Local bistability under microwave heating for spatially mapping disordered superconductors

D. B. Karki — *Beijing Academy of Quantum Information Sciences* (now @ *Argonne National Laboratory*)

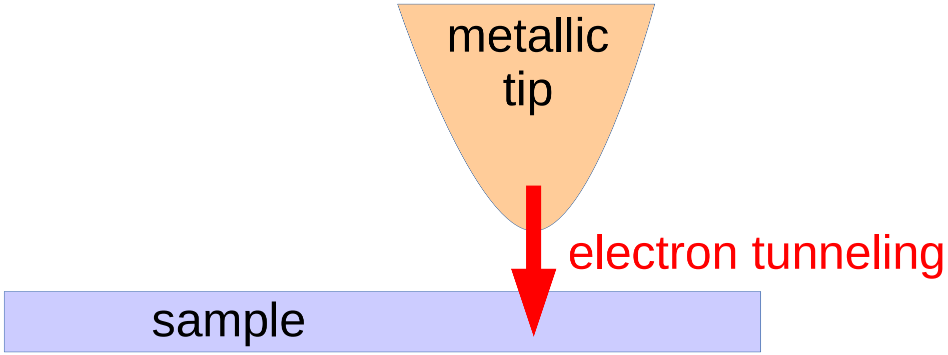
R. S. Whitney, **D. M. Basko** — *LPMMC, Université Grenoble Alpes & CNRS*

Phys. Rev. B **106**, 155419 (2022)

Outline

- Motivation: local probes
- Model: 2D heat transport equation
- Local bistability
- Spatial resolution

Scanning tunneling microscopy (STM)

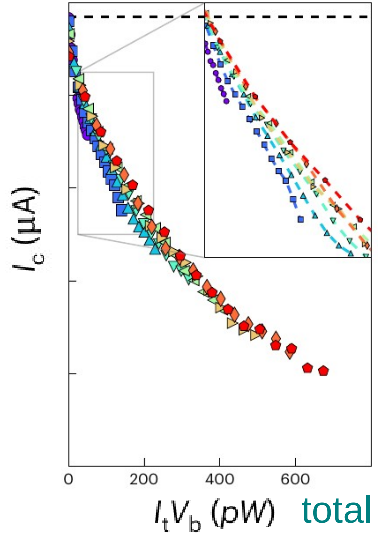


G. Binnig *et al*., Appl. Phys. Lett. **40**, 178 (1982)
J. Tersoff and D. R. Hamann, PRB **31**, 805 (1985)

- measure tunneling current
- probe electronic states on the surface
- high spatial resolution (atomic scale)
- non-invasive probe

Scanning critical current microscopy:

- sample = superconducting wire
- perturb the sample by the STM current
- measure the critical current
- spatial resolution ~ coherence length
- invasive probe



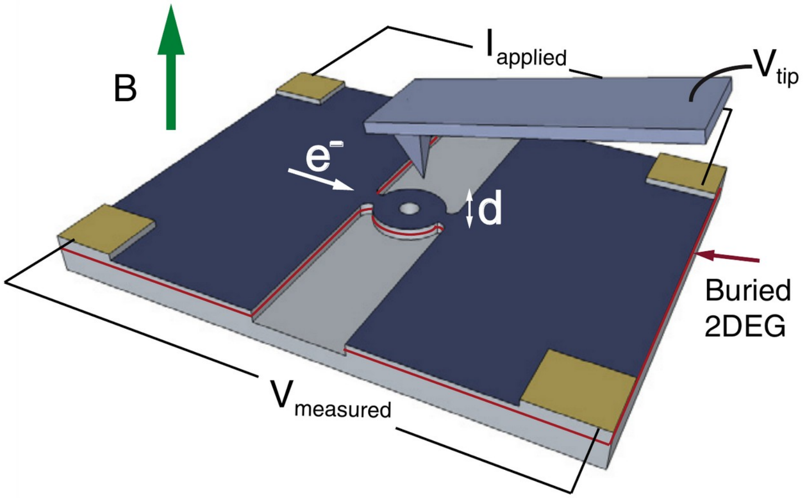
total injected power

T. Jalabert *et al*.,
Nature Phys. (2023)

local heating
by injected hot electrons

Scanning gate microscopy (SGM)

