAL REVIEW B **102**, 024504 (2020)

Editors' Suggestion

**Spectroscopic evidence for strong correlations between local superconducting gap and local Altshuler-Aronov density of states suppression in ultrathin NbN films**

C. Carbillet,<sup>1</sup> V. Cherkez,<sup>1</sup> M. A. Skvortsov,<sup>2,3,\*</sup> M. V. Feigel'man,<sup>3,2</sup> F. Debontridder,<sup>1</sup> L. B. Ioffe,<sup>4,3</sup> V. S. Stolyarov,<sup>1,5,6</sup> K. Ilin,<sup>7</sup> M. Siegel,<sup>7</sup> C. Noûs,<sup>8</sup> D. Roditchev,<sup>1,9</sup> T. Cren,<sup>1</sup> and C. Brun<sup>1,†</sup>

## 2. Case of weak disorder in Pb single atomic layer

PHYSICAL REVIEW B **107**, 174508 (2023)

Editors' Suggestion

**Local density of states fluctuations in a two-dimensional superconductor as a probe of quantum diffusion**

Mathieu Lizée,<sup>1,2,\*</sup> Matthias Stosiek,<sup>3</sup> Igor Burmistrov,<sup>4,5,†</sup> Tristan Cren,<sup>2</sup> and Christophe Brun<sup>2,‡</sup>



# Investigating LDOS behavior at the local scale

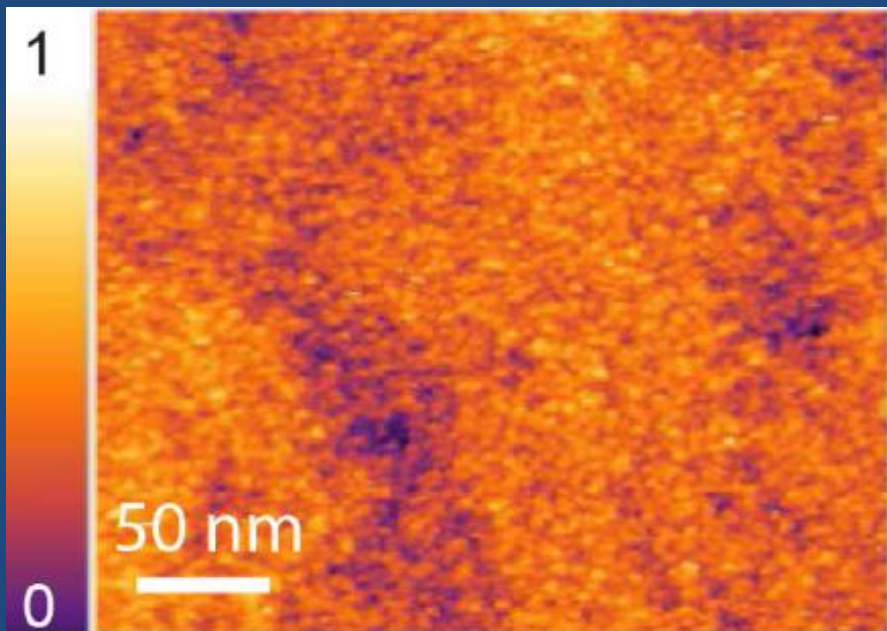
NbN film moderate disorder

$$k_F \ell_e \approx 2 - 3$$

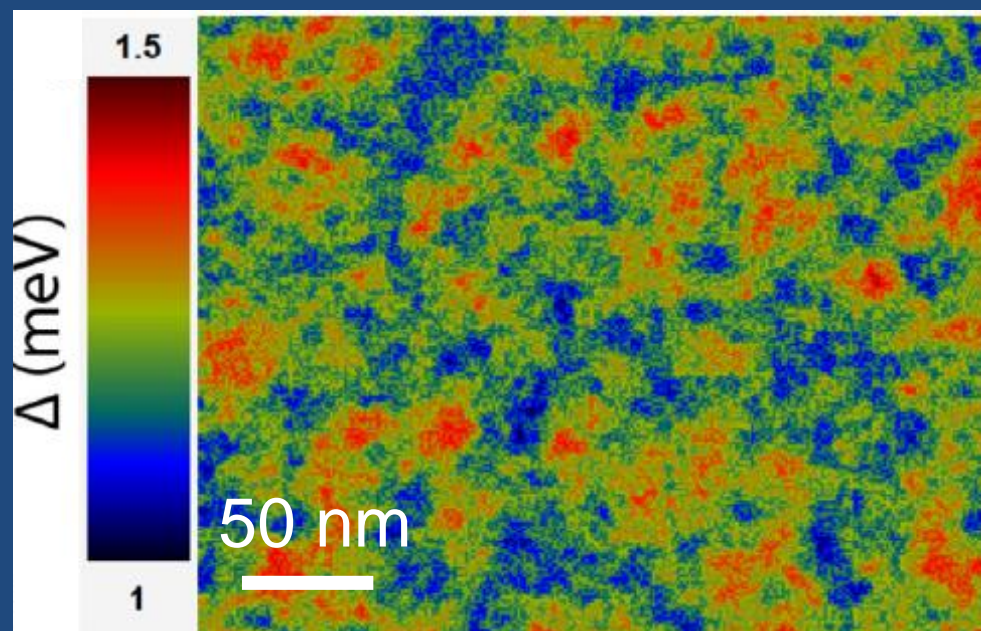
$$T_c \approx 0.25 T_{c-bulk}$$

STM topography

Gap map



$z(\vec{r})$



$|\Delta|(\vec{r})$

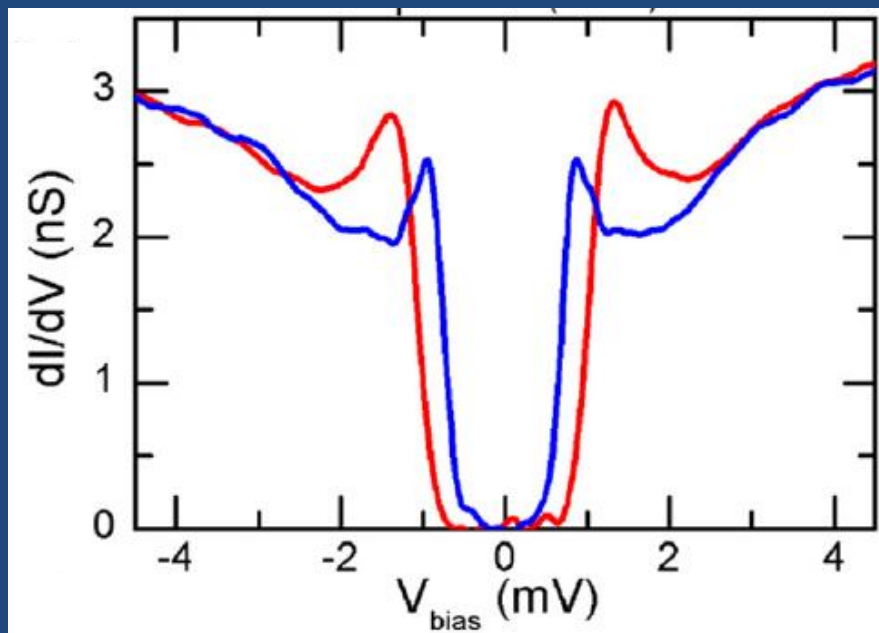
$$l_{|\Delta|} \approx 27 \text{ nm} \gg \xi \approx 5 \text{ nm} \approx l_{\text{grain}}$$

Carbillet et al PRB 102, 024504 (2020)

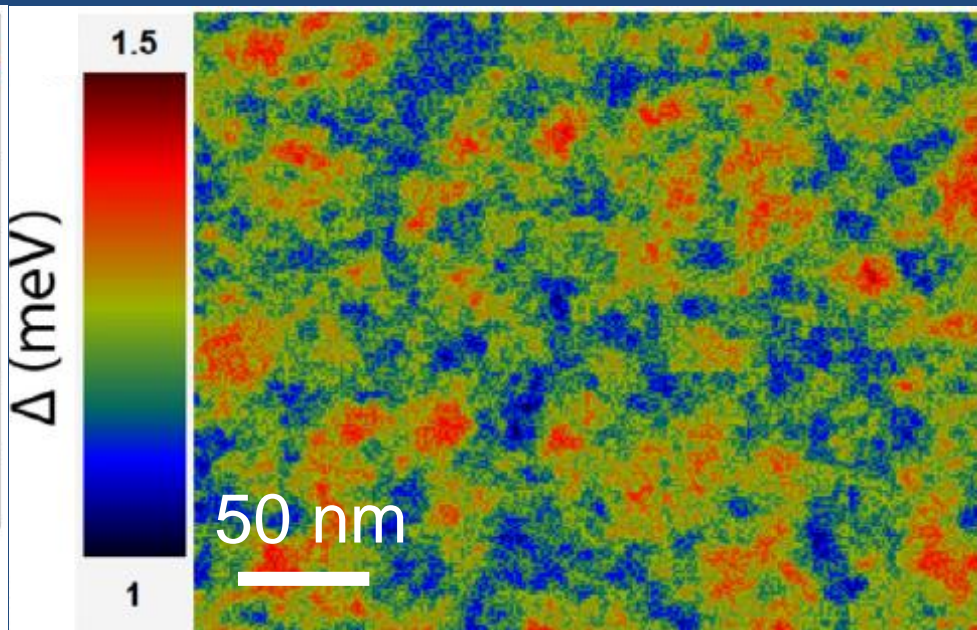
# Investigating LDOS behavior at the local scale

NbN film moderate disorder  $k_F \ell_e \approx 2 - 3$   $T_c \approx 0.25 T_{c-bulk}$

Local  $dI/dV$  spectra



Gap map



$$|\Delta|(\vec{r})$$

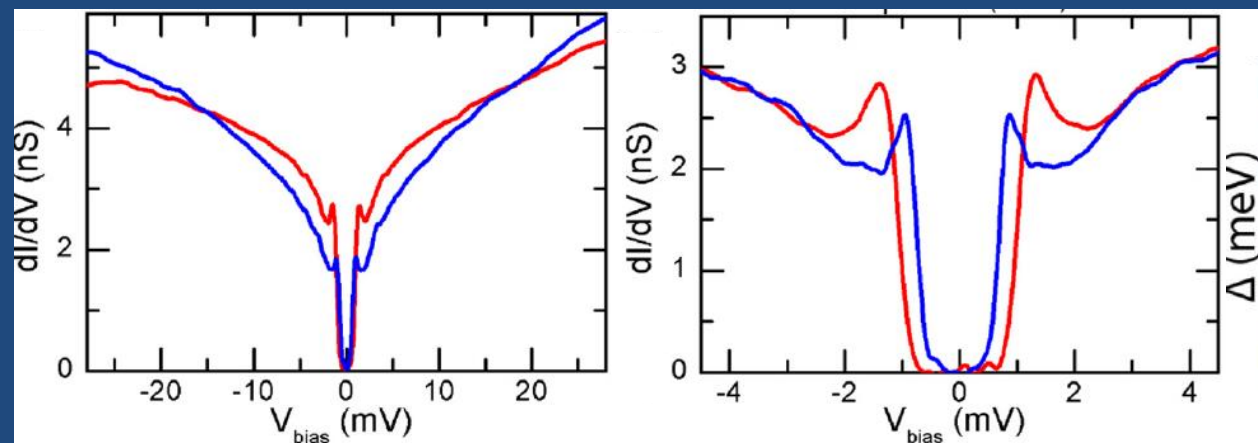
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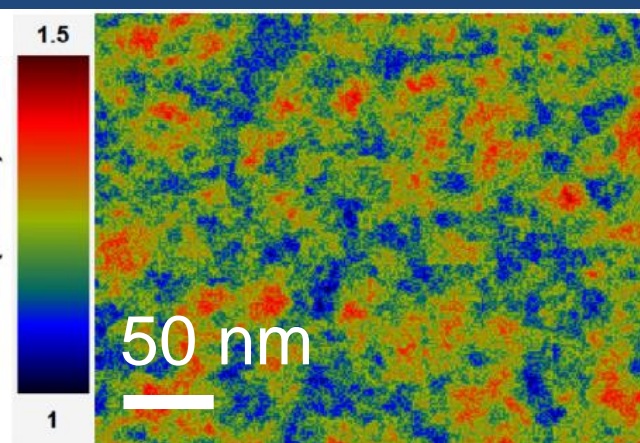
# Investigating LDOS behavior at the local scale

NbN film moderate disorder  $k_F \ell_e \approx 2 - 3$   $T_c \approx 0.25 T_{c-bulk}$

## Local dI/dV spectra



## Gap map



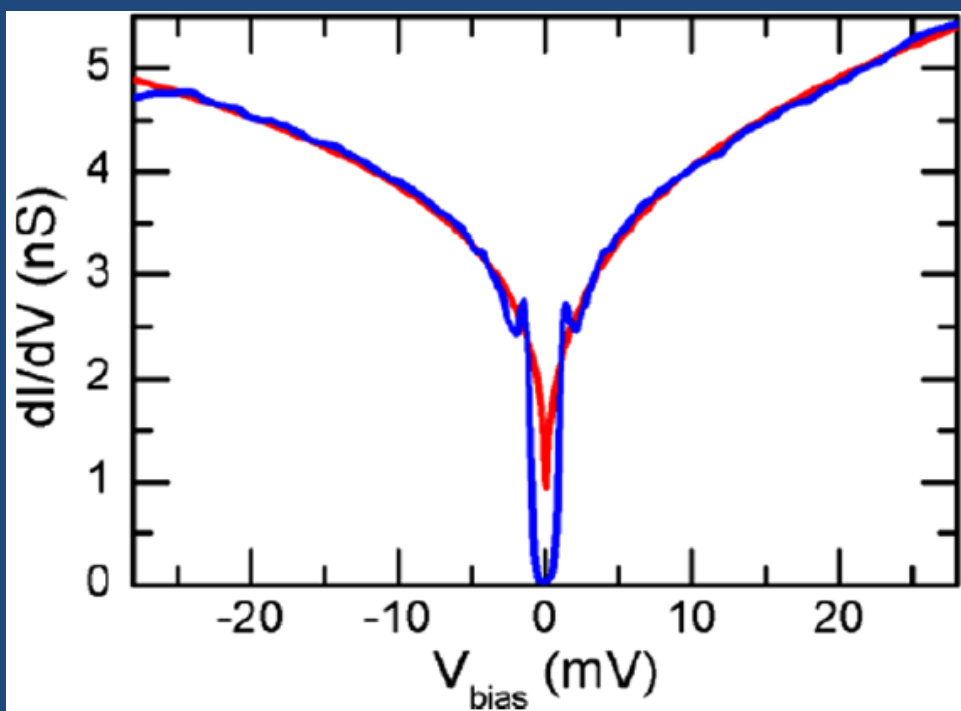
$$|\Delta|(\vec{r})$$

➔ Strong locally varying Altshuler-Aronov background

# Investigating LDOS behavior at the local scale

NbN film moderate disorder  $k_F \ell_e \approx 2 - 3$   $T_c \approx 0.25 T_{c-bulk}$

local  $dI/dV$  spectra at location  $\vec{r}$



red fit:

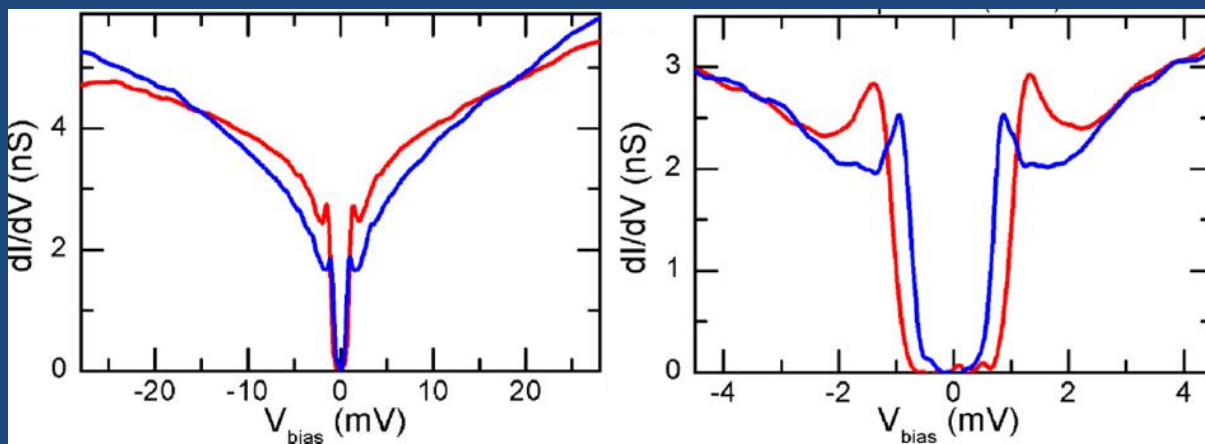
$$dI/dV(\vec{r}, V) = bV^{\alpha(\vec{r})}$$

➔ Power law fitting of the Altshuler-Aronov background



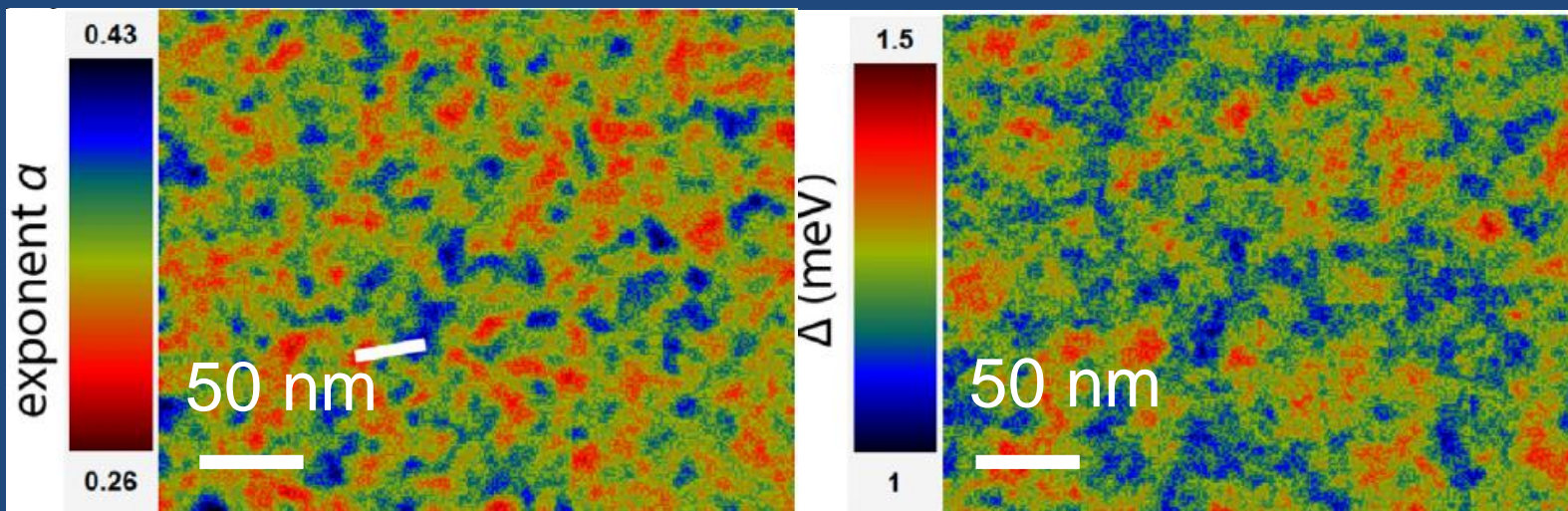
# Investigating LDOS behavior at the local scale

## Local $dI/dV$ spectra



Exponent map

Gap map



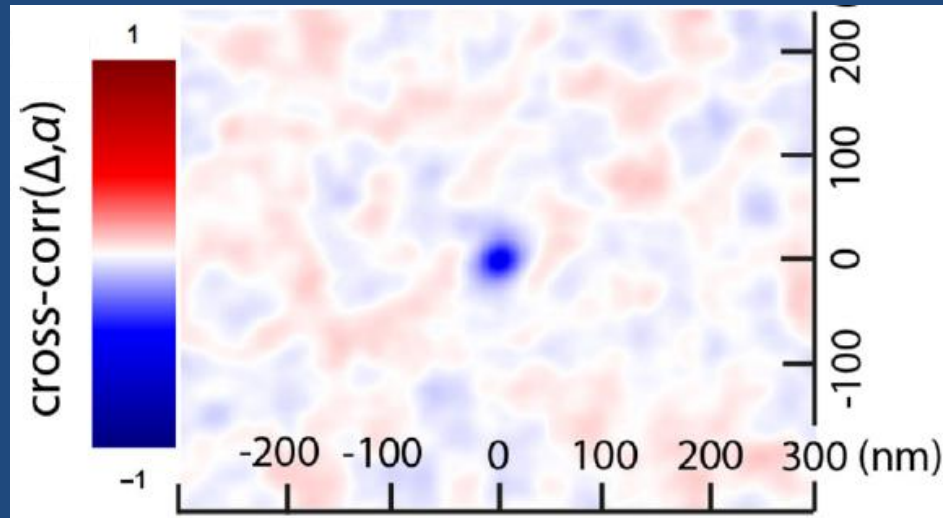
$\alpha(\vec{r})$

$|\Delta|(\vec{r})$

$l_\alpha \approx 18 \text{ nm}$

$l_{|\Delta|} \approx 27 \text{ nm}$

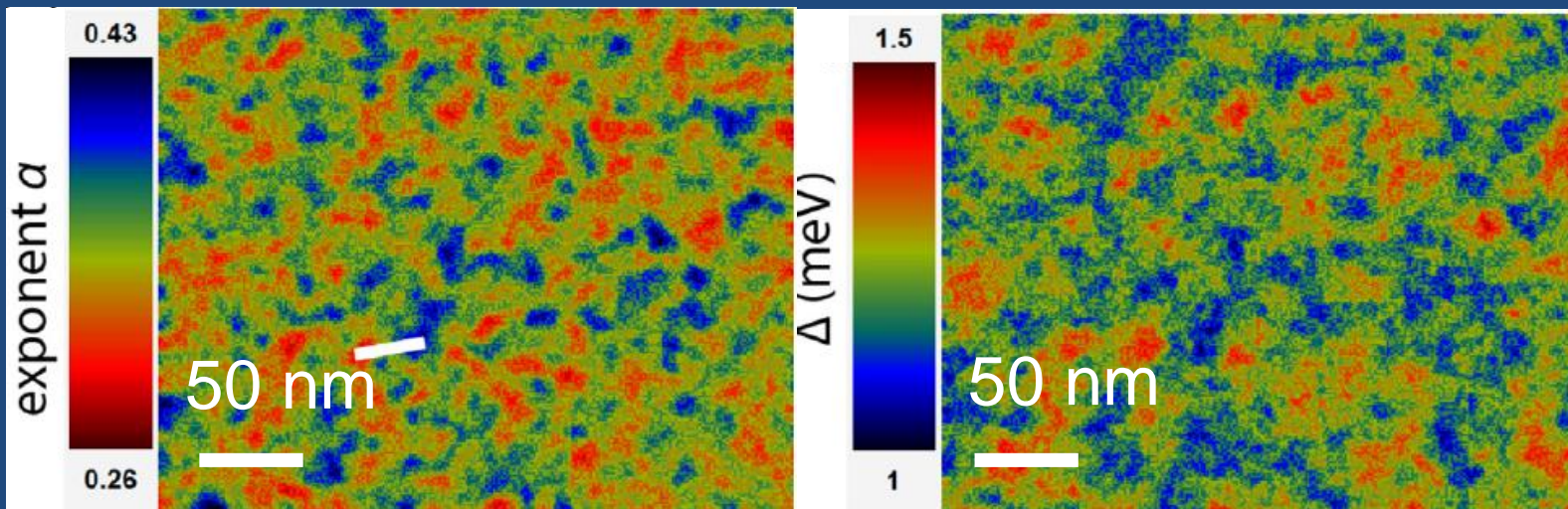
# cross correlation



$$\rho_{cross}(0) = -0.55$$

$$l_{cross} \approx 25 \text{ nm}$$

$$\rho_{cross}(\vec{r}) = \alpha * |\Delta|(\vec{r})$$



$$\alpha(\vec{r})$$

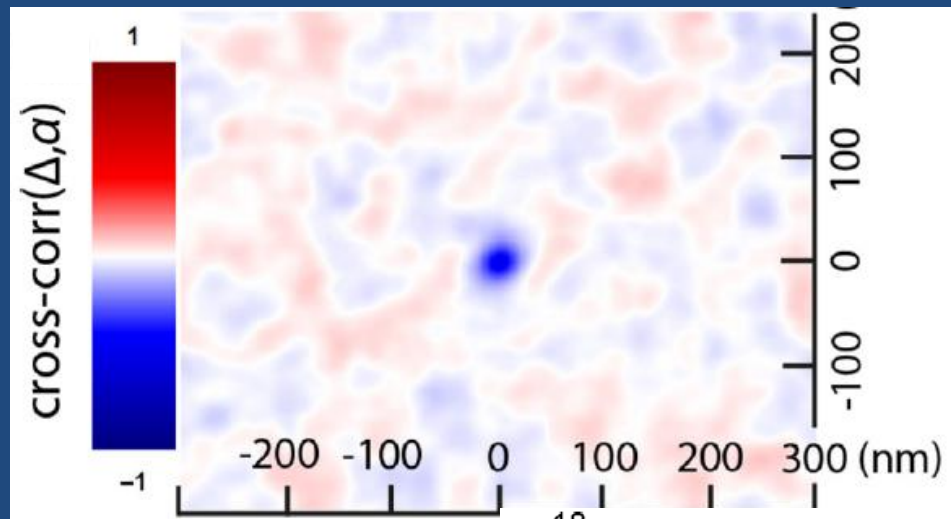
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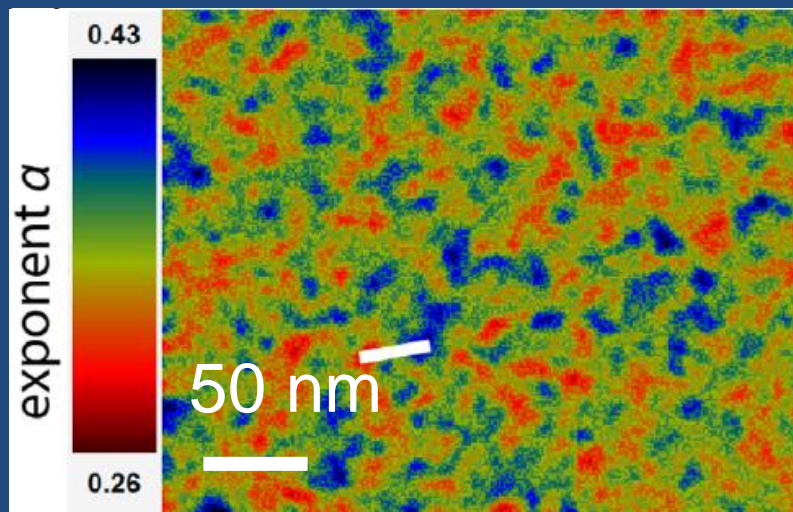
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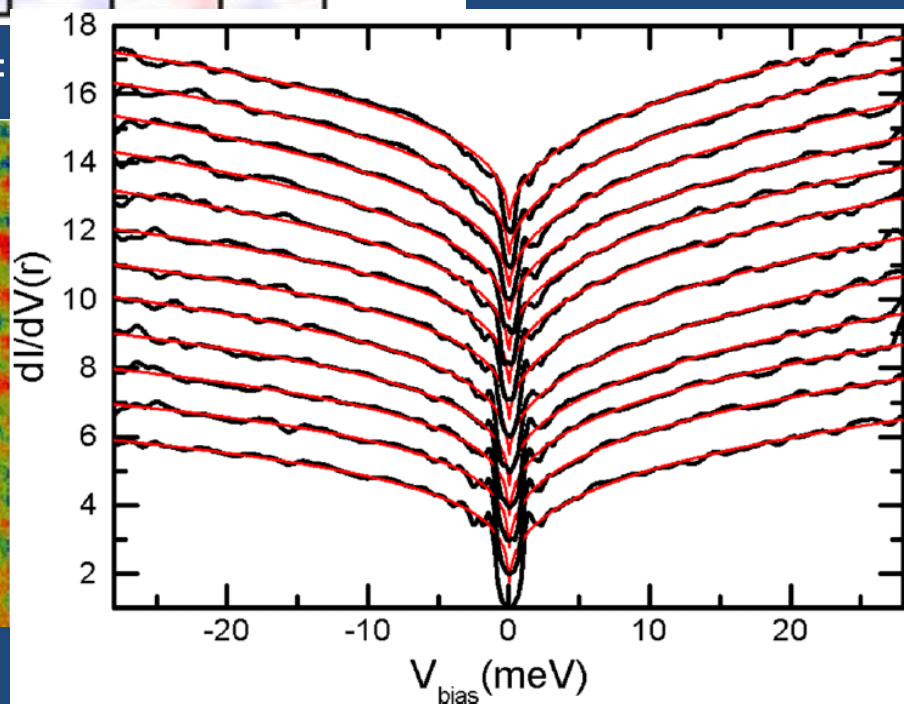
$$l_{cross} \approx 25 \text{ nm}$$

$$\rho_{cross}(\vec{r}) =$$



$$\alpha(\vec{r})$$

$$l_{\alpha} \approx 18 \text{ nm}$$



# Theoretical modelling

suppression of the tunneling DOS induced by Coulomb effect

$$\nu(E) = \nu_0 e^{-S(E)}$$

$$S(E) = \frac{2}{R_Q} \int_E^{1/\tau} \frac{d\omega}{\omega} R(\omega)$$

$$\text{with } R_Q = h/e^2$$

$R(\hbar\omega)$  spreading resistance between the diffusive scale  $r_{in}(\hbar\omega)$  and field propagation scale  $r_{out}(\hbar\omega)$