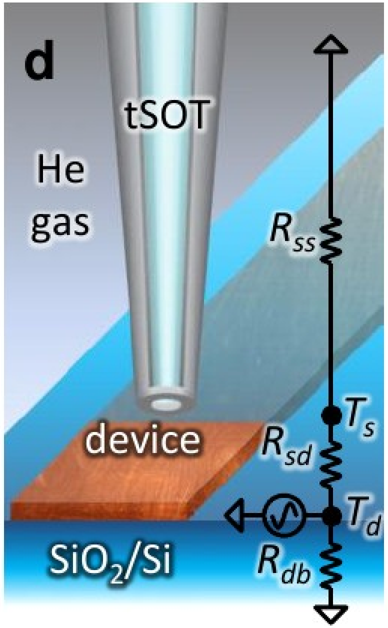
H. Sellier *et al.*,
Semicond. Sci. Technol. **26** (2011) 064008 review paper



- sample = buried 2DEG (inaccessible to STM)
- electrostatically deplete the 2DEG under the tip
- measure the sample global resistance
- spatial resolution > 10 nm (electrostatics)
- cannot gate a metal or a superconductor

Nanoscale thermal imaging

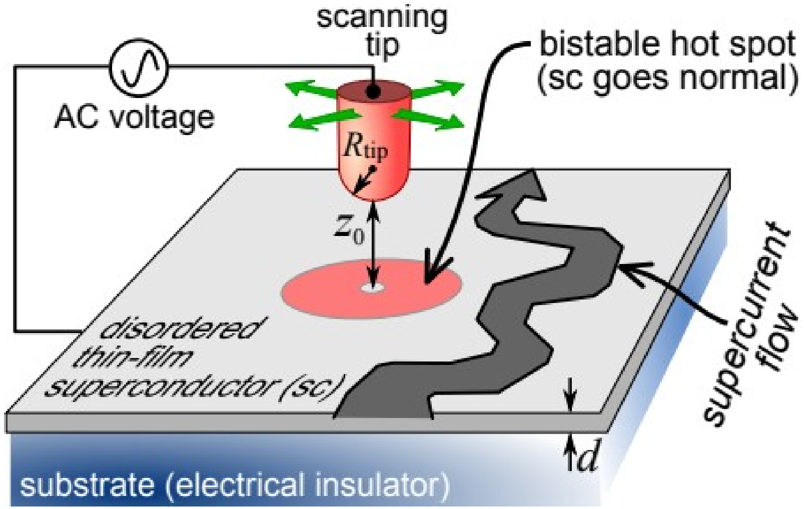
Halbertal *et al.*, Nature **539**, 407 (2016)



- SQUID-on-tip thermometer
- measures the temperature of the gas above the sample
- probes the heat flow from electrons to phonons
- spatial resolution ~ 50 nm

Is it possible to locally heat up a superconductor without contacting it?

Our theoretical proposal:



Might prove useful to map out
the supercurrent spatial pattern
in strongly disordered superconductors

(move the heating tip,
measure the global kinetic inductance)

What is the spatial scale
of the overheated region?

tip size ~ 50–100 nm
superconducting coherence length ~ 5 nm
(strongly disordered NbN, InO_x)

2D heat transport equation

Assume: electrons in **local equilibrium** at temperature $T_\mathrm{e}(\mathbf{r})$

equilibrium superconducting gap $\Delta(T_\mathrm{e}(\mathbf{r}))$

equilibrium quasiparticle distribution $f(\epsilon, T_e(\mathbf{r}))$

phonons at a fixed temperature T_ph

$$c(T_\mathrm{e})d\,\frac{\partial T_\mathrm{e}}{\partial t} = \boldsymbol{\nabla}\cdot[\kappa(T_\mathrm{e})d\,\boldsymbol{\nabla}T_\mathrm{e}] - Q(T_\mathrm{e},T_\mathrm{ph})d + |\mathbf{j}_\omega(\mathbf{r})|^2 d\,\mathrm{Re}\,\frac{1}{\sigma(\omega,T_\mathrm{e})}$$

2D heat transport equation

layer thickness

bulk specific heat

Will not matter in the stationary situation

$$c(T_e)d\,\frac{\partial T_e}{\partial t} = \boldsymbol{\nabla}\cdot\left[\kappa(T_e)d\,\boldsymbol{\nabla}T_e\right] - Q(T_e, T_{ph})d + |\mathbf{j}_\omega(\mathbf{r})|^2 d\,\mathrm{Re}\,\frac{1}{\sigma(\omega, T_e)}$$

2D heat transport equation

bulk thermal conductivity

layer thickness

$$c(T_\mathrm{e})d\,\frac{\partial T_\mathrm{e}}{\partial t} = \boldsymbol{\nabla}\cdot[\kappa(T_\mathrm{e})d\,\boldsymbol{\nabla}T_\mathrm{e}] - Q(T_\mathrm{e},T_\mathrm{ph})d + |\mathbf{j}_\omega(\mathbf{r})|^2 d\,\mathrm{Re}\,\frac{1}{\sigma(\omega,T_\mathrm{e})}$$

normal-state electrical conductivity

$$\kappa(T_\mathrm{e}) = \frac{\sigma_N}{e^2}\int_{\Delta(T_\mathrm{e})}^{\infty}\frac{\epsilon^2\,d\epsilon}{2T_\mathrm{e}^2\cosh^2[\epsilon/(2T_\mathrm{e})]}$$

Landau & Lifshitz, vol. 10

2D heat transport equation

electron-phonon cooling power
(per unit volume)

T. Tsuneto, *Phys. Rev.* **121**, 402 (1961)
A. Schmid, *Z. Physik* **259**, 421 (1973)
S. B. Kaplan *et al.*, PRB **14**, 4854 (1976)
. . .

phonon wavelegth << electron mean free path
($\Sigma T_{\mathrm{e}}^5 - \Sigma T_{\mathrm{ph}}^5$ in the normal state)

or

phonon wavelegth >> electron mean free path
($\Sigma T_{\mathrm{e}}^6 - \Sigma T_{\mathrm{ph}}^6$ in the normal state)

no big difference in practice (see below)

$$c(T_{\mathrm{e}})d\,\frac{\partial T_{\mathrm{e}}}{\partial t} = \boldsymbol{\nabla}\cdot\left[\kappa(T_{\mathrm{e}})d\,\boldsymbol{\nabla} T_{\mathrm{e}}\right] - Q(T_{\mathrm{e}},T_{\mathrm{ph}})d + |\mathbf{j}_\omega(\mathbf{r})|^2 d\,\mathrm{Re}\,\frac{1}{\sigma(\omega,T_{\mathrm{e}})}$$

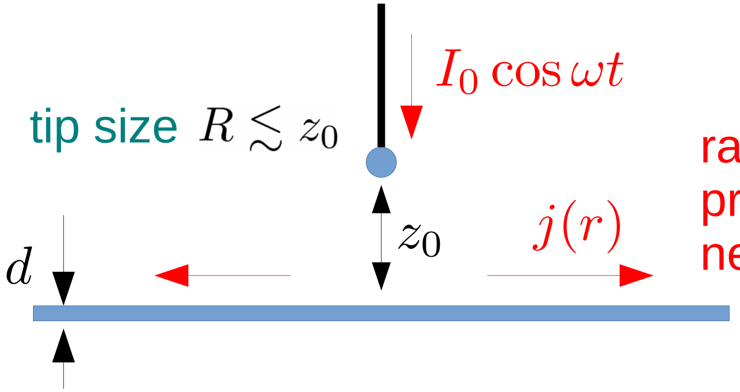
e-ph coupling Σ

$$Q(T_{\mathrm{e}},T_{\mathrm{ph}}) = \frac{\Sigma}{4(n-1)!\,\zeta(n)}\int_{-\infty}^{\infty} d\epsilon \int_0^{\infty} d\Omega\,\Omega^{n-2}\left(\coth\frac{\Omega}{2T_{\mathrm{e}}} - \coth\frac{\Omega}{2T_{\mathrm{ph}}}\right)\left(\tanh\frac{\epsilon+\Omega}{2T_{\mathrm{e}}} - \tanh\frac{\epsilon}{2T_{\mathrm{e}}}\right)\times$$

$$\times\,\frac{\Theta(|\epsilon|-\Delta)}{\sqrt{\epsilon^2-\Delta^2}}\,\frac{\Theta(|\epsilon+\Omega|-\Delta)}{\sqrt{(\epsilon+\Omega)^2-\Delta^2}}\left[\epsilon(\epsilon+\Omega)-\Delta^2\right]\left[\epsilon(\epsilon+\Omega)\right]$$

n = 5 or 6

2D heat transport equation



radial currents
producing the surface charge density
needed to screen a point charge at a distance z_0

$$\frac{I_0^2}{8\pi^2 d^2} \frac{r^2}{(z_0^2 + r^2)(\sqrt{z_0^2 + r^2} + z_0)^2}$$

$$c(T_\mathrm{e})d \frac{\partial T_\mathrm{e}}{\partial t} = \boldsymbol{\nabla} \cdot [\kappa(T_\mathrm{e})d \, \boldsymbol{\nabla} T_\mathrm{e}] - Q(T_\mathrm{e}, T_\mathrm{ph})d + |\mathbf{j}_\omega(\mathbf{r})|^2 d \, \mathrm{Re} \frac{1}{\sigma(\omega, T_\mathrm{e})}$$

material parameters: $T_c, \ \sigma_N d, \ \Sigma d$

ac conductivity
Mattis & Bardeen
Phys. Rev. **111**, 412 (1958)

includes
- the supercurrent
- the quasiparticle current

Local bistability

$$c(T_e)d\frac{\partial T_e}{\partial t} = \boldsymbol{\nabla} \cdot [\kappa(T_e)d\,\boldsymbol{\nabla} T_e] - Q(T_e, T_{ph})d + |\mathbf{j}_\omega(\mathbf{r})|^2 d\,\mathrm{Re}\,\frac{1}{\sigma(\omega, T_e)}$$