# Theoretical modelling

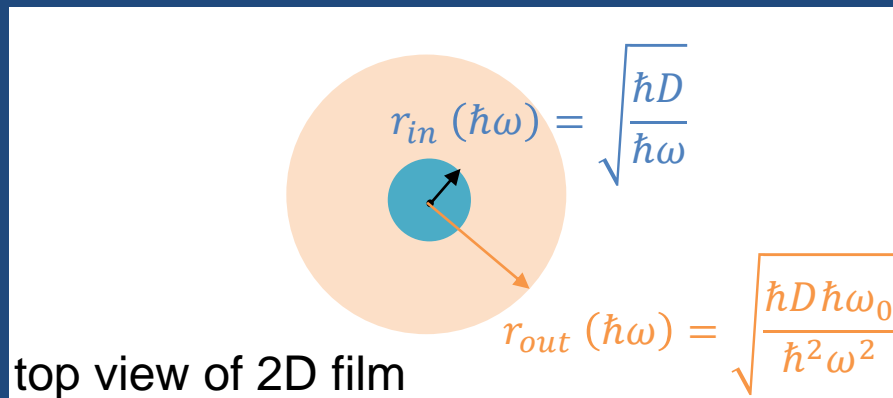
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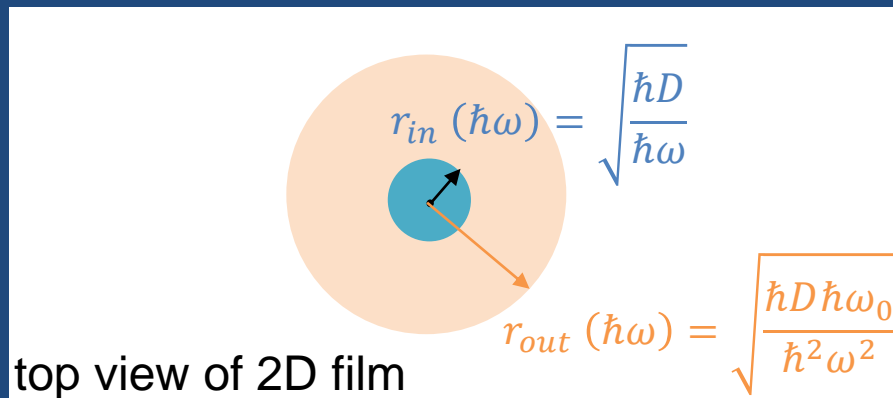
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$R(\hbar\omega)$  spreading resistance between the diffusive scale  $r_{in}(\hbar\omega)$  and field propagation scale  $r_{out}(\hbar\omega)$



$$R(\hbar\omega) = \frac{R_{\square}}{2\pi} \ln \frac{r_{in}(\hbar\omega)}{r_{out}(\hbar\omega)}$$

$$R(\hbar\omega) = \frac{R_{\square}}{2\pi} \ln \frac{\omega_0}{\omega}$$

## Theoretical modelling

suppression of the tunneling DOS induced by Coulomb effect

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➔ Homogeneous case:  $\nu(E) \propto E^{\alpha(E)}$

$$\alpha(E) = \frac{R_{\square}}{2\pi R_Q} \ln \frac{\hbar\omega_0}{E}$$

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$$\hbar/\tau \approx 0.3 \text{ eV}$$

$$\hbar\omega_0 \approx 10 \text{ eV}$$

$$R_{\square} \approx 7 \text{ k}\Omega$$

$$\alpha_{th}(E) \approx 0.29$$

$$E = 5 - 30 \text{ meV}$$

# Theoretical modelling

suppression of the tunneling DOS induced by Coulomb effect

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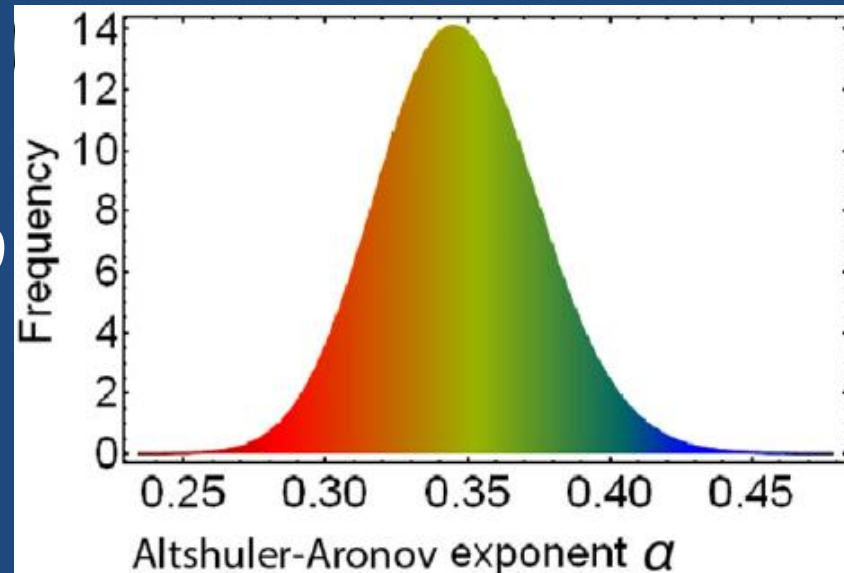
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$$E = 5 - 30 \text{ meV}$$

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$\alpha(\vec{r})$  experimental distribution



# Theoretical modelling

Inhomogeneous case:

➔ Assumption that fluctuations of  $\alpha(\vec{r})$  and  $|\Delta|(\vec{r})$  originate from fluctuations in 2D resistivity:

$$\rho(\vec{r}) = R_{\square} + \delta\rho(\vec{r})$$

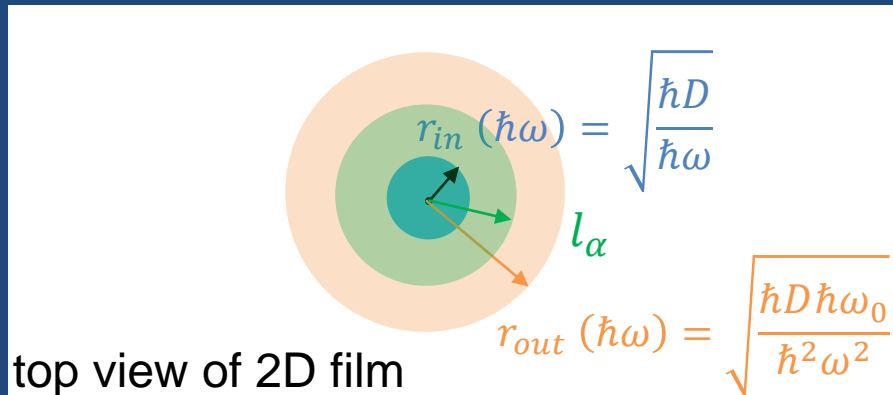
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$$\rho(\vec{r}) = R_{\square} + \delta\rho(\vec{r})$$

➔ The spreading resistance possess a local term for  $|\vec{r} - \vec{r}_0| < l_{\alpha}$



$$R(\vec{r}, \hbar\omega) = \frac{\rho(\vec{r})}{2\pi} \ln \frac{l_{\alpha}}{r_{in}(\hbar\omega)} + \frac{R_{\square}}{2\pi} \ln \frac{r_{out}(\hbar\omega)}{l_{\alpha}}$$

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Inhomogeneous case:

Assumption that fluctuations of  $\alpha(\vec{r})$  and  $|\Delta|(\vec{r})$  originate from fluctuations in 2D resistivity:

$$\rho(\vec{r}) = R_{\square} + \delta\rho(\vec{r})$$

$$\delta\alpha(\vec{r}, E) = \frac{\delta\rho(\vec{r})}{2\pi R_Q} \ln \frac{E}{\hbar D / l_{\alpha}^2}$$

$$\longrightarrow \alpha(\vec{r}) = \langle \alpha \rangle + \delta\alpha(\vec{r})$$



# Theoretical modelling

Inhomogeneous case:

Assumption that fluctuations of  $\alpha(\vec{r})$  and  $|\Delta|(\vec{r})$  originate from fluctuations in 2D resistivity:

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$$\alpha(\vec{r}) = \langle\alpha\rangle + \delta\alpha(\vec{r})$$

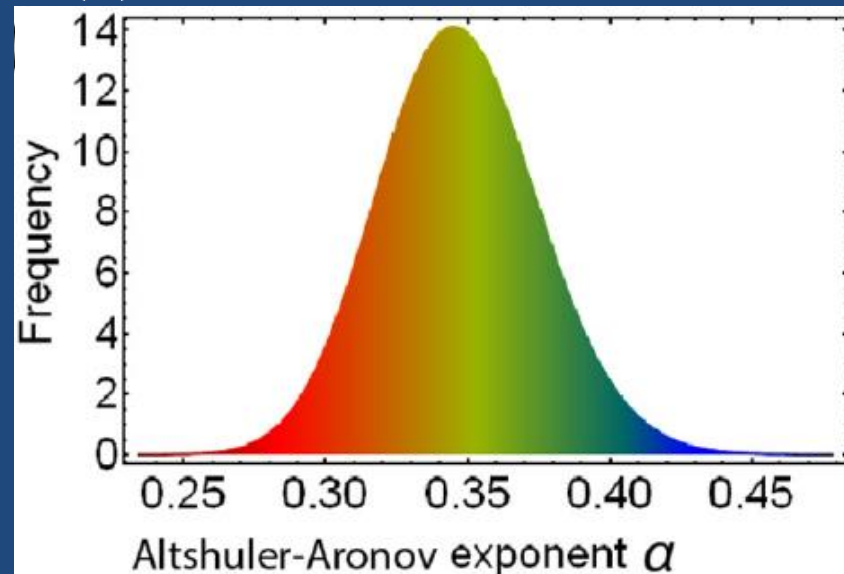
$$\delta\alpha(\vec{r}, E) = \frac{\delta\rho(\vec{r})}{2\pi R_Q} \ln \frac{E}{\hbar D / l_{\alpha}^2}$$

$$l_{\alpha} \approx 18 \text{ nm}$$

$$\sigma_{\alpha} \approx 0.03 \quad \longrightarrow \quad \sigma_{\rho} ?$$

$$D \approx 0.5 \text{ cm}^2/\text{s}$$

$\alpha(\vec{r})$  experimental distribution



# Theoretical modelling

Inhomogeneous case:

Assumption that fluctuations of  $\alpha(\vec{r})$  and  $|\Delta|(\vec{r})$  originate from fluctuations in 2D resistivity:

$$\rho(\vec{r}) = R_{\square} + \delta\rho(\vec{r})$$

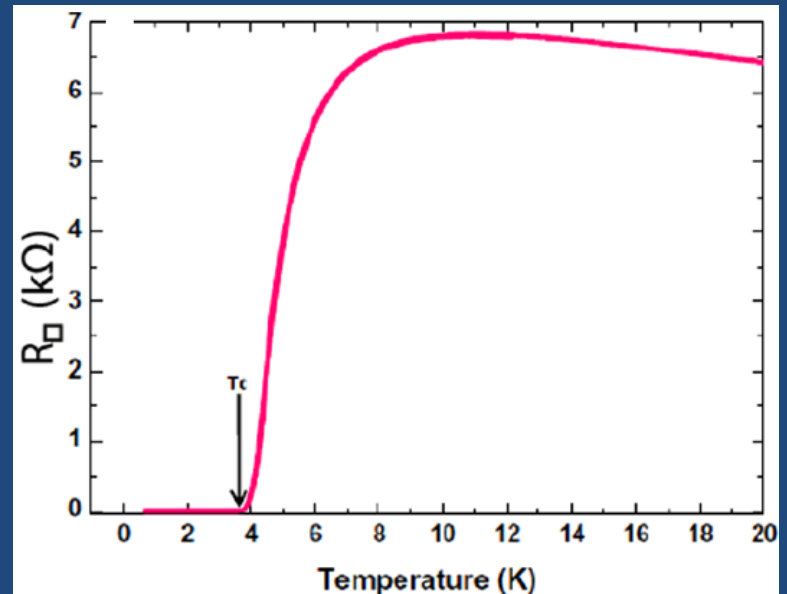
$$\alpha(\vec{r}) = \langle\alpha\rangle + \delta\alpha(\vec{r})$$

$$\delta\alpha(\vec{r}, E) = \frac{\delta\rho(\vec{r})}{2\pi R_Q} \ln \frac{E}{\hbar D / l_{\alpha}^2}$$

$$l_{\alpha} \approx 18 \text{ nm}$$

$$\sigma_{\alpha} \approx 0.03 \quad \longrightarrow \quad \sigma_{\rho} \approx 1.1 \text{ k}\Omega$$

$$D \approx 0.5 \text{ cm}^2/\text{s} \quad \sigma_{\rho} / R_{\square}^{\text{max}} \approx 16\%$$



In situ R(T)

# Theoretical modelling

Inhomogeneous case:

local Finkelstein picture to explain of  $|\Delta|(\vec{r})$  fluctuations

$$\rho(\vec{r}) = R_{\square} + \delta\rho(\vec{r})$$
$$\Delta(\vec{r}) = \langle\Delta\rangle + \delta\Delta(\vec{r})$$
$$\delta\Delta(\vec{r}) = -\frac{\delta\rho(\vec{r})}{6\pi R_Q} \ln^3 \frac{\hbar\omega_D}{\Delta_{bare}}$$

# Theoretical modelling

Inhomogeneous case:

local Finkelstein picture to explain of  $|\Delta|(\vec{r})$  fluctuations

$$\rho(\vec{r}) = R_{\square} + \delta\rho(\vec{r})$$
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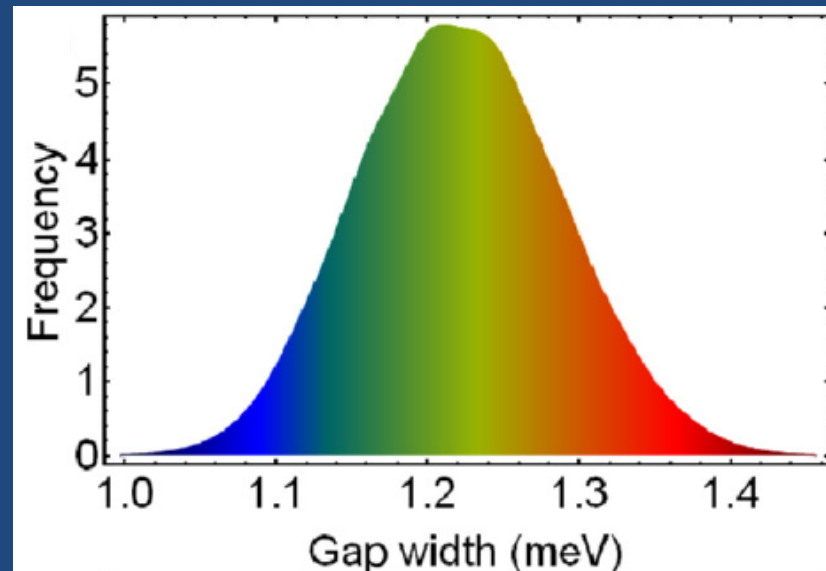
$$\sigma_{\rho} \approx 1.1 \text{ k}\Omega$$

$$\langle\Delta\rangle \approx 1.22 \text{ meV}$$

$$\hbar\omega_D \approx 300 \text{ K} \quad \longrightarrow \quad \sigma_{\Delta} \approx 0.09 \text{ meV}$$

$$\Delta_{bare} \approx 2.85 \text{ meV} \quad \sigma_{\Delta}/\langle\Delta\rangle \approx 7.4\%$$

$\Delta(\vec{r})$  experimental distribution



## Conclusion of part 1

Moderate disorder in ultrathin NbN films  $\left\{ \begin{array}{l} k_F \ell_e \approx 2 - 3 \\ T_c \approx 0.25 T_{c-bulk} \end{array} \right.$

Emergent electronic inhomogeneities explained by a local Finkelstein picture: meV scale and tens meV scale linked

$$\begin{array}{lll} \rho(\vec{r}) = R_{\square} + \delta\rho(\vec{r}) & \sigma_{\rho}/R_{\square}^{max} \approx 16\% & l_{grain} = 2-8 \text{ nm} \\ \alpha(\vec{r}) = \langle\alpha\rangle + \delta\alpha(\vec{r}) & \sigma_{\alpha}/\alpha \approx 10\% & \xi \approx 5 \text{ nm} \\ \Delta(\vec{r}) = \langle\Delta\rangle + \delta\Delta(\vec{r}) & \sigma_{\Delta}/\langle\Delta\rangle \approx 7\% & l_{|\Delta|} \approx 18 \text{ nm} \\ & & l_{|\Delta|} \approx 27 \text{ nm} \end{array}$$

➔ Low  $q < q_D$  matter !!