Local bistability

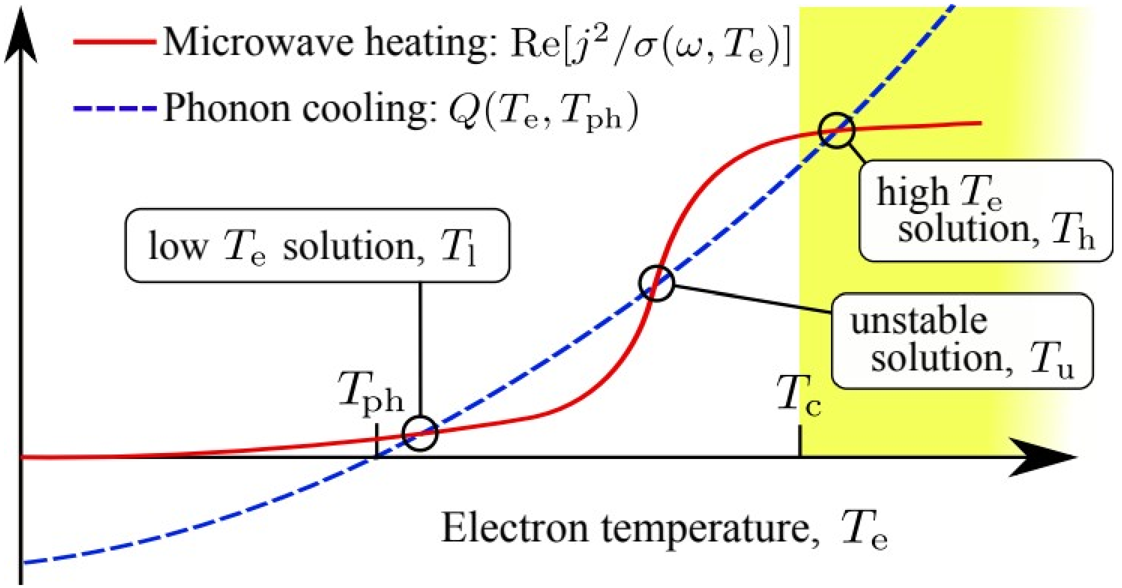
$$Q(T_{\rm e}, T_{\rm ph}) = j^2 \,{\rm Re}\, \frac{1}{\sigma(\omega, T_{\rm e})}$$

algebraic equation to find T_e

Multiple stable solutions for $T_{\rm e}$

Similar predictions in
Zharov, Korotkov, Reznik, *SuST* **5**, 104 (1992)
de Visser, Withington, Goldie,
J. App. Phys. **108**, 114504 (2010)
Thompson et al., *SuST* **26**, 095009 (2013)

Picture in the case
$\hbar\omega < 2\Delta(T = 0)$

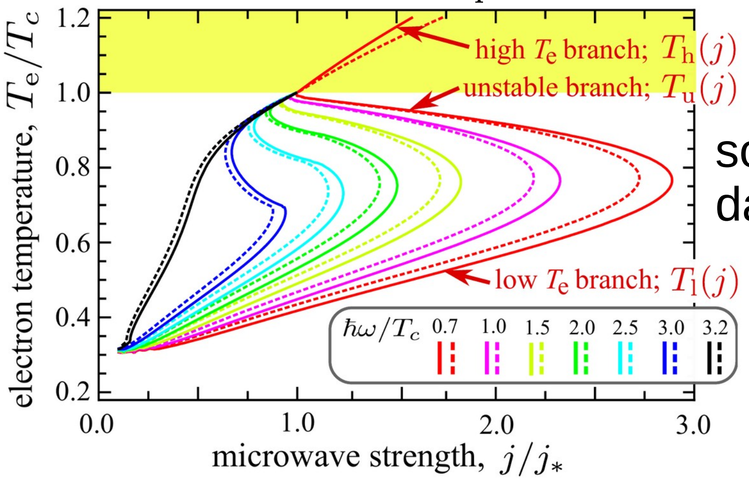


onset of absorption when $2\Delta(T_{\rm e}) \approx \hbar\omega$

Local bistability

$$Q(T_\mathrm{e}, T_\mathrm{ph}) = j^2\,\mathrm{Re}\,\frac{1}{\sigma(\omega, T_\mathrm{e})}$$