Interface resistance of nanocrystals mostly contribute to  $\delta\rho(\vec{r})$   
thus linked to the grain structure

Funding ANR  
Superstripes



# Thanks

## Experiments



Clémentine  
Carbillet  
PhD



Vladimir Cherkez  
Post-doc



Dimitri  
Roditchev



Tristan  
Cren



François  
Debontridder



Christophe  
Brun

## Theory



Misha  
Skvortsov



Misha  
Feigel'man



Lev  
Ioffe

## Growth of NbN films



Kostia Ilin, KIT Germany

# Content

Revealing quantitatively the interplay between disorder and electron-electron interactions in 2D superconductors from LDOS analysis

## 1. Case of moderate disorder in NbN thin films

Carbillet et al PRB 102, 024504 (2020)

## 2. Case of weak disorder in Pb atomic monolayer

Lizée et al. PRB 107, 174508 (2023)

# Motivation

PRL **108**, 017002 (2012)

PHYSICAL REVIEW LETTERS

week ending  
6 JANUARY 2012

Th

## **Enhancement of the Critical Temperature of Superconductors by Anderson Localization**

I. S. Burmistrov,<sup>1</sup> I. V. Gornyi,<sup>2,3,4</sup> and A. D. Mirlin<sup>2,4,5,6</sup>

2D systems, electron-electron interaction in particle-hole and Cooper channels,  $\sigma$ -model renormalization group framework  
Short-range spatial fluctuations of  $\lambda(r)$  over  $l_\lambda \ll \xi$   
this physically corresponds to screened Coulomb interactions



# Motivation

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## **Enhancement of the Critical Temperature of Superconductors by Anderson Localization**

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Short-range spatial fluctuations of  $\lambda(r)$  over  $l_\lambda \ll \xi$

this physically corresponds to screened Coulomb interactions

➔ Extension of the works of Feigel'man et al.:

PRL **98**, 027001 (2007)

PHYSICAL REVIEW LETTERS

week ending  
12 JANUARY 2007

Th

## **Eigenfunction Fractality and Pseudogap State near the Superconductor-Insulator Transition**

M. V. Feigel'man,<sup>1</sup> L. B. Ioffe,<sup>2,1</sup> V. E. Kravtsov,<sup>3,1</sup> and E. A. Yuzbashyan<sup>2</sup>

and also Feigel'man et al. *Annals of physics* **325**, 1390 (2010)

3D systems, close to the mobility edge

# Motivation

PRL **108**, 017002 (2012)

PHYSICAL REVIEW LETTERS

week ending  
6 JANUARY 2012

Th

## **Enhancement of the Critical Temperature of Superconductors by Anderson Localization**

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# Motivation

PRL **108**, 017002 (2012)

PHYSICAL REVIEW LETTERS

week ending  
6 JANUARY 2012

Th

## Enhancement of the Critical Temperature of Superconductors by Anderson Localization

I. S. Burmistrov,<sup>1</sup> I. V. Gornyi,<sup>2,3,4</sup> and A. D. Mirlin<sup>2,4,5,6</sup>

Exp

## Disorder induced multifractal superconductivity in monolayer niobium dichalcogenides

Zhao et al. Nature Phys. 15, 904 (2019)

Exp

## Visualization of Multifractal Superconductivity in a Two-Dimensional Transition Metal Dichalcogenide in the Weak-Disorder Regime

Rubio-Verdu et al. Nano Lett. 20, 5111 (2020)

➔ Puzzling: in both experiments the level disorder  $k_F \ell_e > 10$  is too low for theory to explain  $T_C$  enhancement

# Motivation

PRL **108**, 017002 (2012)

PHYSICAL REVIEW LETTERS

week ending  
6 JANUARY 2012

Th

## **Enhancement of the Critical Temperature of Superconductors by Anderson Localization**

I. S. Burmistrov,<sup>1</sup> I. V. Gornyi,<sup>2,3,4</sup> and A. D. Mirlin<sup>2,4,5,6</sup>

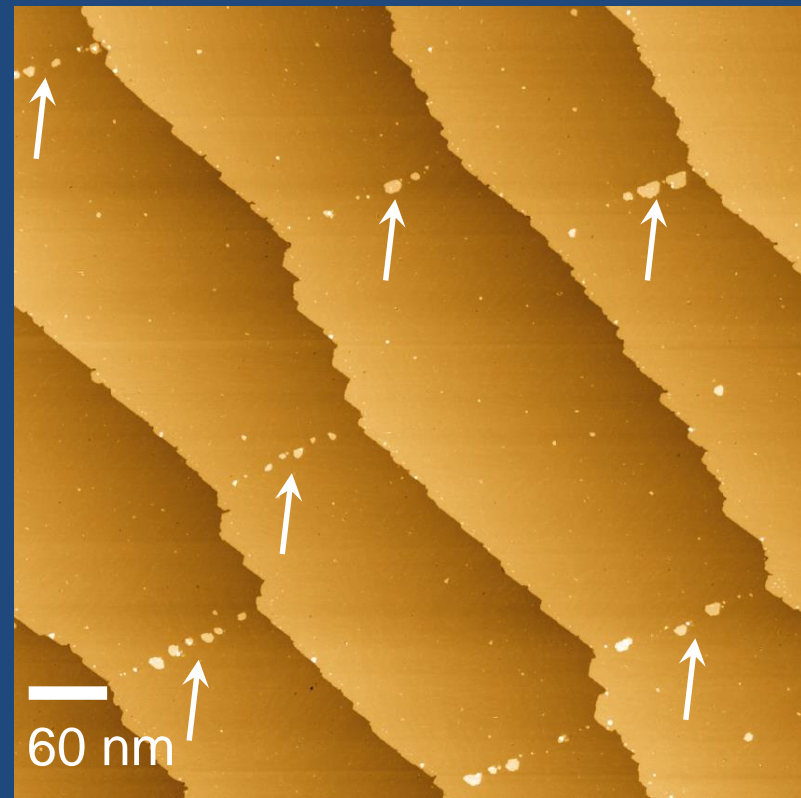
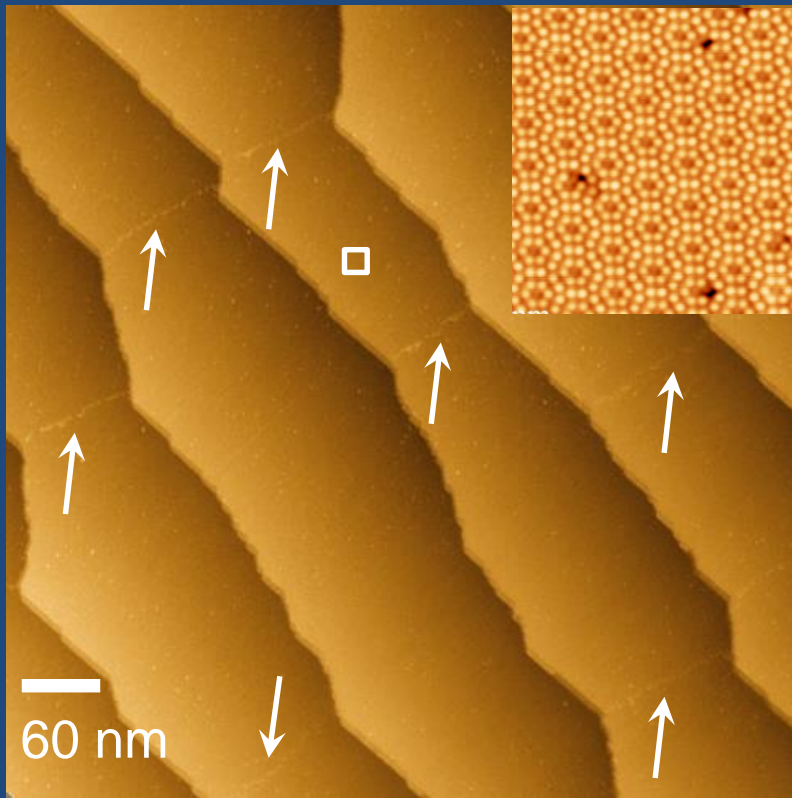
- ➔ Choose an appropriate 2D superconductor to probe first the weak-disorder limit !
- ➔ Check consistency between experiment and theory for LDOS and gap energy spatial fluctuations

# Model 2D system: Pb/Si(111) single atomic layer

ultrahigh vacuum growth:  $P \sim 10^{-11}$  mbar range

Si(111)-7x7

monolayer Pb/Si(111)



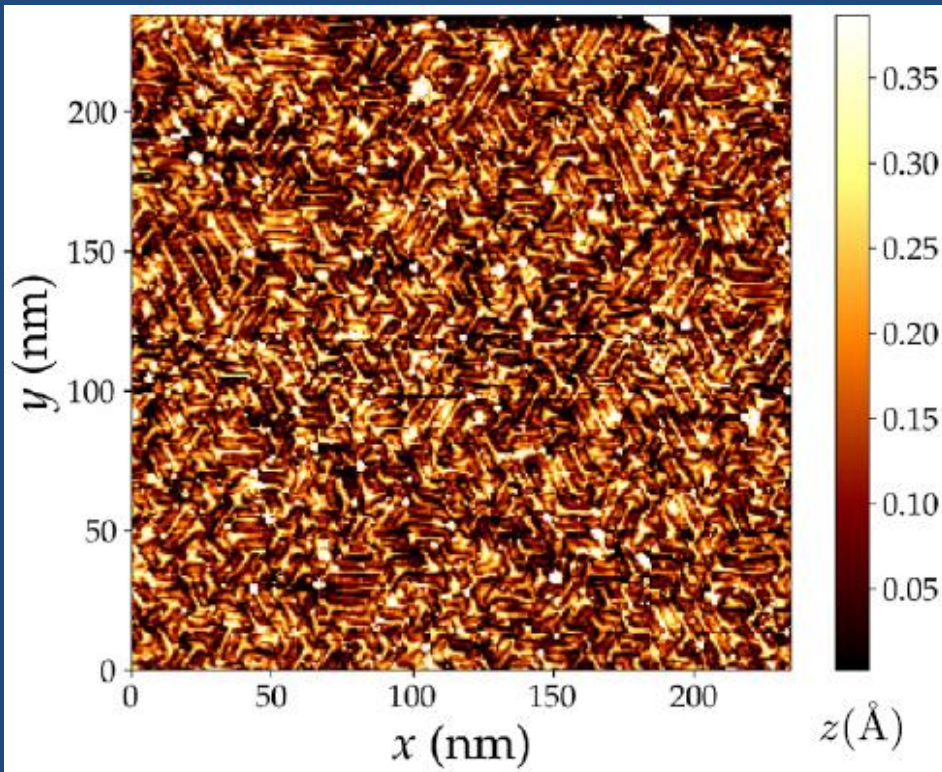
C. Brun et al. Nature Physics 10, 444 (2014)