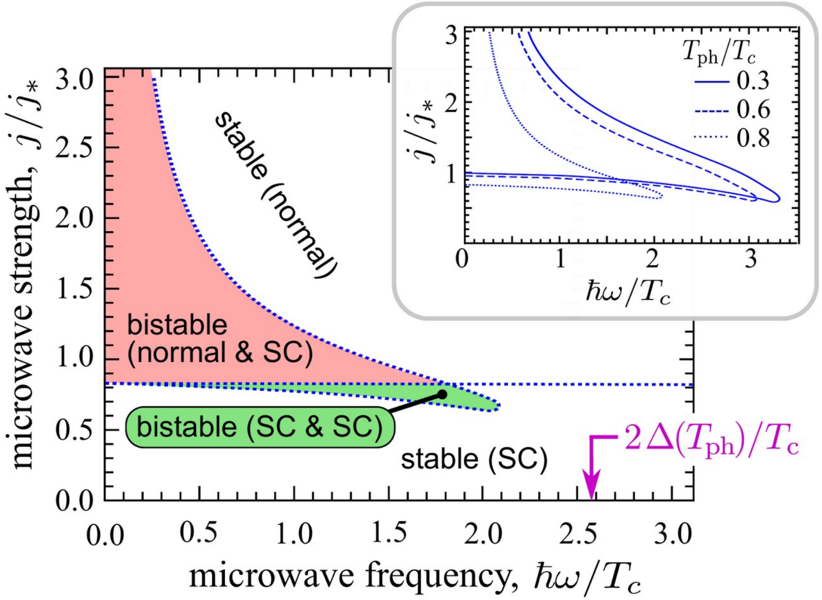
numerical solution at $T_\mathrm{ph} = 0.3\,T_c$



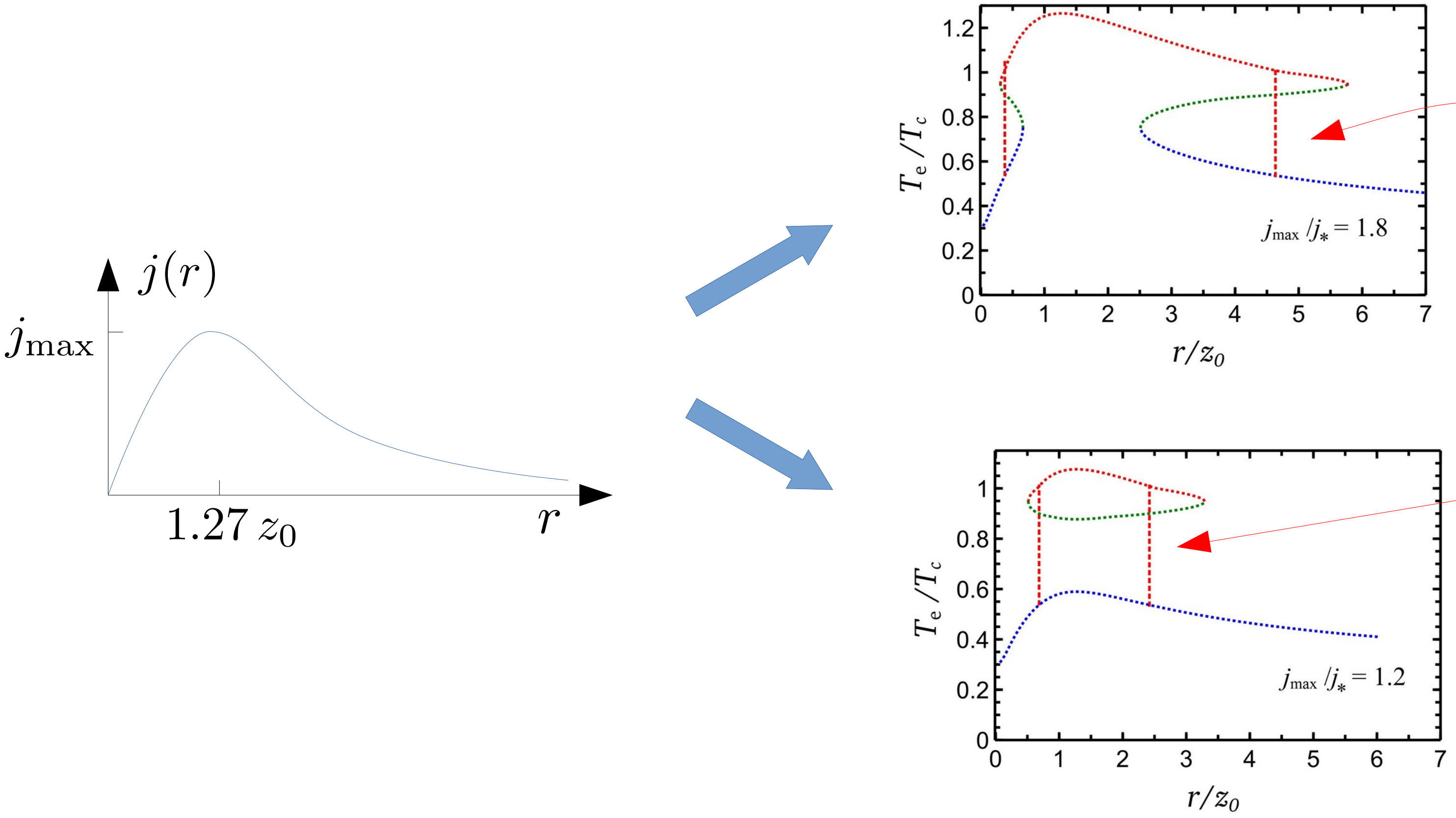
solid: ΣT^5
dashed: ΣT^6

$j_* \equiv \sqrt{\sigma_N \Sigma T_c^n}$ natural unit
current needed to maintain
$T_\mathrm{e} = T_c$ @ $T_\mathrm{ph} = 0$

Phase diagram:



Temperature profile



“domain wall”
(equal-area law)

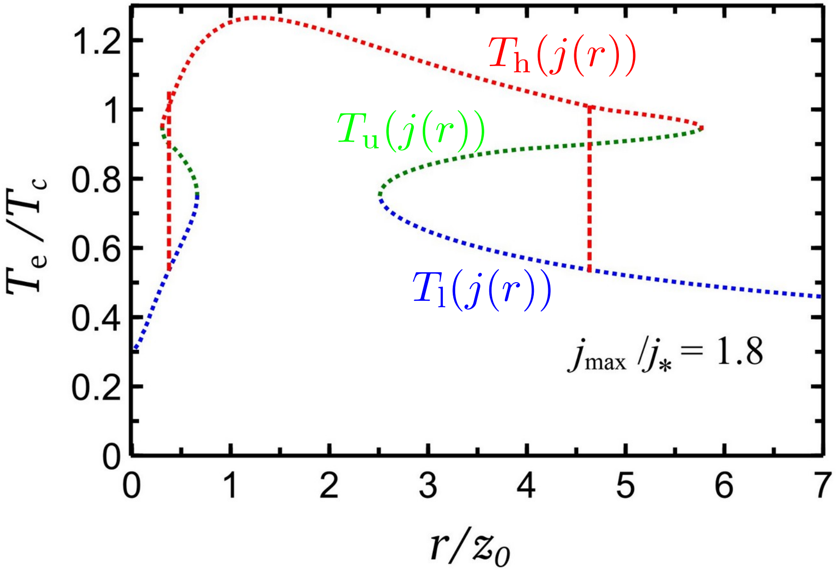
Equal-area law

$$c(T_\mathrm{e})d\,\frac{\partial T_\mathrm{e}}{\partial t} = \boldsymbol{\nabla}\cdot[\kappa(T_\mathrm{e})d\,\boldsymbol{\nabla}T_\mathrm{e}] - Q(T_\mathrm{e},T_\mathrm{ph})d + |\mathbf{j}_\omega(\mathbf{r})|^2 d\,\mathrm{Re}\,\frac{1}{\sigma(\omega,T_\mathrm{e})}$$

$$= \frac{1}{\kappa(T_\mathrm{e})}\,\frac{\delta\mathcal{F}[T_\mathrm{e}(\mathbf{r})]}{\delta T_\mathrm{e}(\mathbf{r})}$$

stationary temperature profile ⟺ minimum of a functional

Gurevich & Mints, Rev. Mod. Phys. **59**, 941 (1987)



neglect the gradients:

$$\int_{T_\mathrm{u}(j(R))}^{T_\mathrm{h}(j(R))}\left[\mathrm{Re}\,\frac{j^2(R)}{\sigma(\omega,T)} - Q(T,T_\mathrm{ph})\right]\kappa(T)\,dT =$$

$$= \int_{T_\mathrm{l}(j(R))}^{T_\mathrm{u}(j(R))}\left[Q(T,T_\mathrm{ph}) - \mathrm{Re}\,\frac{j^2(R)}{\sigma(\omega,T)}\right]\kappa(T)\,dT$$