C. Brun et al. Supercond. Sci. Technol. 30, 013003 (2017)

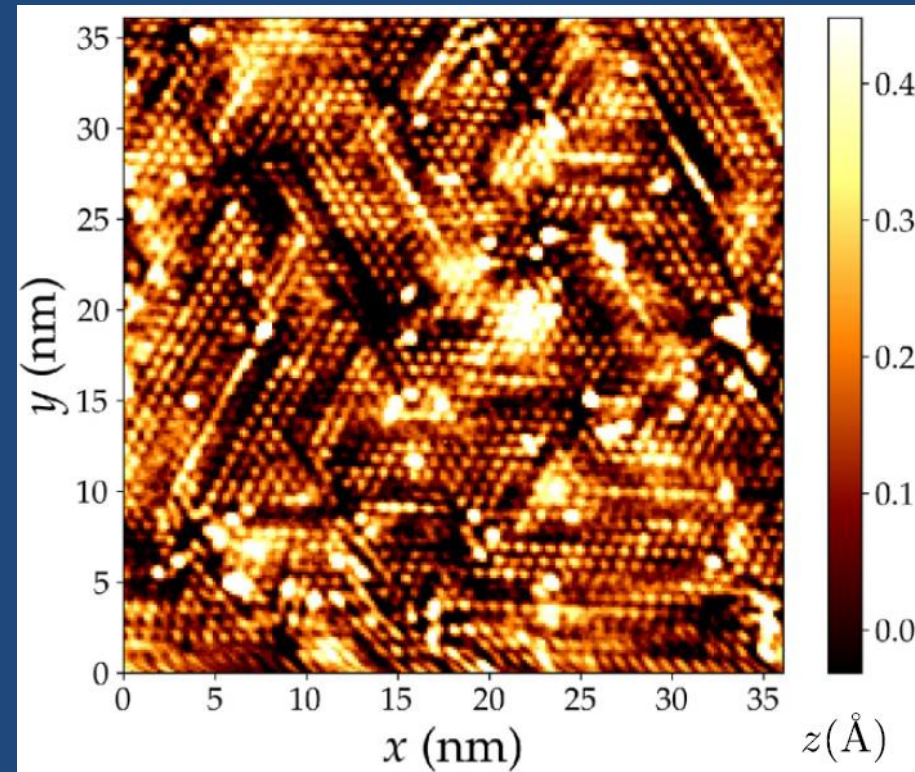


# Model 2D system: Pb/Si(111) single atomic layer

large scale



small scale

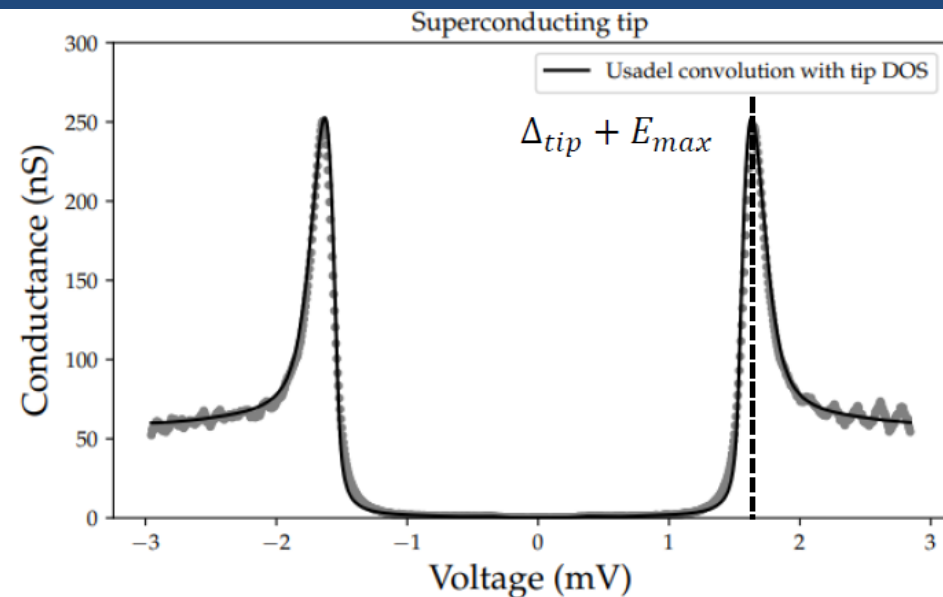
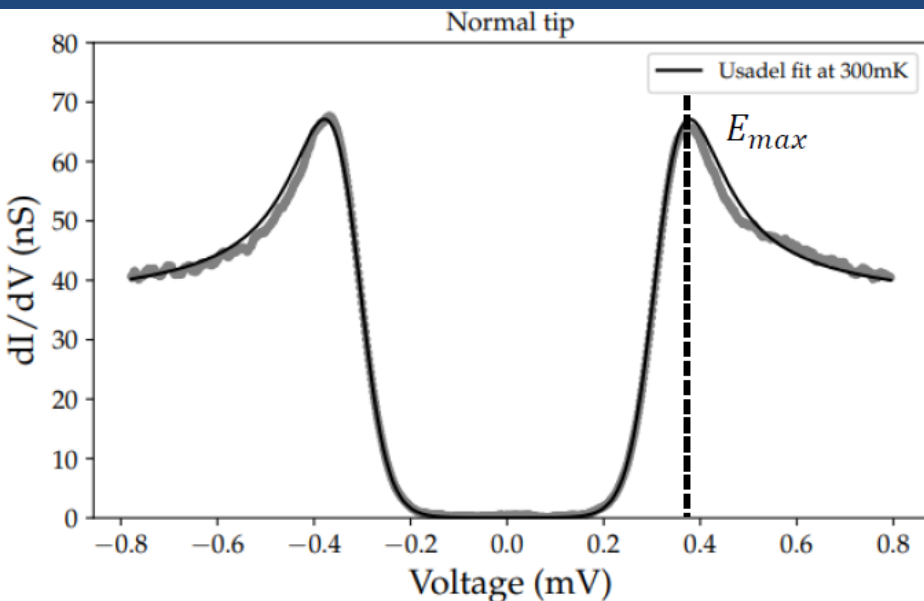


Polycrystalline structure with grain size  $l_{grain} = 2-10$  nm

ARPES: single 2D band with strong Rashba spin-orbit coupling

# Model 2D system: Pb/Si(111) single atomic layer

average superconducting properties at  $T = 0.3$  K

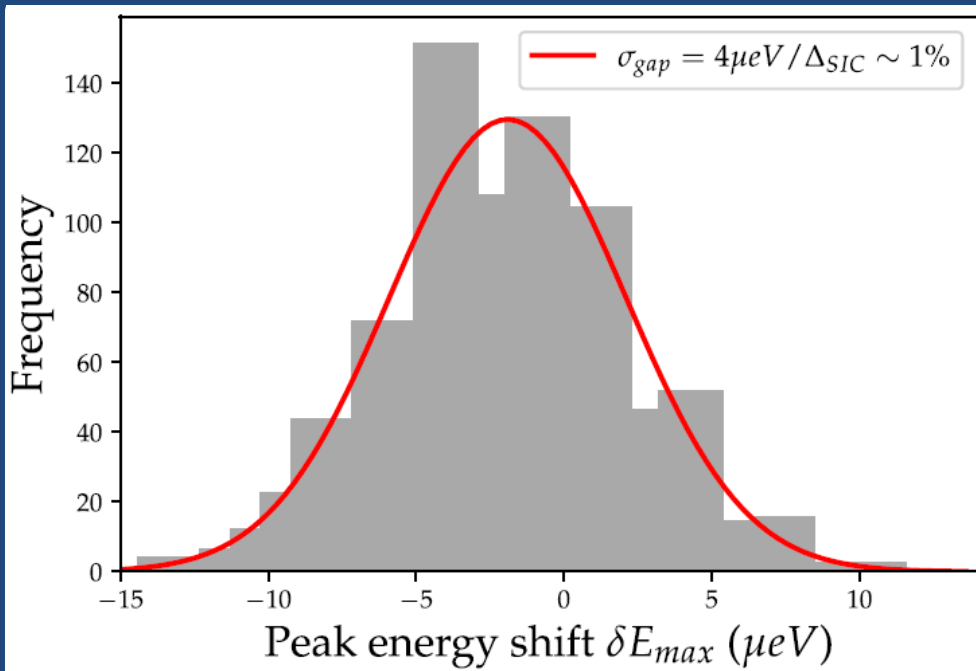


$$\langle \Delta \rangle \approx 350 \mu\text{eV} \quad \Gamma \approx 10 \mu\text{eV} \quad T_c \approx 1.8 \text{ K}$$

$$\text{estimated } k_F \ell_e \approx 30 - 100 \quad \xi \approx 50 \text{ nm} \gg l_{grain}$$

consistent with Zhang et al. Nature Phys. 2010, Yamada et al. PRL 2013, Cherkez et al PRX 2014

very small local energy gap fluctuations at  $T = 0.3$  K



Exp:  $\sigma_{\Delta} / \langle \Delta \rangle \approx 1.1 \%$

Th:  $\sigma_{\Delta} / \langle \Delta \rangle \approx 0.5 - 1 \%$

$$\sigma_{E_{max}} \approx \sqrt{c/g} (\Gamma / \langle \Delta \rangle)^{2/3}$$

with  $c \approx 0.3$

$$\langle \Delta \rangle \approx 350 \mu\text{eV} \quad \sigma_{\Delta} \approx 4 \mu\text{eV}$$

$$\Gamma \approx 10 \mu\text{eV} \quad \text{estimated } g = k_F \ell_e \approx 30 - 100$$

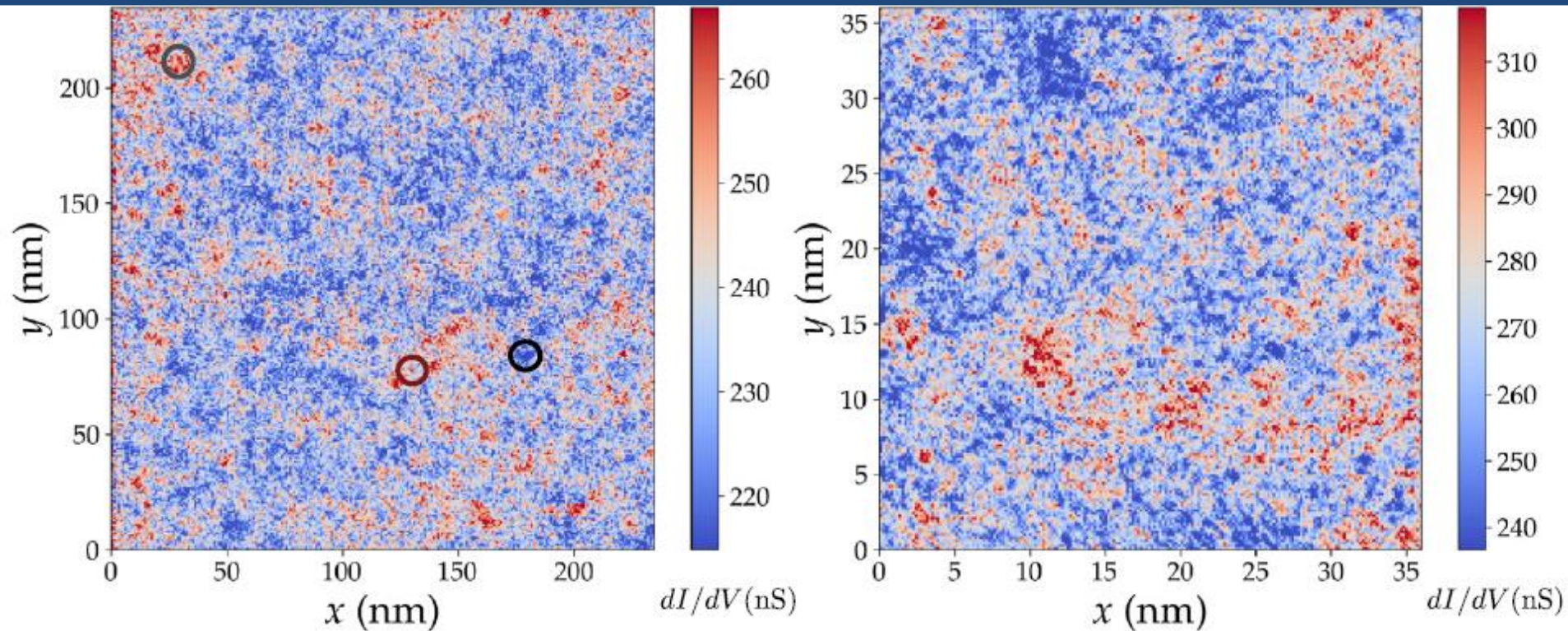


large local density of states fluctuations at  $T = 0.3$  K

large scale

$$E = E_{max}$$

small scale



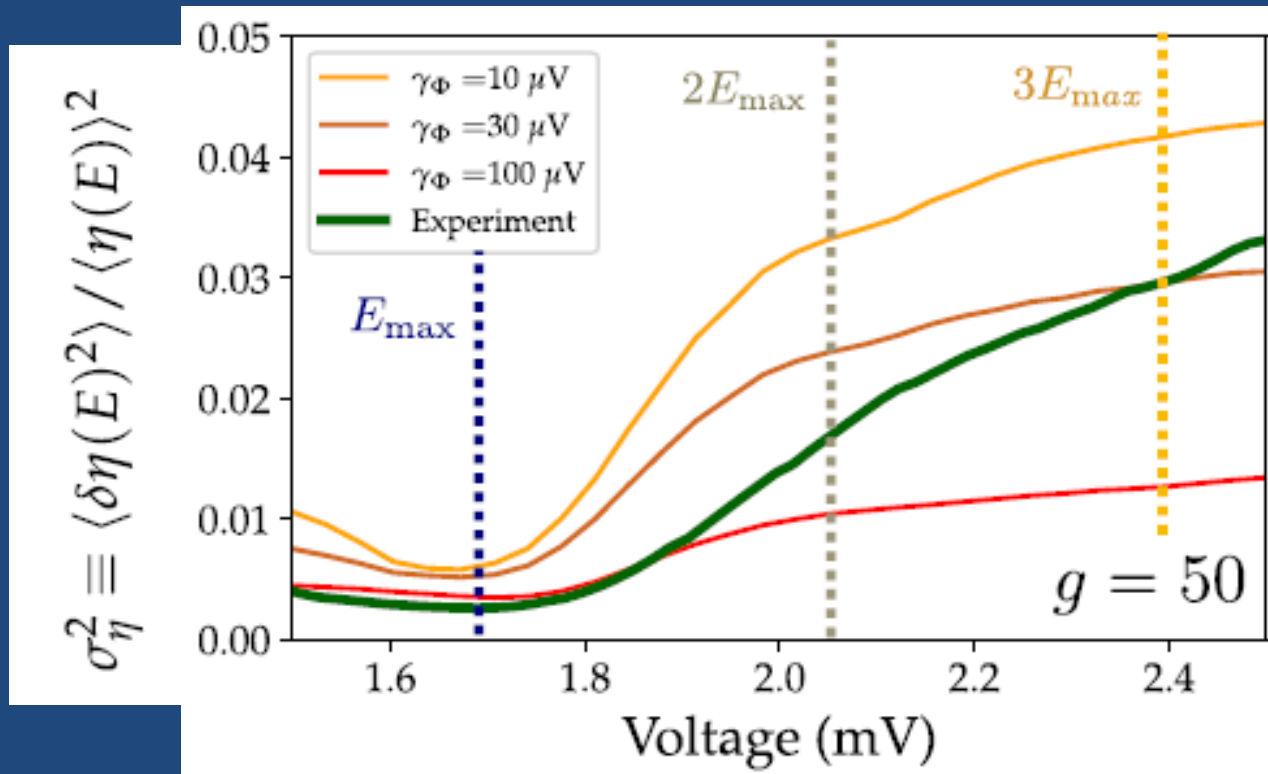
reminder:  $\xi(0) \approx 50$  nm !!



reveal the multifractality of the underlying wavefunctions

quantifying local density of states fluctuations at  $T = 0.3$  K

normalized variance of  $\eta(E) \propto dI/dV(E)$



reminder:  $g = k_F \ell_e \approx 30 - 100$

$\sigma_{\eta\text{-exp}}(E) \in [6 - 20]\%$

➔ quasi-quantitative agreement  $\gamma_\phi = \hbar D / l_\phi^2$

## Conclusion of part 2

weak disorder in a Pb/Si(111) 2D single layer  $k_F \ell_e \approx 30 - 100$

$$l_{grain} = 2-10 \text{ nm}$$

screening of e-e interaction: spatial fluctuations of  $\lambda(r)$  over  $l_\lambda \ll \xi$

relative LDOS fluctuations  $\approx 10 \%$        $\sigma_\Delta / \langle \Delta \rangle \approx 1 \%$

quantitative agreement with multifractal 2D superconductivity

Lizée et al. PRB 107, 174508 (2023)