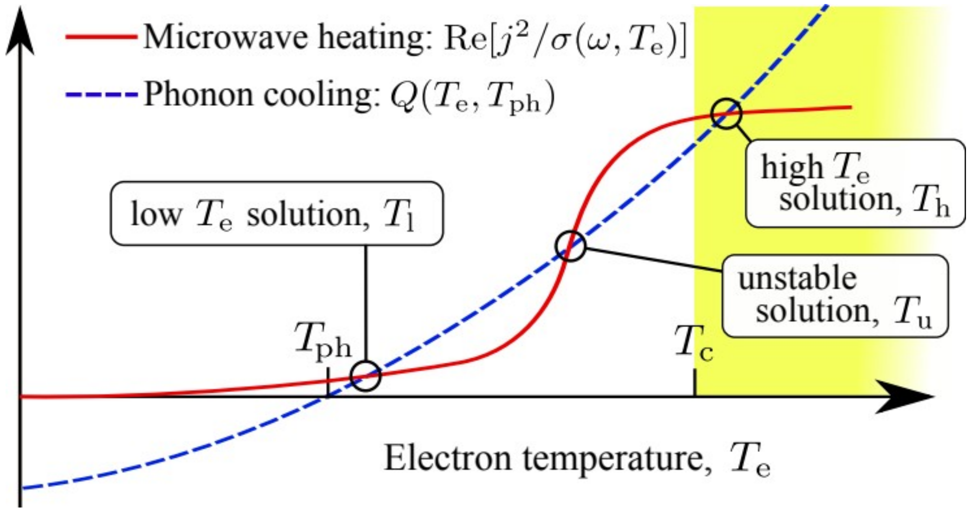
Equal-area law

$$c(T_\mathrm{e})d\,\frac{\partial T_\mathrm{e}}{\partial t} = \boldsymbol{\nabla}\cdot[\kappa(T_\mathrm{e})d\,\boldsymbol{\nabla}T_\mathrm{e}] - Q(T_\mathrm{e},T_\mathrm{ph})d + |\mathbf{j}_\omega(\mathbf{r})|^2 d\,\mathrm{Re}\,\frac{1}{\sigma(\omega,T_\mathrm{e})}$$

$$= \frac{1}{\kappa(T_\mathrm{e})}\,\frac{\delta\mathcal{F}[T_\mathrm{e}(\mathbf{r})]}{\delta T_\mathrm{e}(\mathbf{r})}$$

stationary temperature profile ⟺ minimum of a functional

Gurevich & Mints, Rev. Mod. Phys. **59**, 941 (1987)



neglect the gradients:

$$\int_{T_\mathrm{u}(j(R))}^{T_\mathrm{h}(j(R))}\left[\mathrm{Re}\,\frac{j^2(R)}{\sigma(\omega,T)} - Q(T,T_\mathrm{ph})\right]\kappa(T)\,dT =$$

$$= \int_{T_\mathrm{l}(j(R))}^{T_\mathrm{u}(j(R))}\left[Q(T,T_\mathrm{ph}) - \mathrm{Re}\,\frac{j^2(R)}{\sigma(\omega,T)}\right]\kappa(T)\,dT$$

Role of thermal conductivity

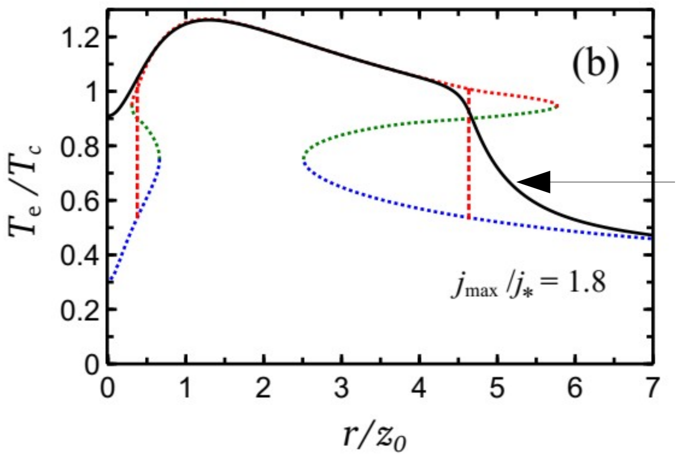
smearing of temperature jumps

$$\nabla \cdot [\kappa(T_e)\,\nabla T_e] - Q(T_e, T_{ph}) + |\mathbf{j}_\omega(\mathbf{r})|^2 \,\mathrm{Re}\,\frac{1}{\sigma(\omega, T_e)} = 0$$

Thermal relaxation length $\Lambda = [\kappa(T_c)/(n\Sigma T_c^{n-1})]$

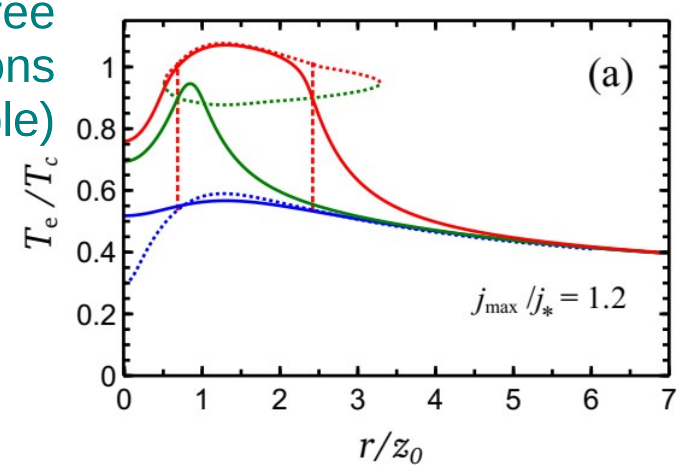
15–20 nm
for strongly disordered NbN, InO_x

Numerical solution for $\Lambda/z_0 = 0.16$



one solution for $T_e(r)$

three solutions (one unstable)



Typical numbers

$\underline{\mathrm{InO}_x}$

$T_c = 3.5\ \mathrm{K}$

$1/\sigma_N = 5 \times 10^{-5}\,\Omega \cdot \mathrm{m}$

$Q = \Sigma(T_{\mathrm{e}}^6 - T_{\mathrm{ph}}^6)$

$\Sigma = 2 \times 10^9\,\mathrm{W} \cdot \mathrm{K}^{-6} \cdot \mathrm{m}^{-3}$

$\Lambda = 17\,\mathrm{nm}$

$\underline{\mathrm{NbN}}$

$T_c = 10\ \mathrm{K}$

$1/\sigma_N = 4 \times 10^{-6}\,\Omega \cdot \mathrm{m}$

$Q = \Sigma(T_{\mathrm{e}}^5 - T_{\mathrm{ph}}^5)$

$\Sigma = 5 \times 10^9\,\mathrm{W} \cdot \mathrm{K}^{-5} \cdot \mathrm{m}^{-3}$

$\Lambda = 16\,\mathrm{nm}$

Power needed to maintain $j_{\mathrm{max}} = j_*$ for $d = 10\,\mathrm{nm}, \quad z_0 = 100\,\mathrm{nm}$

a few nanoWatts

a few microWatts

Summary

1. AC voltage on a microwave tip ➡ hot spot in a superconducting layer
   spot size ~ tip-plane distance ~ tip size ~ 100 nm
   a normal ring inside the spot

2. Short thermal relaxation length + local heating bistability
   ➡ sharp boundary of the hot spot (~ thermal relaxation length)

3. Possible global bistability (multiple stable solutions for $T_{\mathrm{e}}(r)$)
   ➡ possible hysteresis in the drive power

4. Might serve as an invasive local probe to map out supercurrent pattern

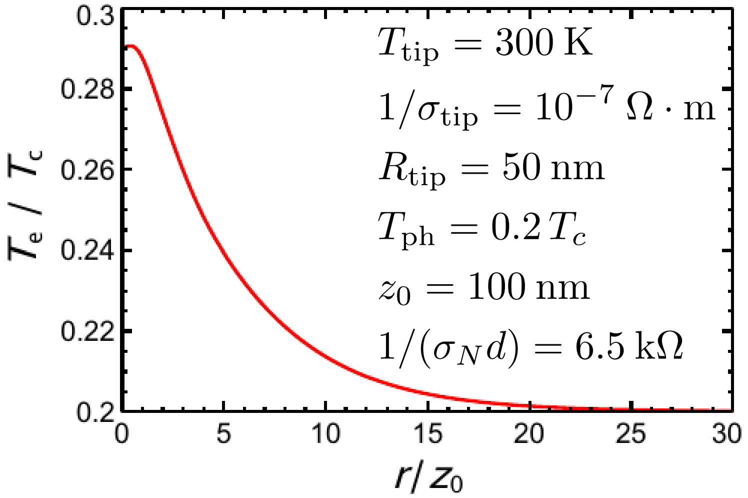
Karki, Whitney, Basko, *PRB* **106**, 155419 (2022)

Heating by a thermal tip?

External microwave drive → Thermal fluctuating currents (Nyquist-Johnson)
Electrostatic + magnetostatic coupling

Fixed drive frequency → Thermal frequency spectrum

Transferred power $\propto T_{\mathrm{tip}}^4 - T_{\mathrm{e}}^4(r)$



One parameter (T_{tip}) governs both

- the effective drive strength
- the effective drive frequency

No bistability, weak overheating, large spot