Funding ANR  
Rodesis



# Thanks

## Experiments, data analysis



Mathieu Lizée  
student



Tristan Cren



Christophe  
Brun

## Theory

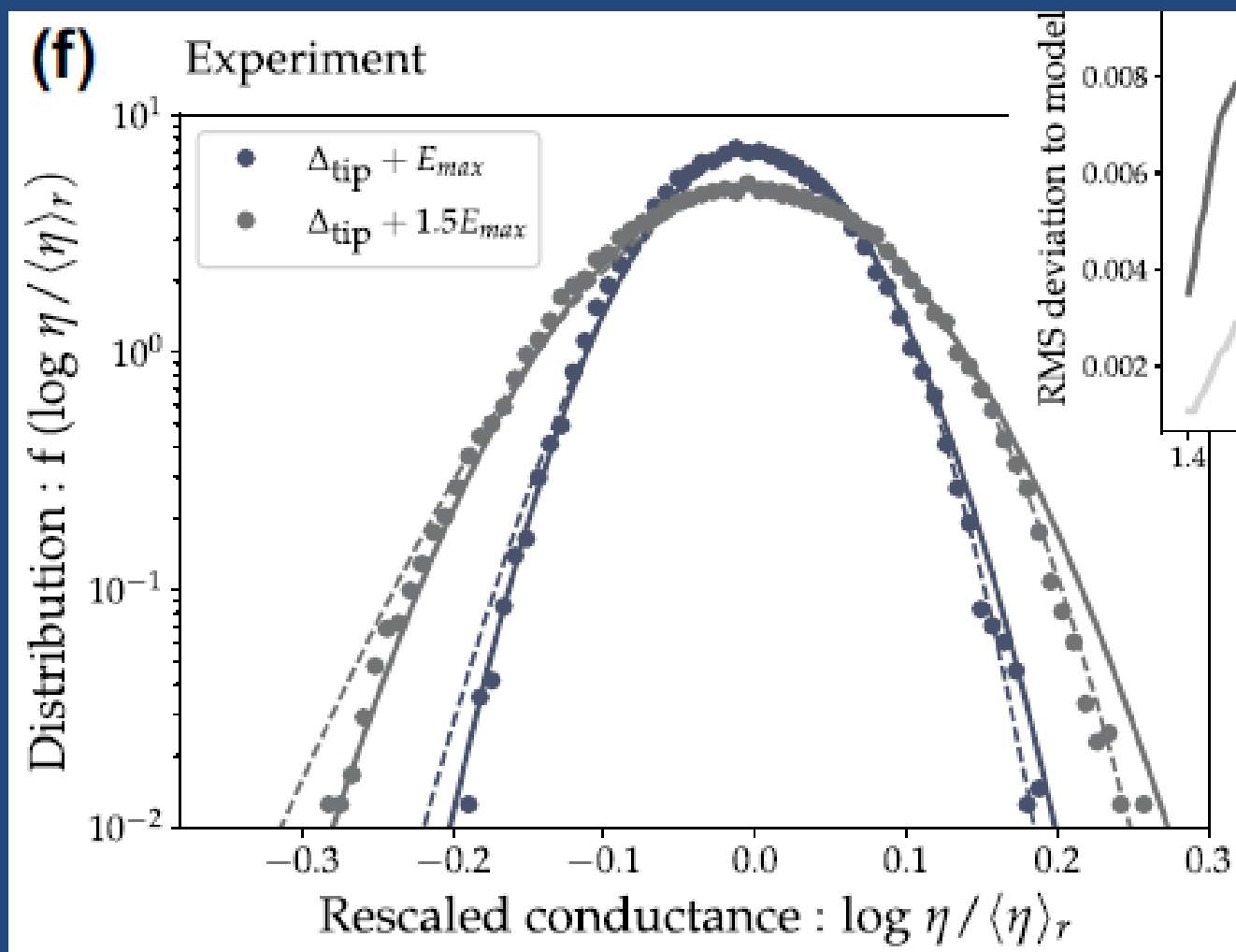


Igor Burmistrov  
Landau Institute  
Russia

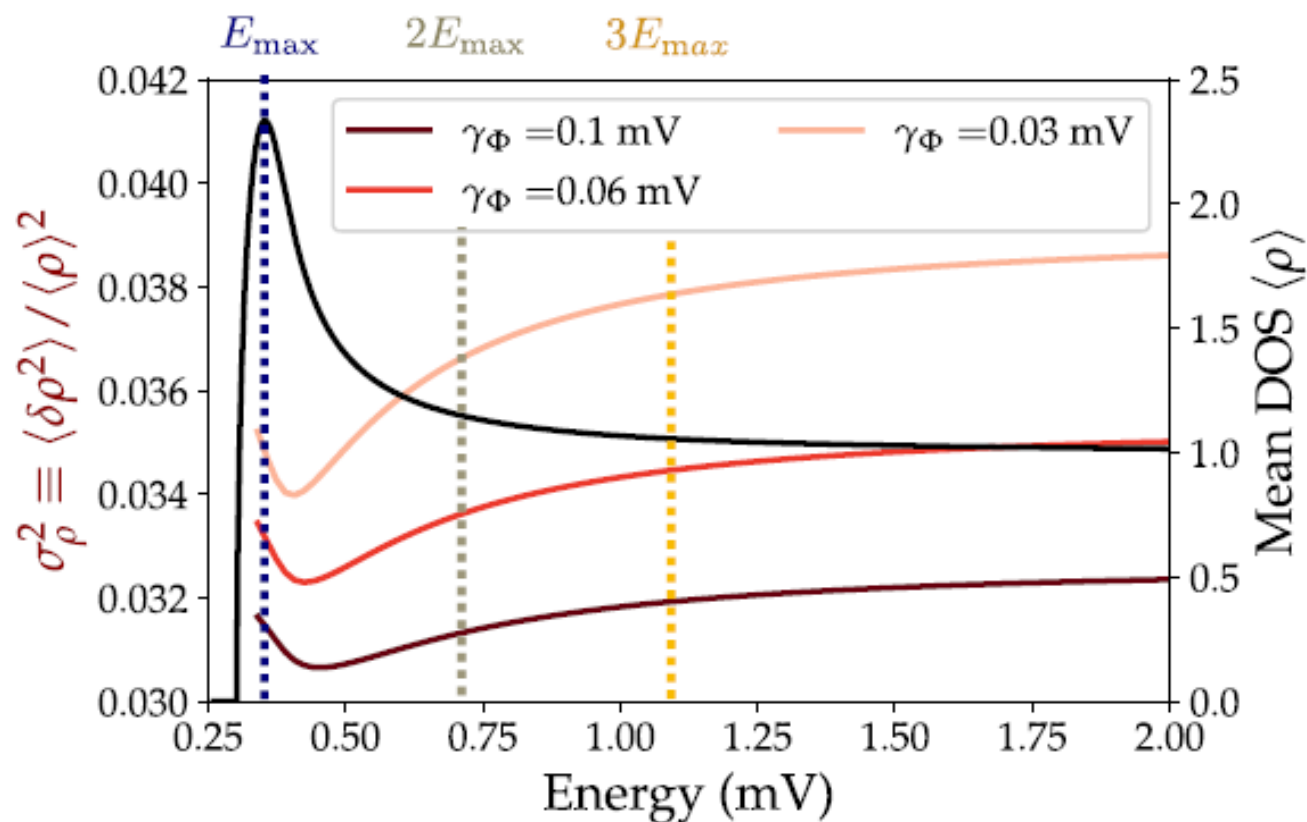


Matthias Stosiek  
Post-doc  
Japan

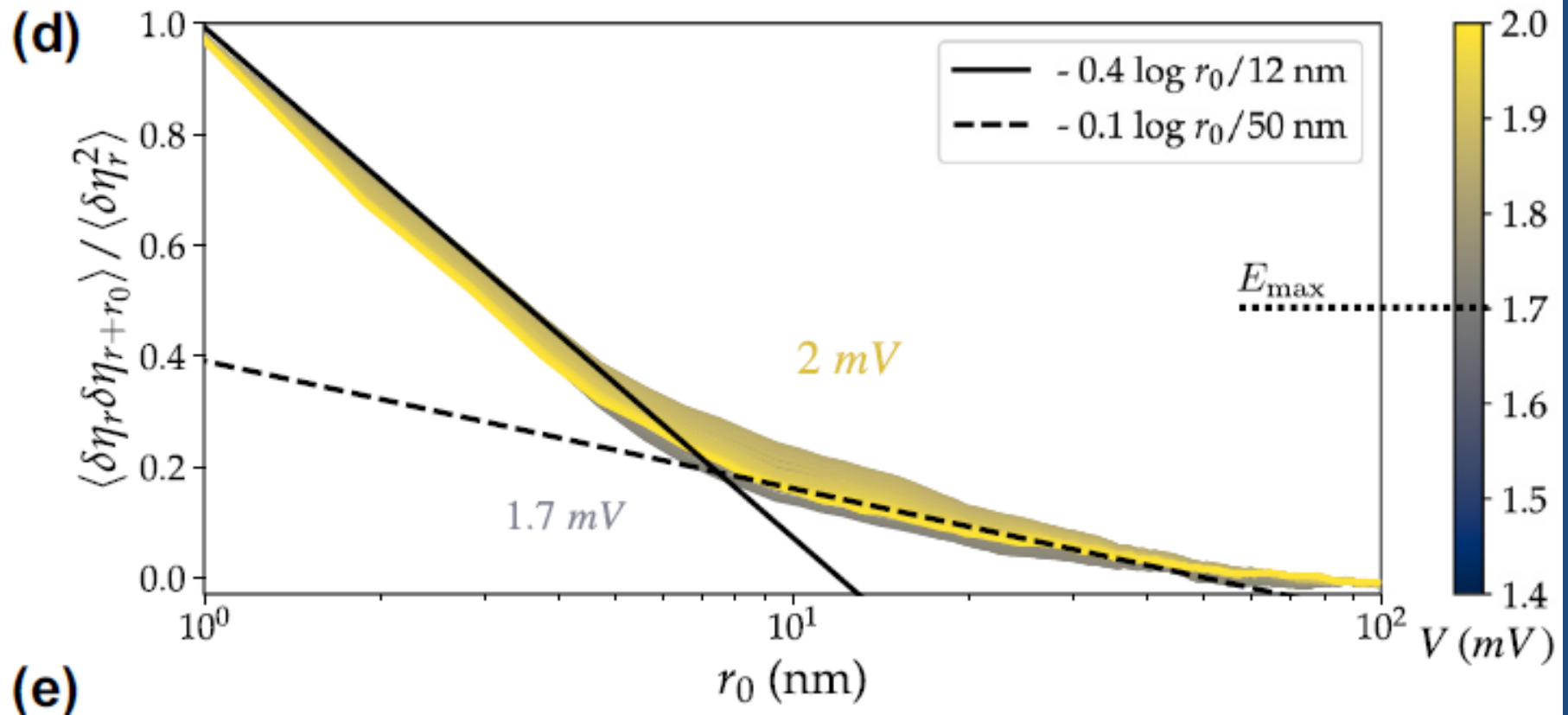




$$\langle \delta\rho(E_1, \mathbf{r}_1) \delta\rho(E_2, \mathbf{r}_2) \rangle = \frac{\rho_0^2}{2\pi g} \operatorname{Re} \left\{ [1 + X_{E_1} X_{E_2}^*] K_0 \left( R \sqrt{\frac{2\gamma_\Phi - iE_1/X_{E_1} + iE_2/X_{E_2}^*}{\hbar D}} \right) \right. \\ \left. - [1 - X_{E_1} X_{E_2}] K_0 \left( R \sqrt{\frac{2\gamma_\Phi - iE_1/X_{E_1} - iE_2/X_{E_2}}{\hbar D}} \right) \right\},$$

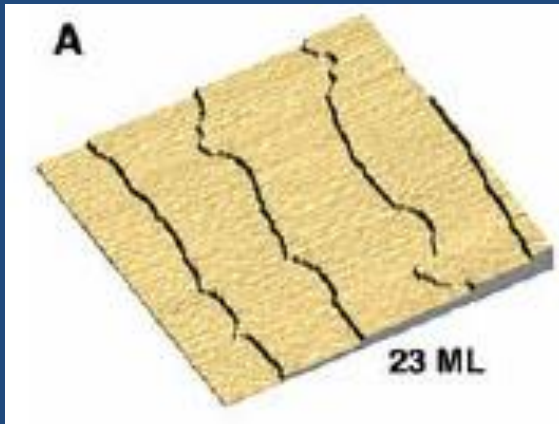


# Experimental dependence of auto-correlation

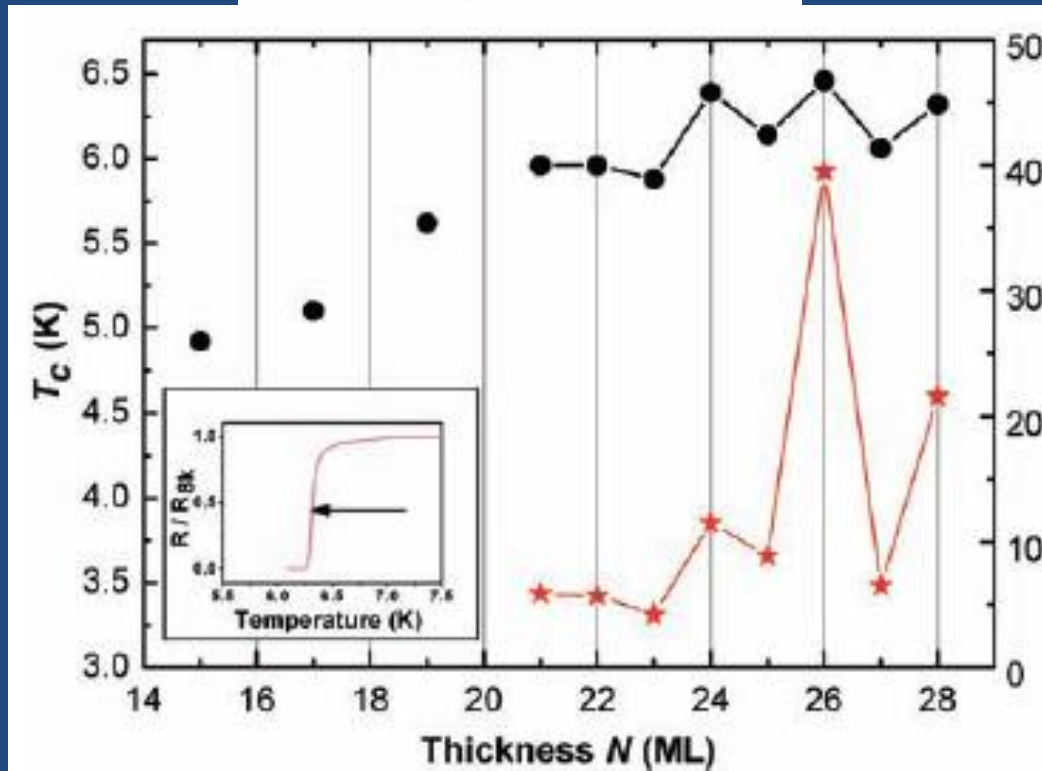


# Reducing the thickness of crystalline thin films

2x2 $\mu\text{m}^2$



Pb films grown on Si(111)  
in ultrahigh vacuum



Not exhaustive !

*Strongin et al. PRL 1973*

*Pfennigstorf et al. PRB 2002*

*Ozer et al. Nat. Phys. 2006*

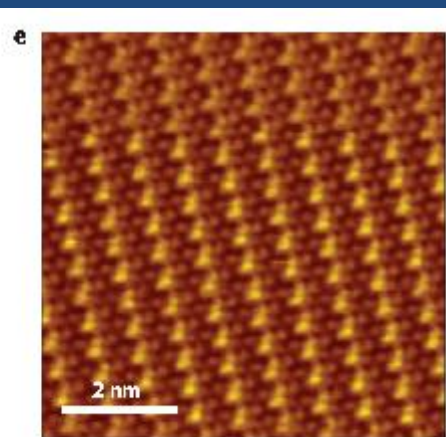
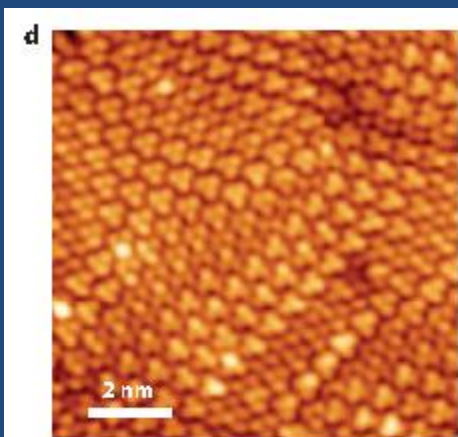
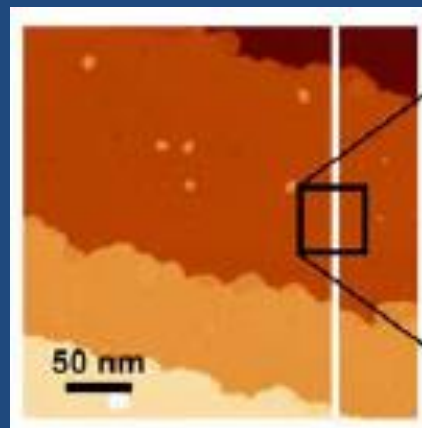
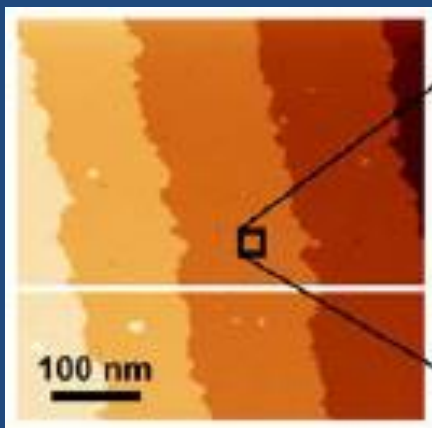
*Eom et al. PRL 2006*

*Brun et al. PRL 2009*

*Qin et al. science 2009*

*Guo et al. Science 2004*

# Superconductivity in 1 atomic layer of Pb/Si(111)



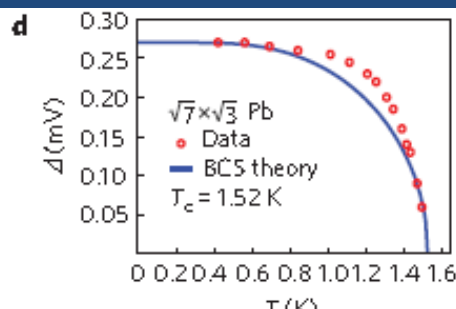
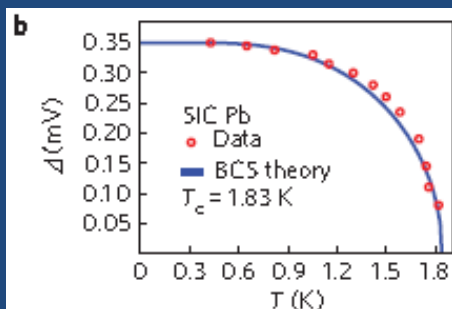
$\sqrt{7} \times \sqrt{3}$

Striped-incommensurate (SIC)

Coverage: 1.30 Pb atom for 1 Si

Coverage: 1.20 Pb atom for 1 Si

$T_c = 1.8 \text{ K}$

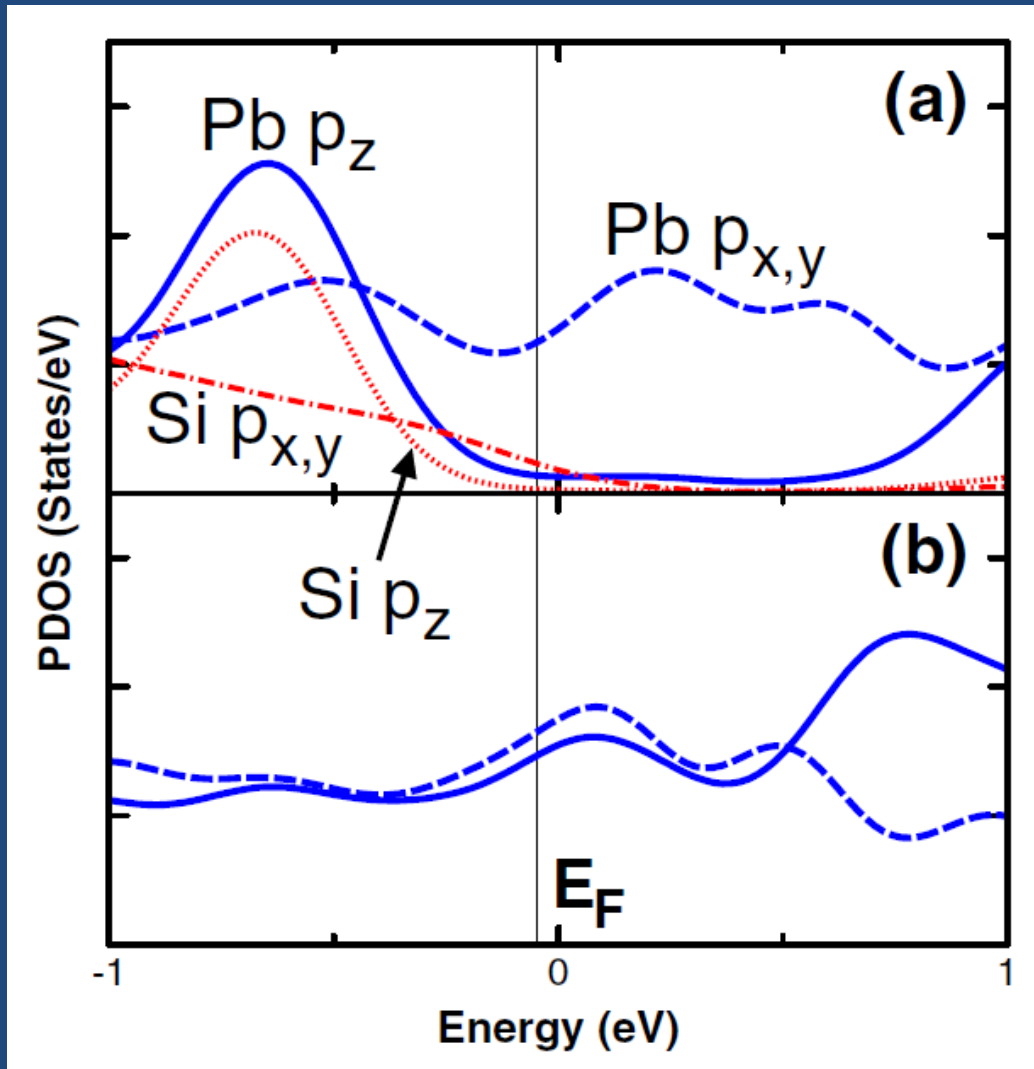


$T_c = 1.5 \text{ K}$

*Zhang et al. Nat. Phys. 6, 104 (2010)*



# DFT : decoupling of surface states from bulk at $E_F$



$\sqrt{7} \times \sqrt{3}$ -Pb/Si(111)

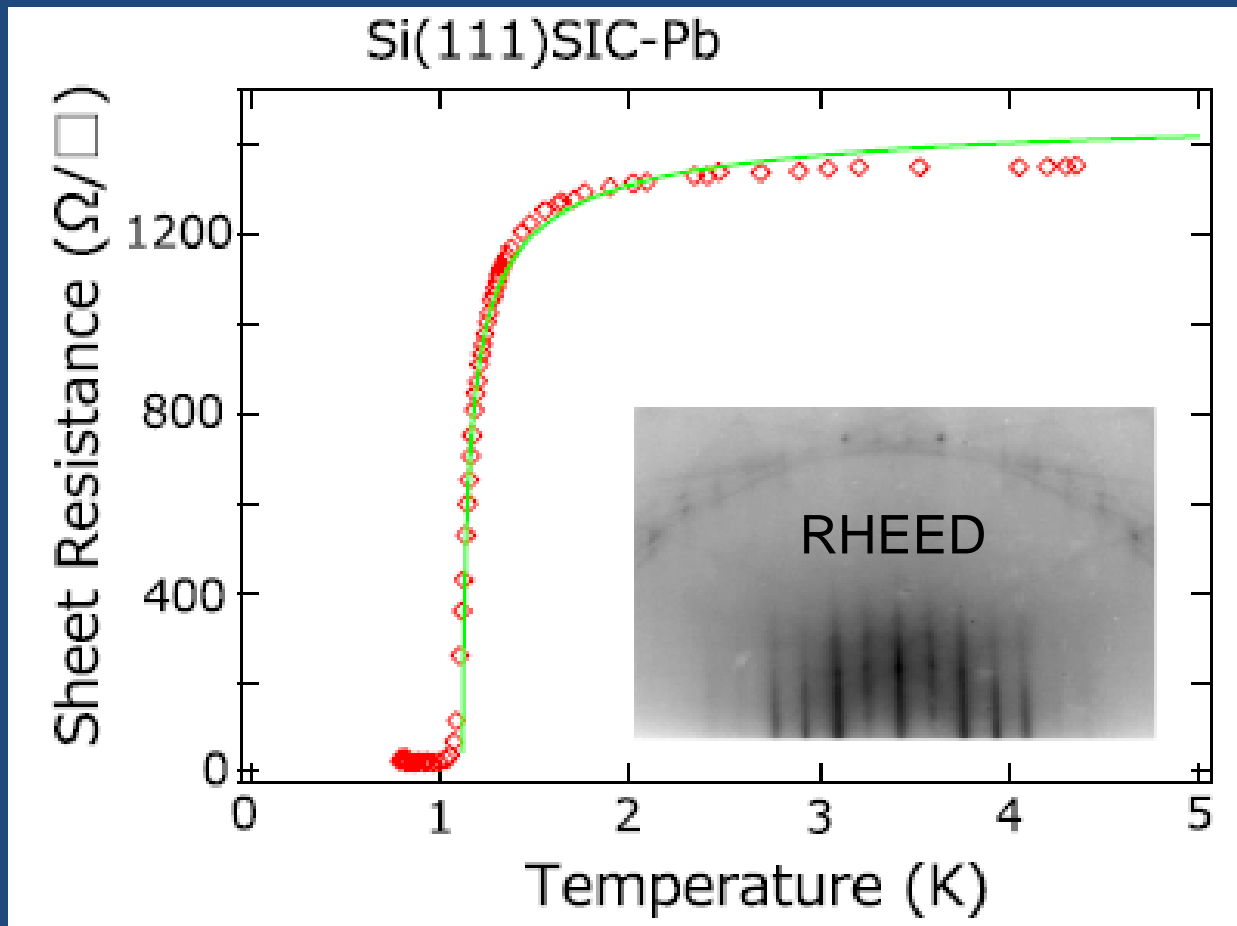
Free standing  $\sqrt{7} \times \sqrt{3}$ -Pb

Small changes of the Pb-6 $p_{x,y}$  states due to the substrate

*DFT : Jung and Kang Surf. Sci 601, 555 (2011)*

# Macroscopic resistivity of 1 atomic layer of Pb/Si(111)

Micro-four point probes – spacing = 20 microns



$$T_{c\text{-transport}} = 1.1\text{K} < T_{c\text{-STM}} = 1.8\text{K}$$

*Uchihashi et al. PRL 107, 207001 (2011) (In monolayer)*

*Yamada et al. PRL 110, 237001 (2013) (Pb monolayer)*

# Existence of strong Rashba SOC in Pb/Si(111)

- Strong spin-orbit coupling in Pb/Si(111) layer
- 3D inversion always broken in a surface layer

→ Split-off in (x,y) plane of the 2D electron bands

$$H_{so} = \alpha(\boldsymbol{\sigma} \times \mathbf{p}) \cdot \mathbf{n}$$

Spins put in plane by  $H_{so}$

Spins  $\perp$  to  $\mathbf{p}$

# Existence of strong Rashba SOC in Pb/Si(111)

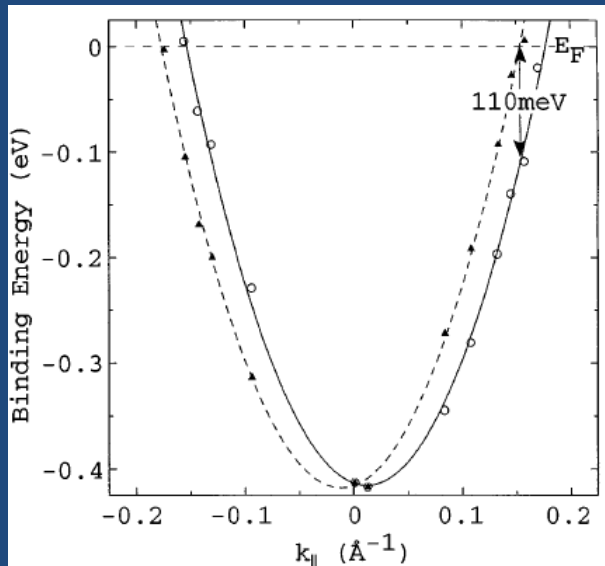
- Strong spin-orbit coupling in Pb/Si(111) layer
- 3D inversion always broken in a surface layer

➔ Split-off in (x,y) plane of the 2D electron bands

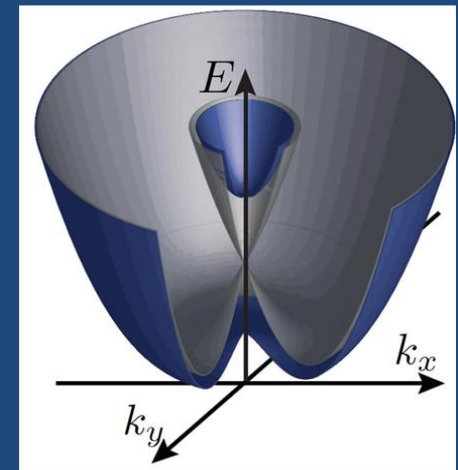
$$H_{SO} = \alpha(\boldsymbol{\sigma} \times \mathbf{p}) \cdot \mathbf{n}$$

Spins put in plane by  $H_{SO}$

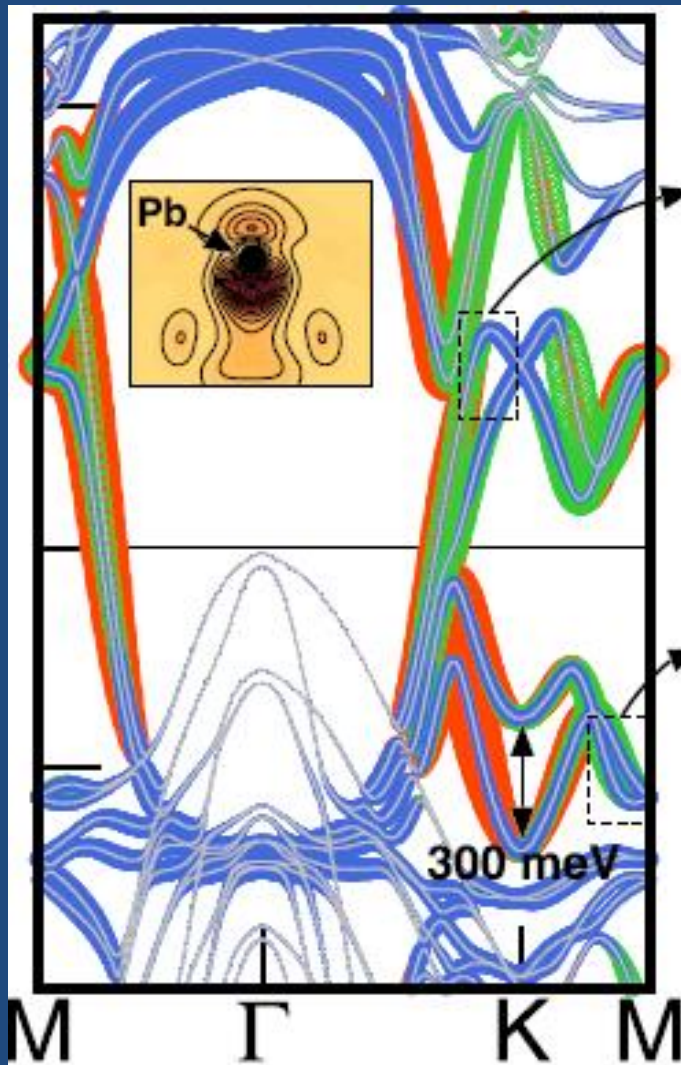
Spins  $\perp$  to  $\mathbf{p}$



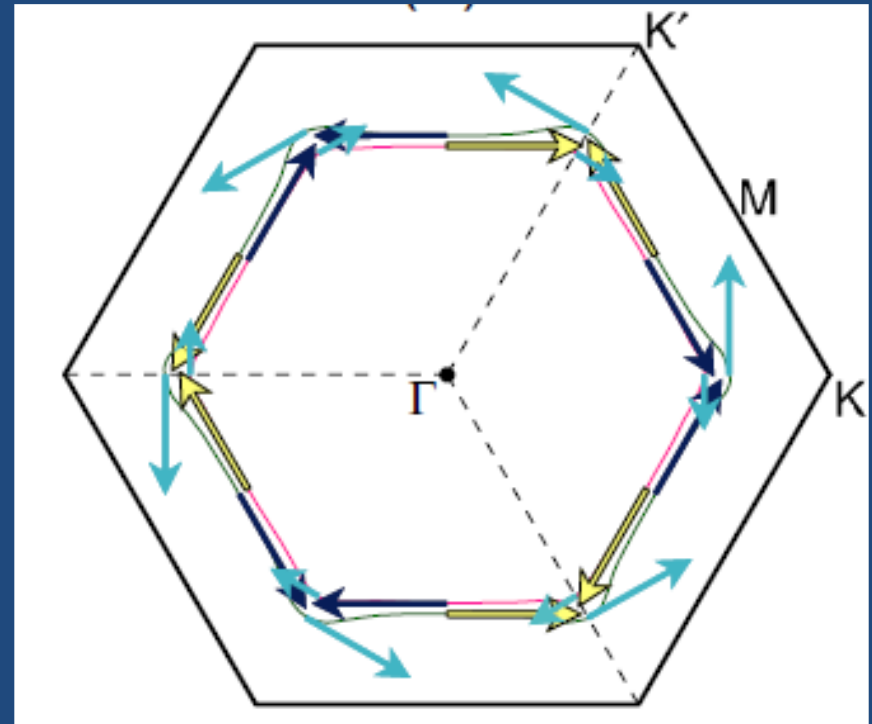
Au(111)



# DFT+SOC and spin-ARPES of the Pb-SiC monolayer



Fermi surface



Helical spin texture

# Summary of the electronic properties of the Pb-SiC monolayer

1 Rashba split band (Pb  $6p_{x,y}$  orbitals)

$$m \sim 1.2 m_0$$

$$E_F \sim 0.8 \text{ eV}$$

$$k_F \sim 1.36 \text{ \AA}^{-1}$$

$$\lambda_F \sim 4.6 \text{ \AA}$$



True 2D electronic confinement  
 $6p_{x,y}$  states at  $E_F$

$$\Delta \ll 2|\alpha|p_F \ll E_F$$

$$0.3\text{meV} \quad 100\text{meV} \quad 800\text{meV}$$

$$\xi \sim 50 \text{ nm}$$

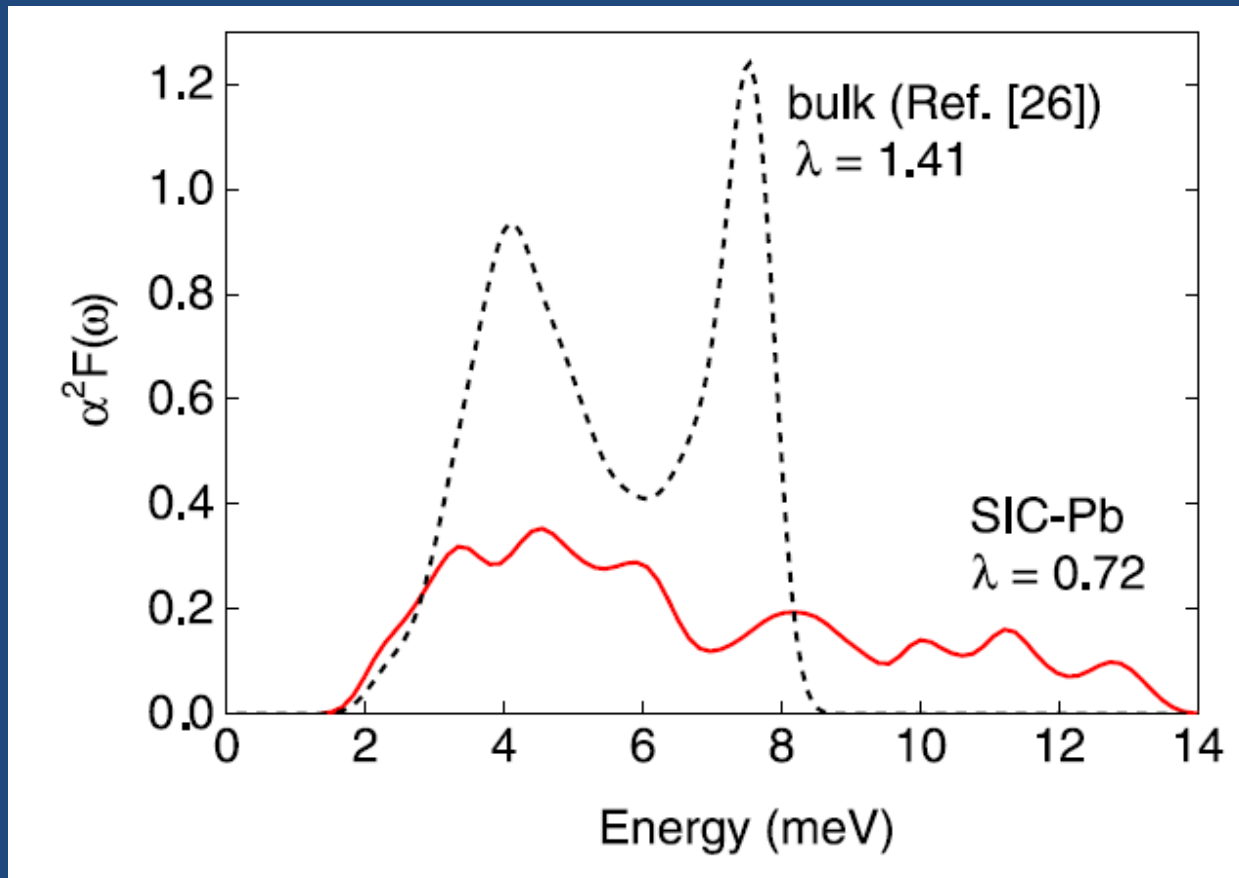
$$\ell_e \sim 5 \text{ nm}, \quad k_F \ell_e > 20 \text{ very far from SIT !}$$

Mean-level spacing of single-electron states in a coherence volume

$$\delta = E_F / (\xi/a)^2 \sim 0.1 \text{ meV} \sim \Delta/3$$



# Electron-phonon coupling from DFT



Assumptions: perfect  $\sqrt{3} \times \sqrt{3}$  Pb/Si(111) - No SOC - Fixed Si atoms

*Noffsinger and Cohen Solid State Commun. 151, 421 (2011)*

# Vortices in SIC

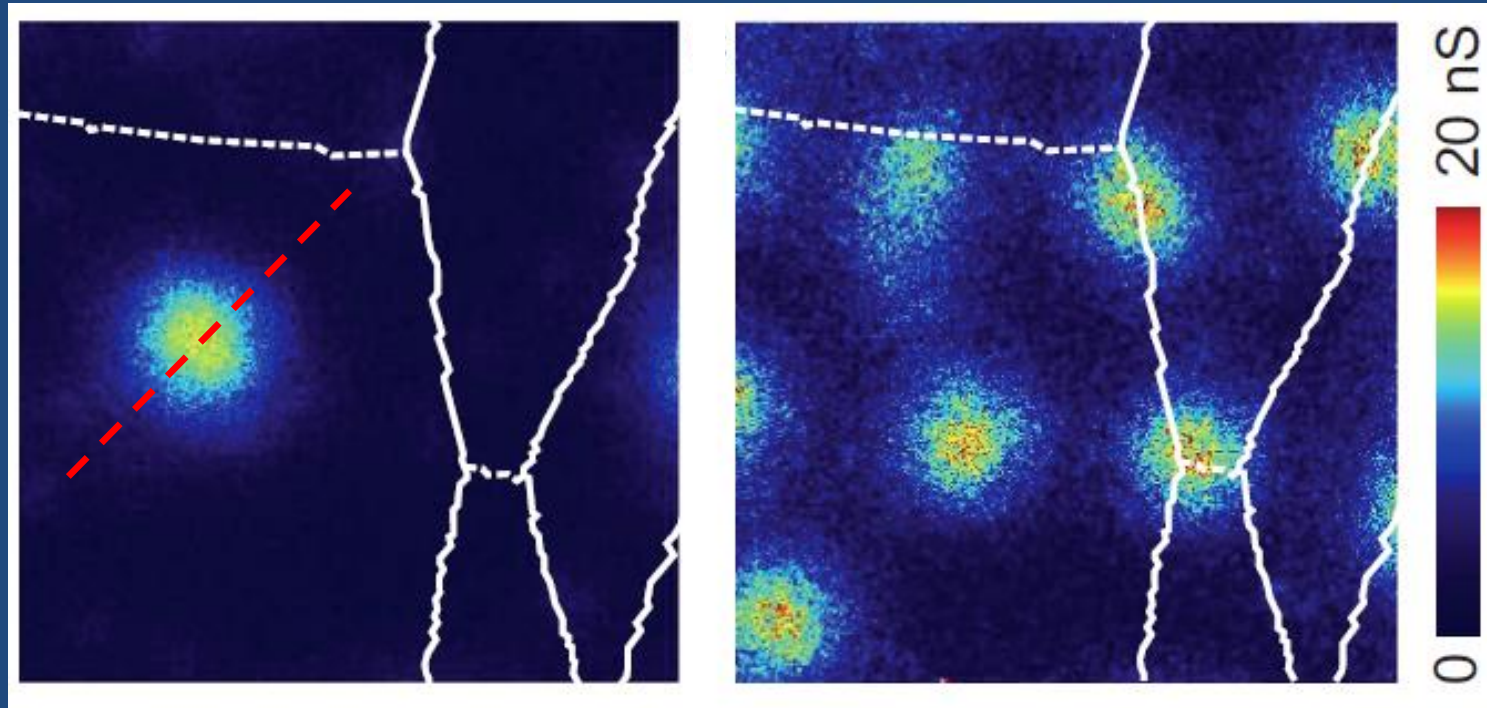
$T_c \sim 1.8\text{K}$

dI/dV (V=0) map

dI/dV (V=0) map

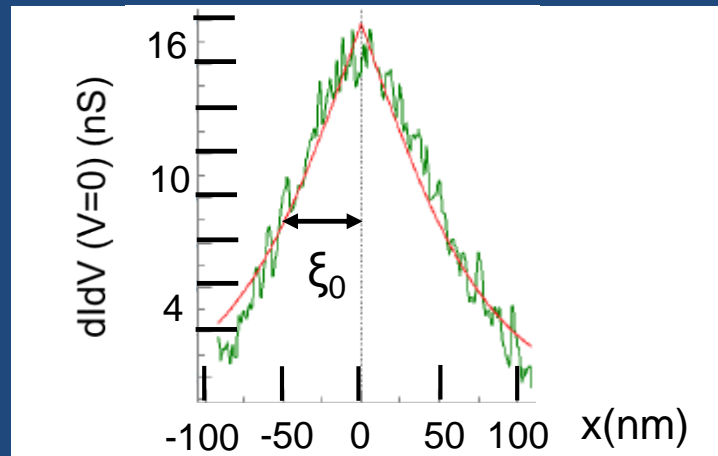
$T=0.3\text{K}$

$600 \times 600 \text{ nm}^2$



$B=10\text{mT}$

$B=40\text{mT}$



Justification: see 3D  
Usadel calculations  
by Kupriyanov & Golubov

*Nature Commun.* 9, 2277,  
2018

$$\xi_0 = (\hbar D / \Delta)^{1/2} \approx 47 \text{ nm}$$

*Nat. Phys.* 10, 444 (2014)

Very good agreement also with:

Cherkez et al PRX 2014